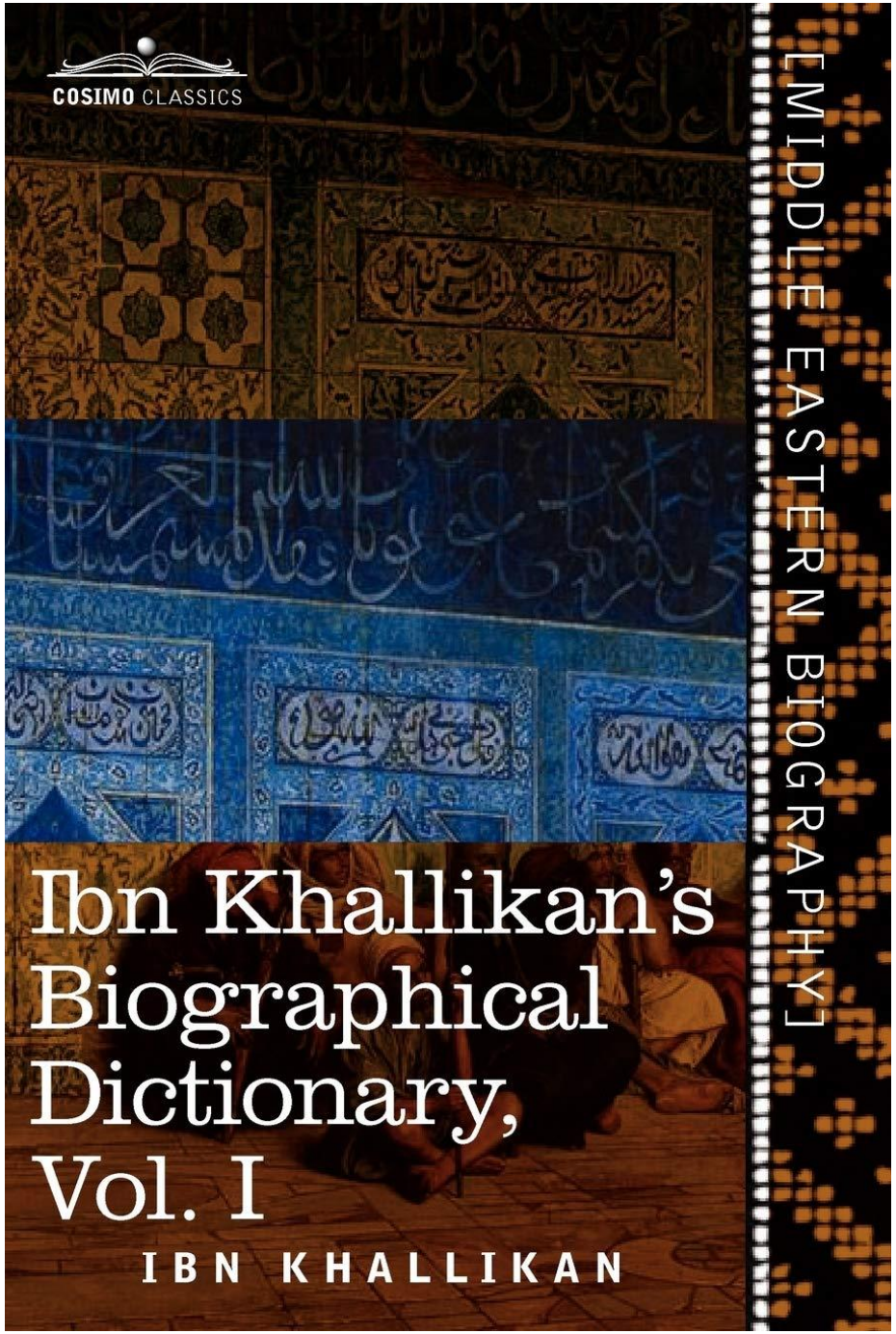
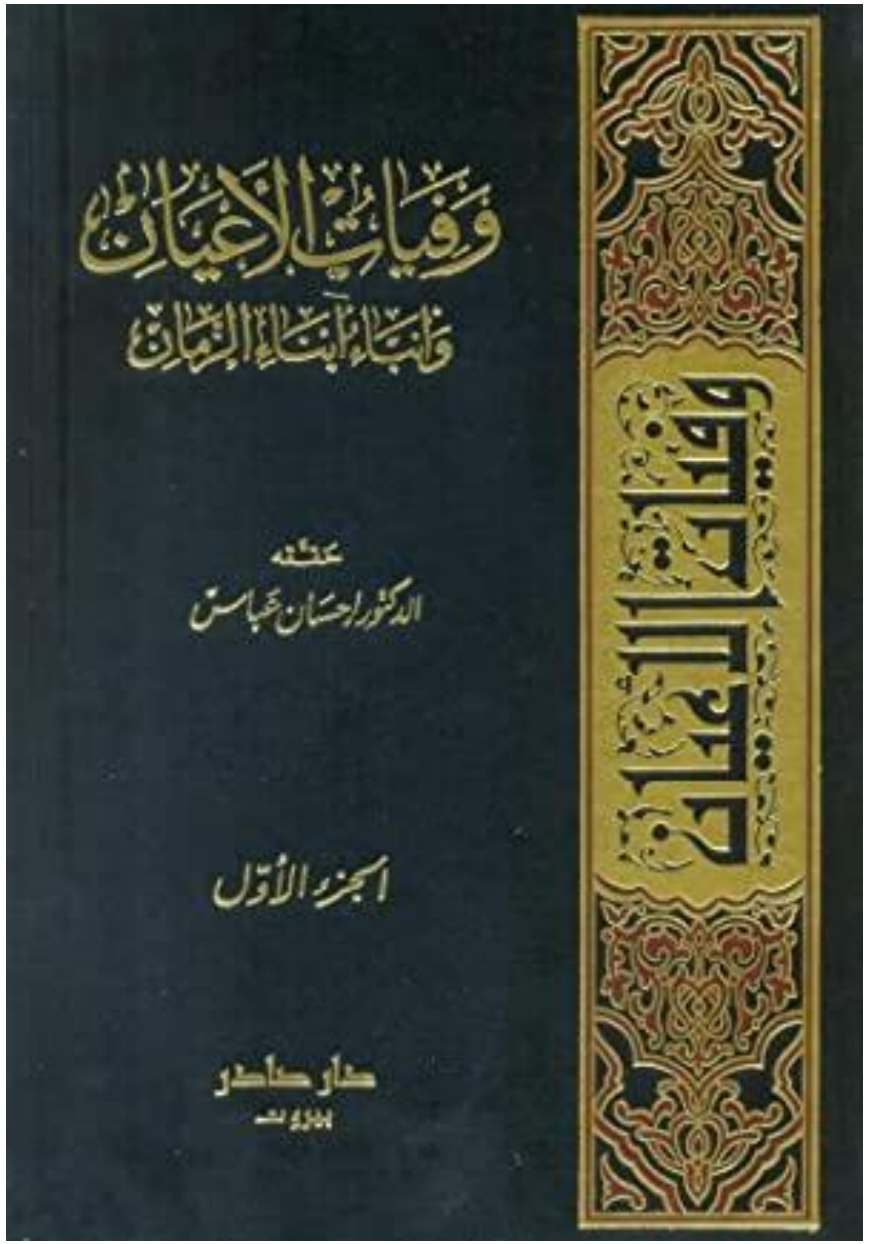
Wheat on a Chessboard

First Recorded by Ibn Khallikan, 1256



Aḥmad bin Muḥammad bin Ibrāhīm Abu ’l-ʿAbbās Shams al-Dīn al-Barmakī al-Irbilī al-Shāfiʿī, ibn Khallikān (1211 – 1282) was a 13th century Islamic scholar who compiled a biographical encyclopedia of Muslim scholars and important men in Muslim history.

He was born in Erbil, studied in Arbil, Aleppo and Damascus, and took up jurisprudence in Mosul and also Cairo.



This story was first recorded in 1256 in Ibn Khallikan’s biographical encyclopedia.

The Indian King Shirham asked Grand Vizier Sissa ben Dhair what reward he wanted for inventing the game of chess.

^ Ancient Chessboard,
Indus Valley, 1500 BC

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Sissa responds: “Majesty, I would be happy if you were to give me a grain of wheat to place on the first square of the chessboard, and two grains of wheat to place on the second square, four grains on the third, eight grains of wheat to place on the fourth, and so on for the sixty-four squares.”

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“And is that all you wish, Sissa, you fool?” the astonished King asked.

Turns out that the fool is not Sissa.

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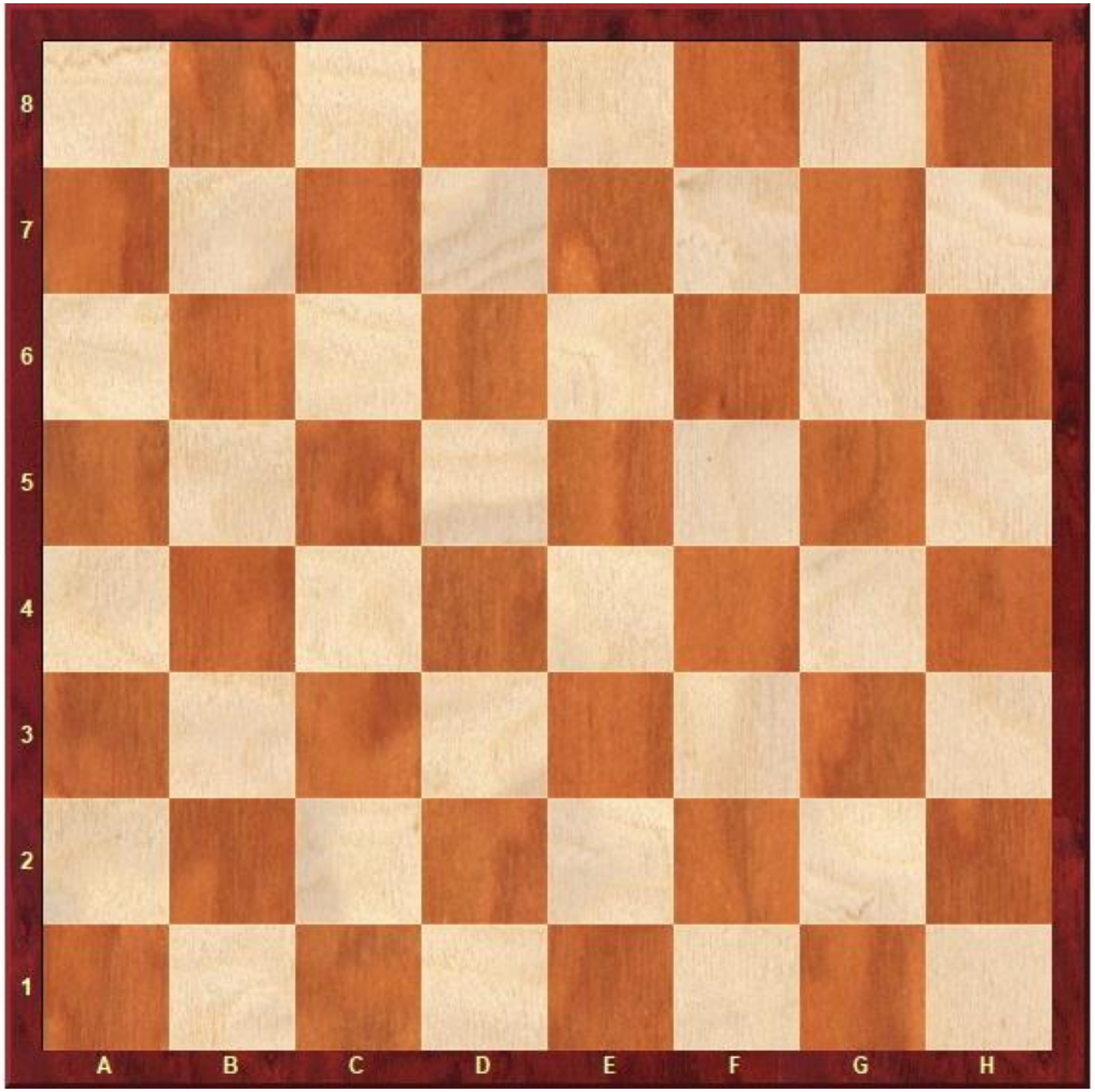
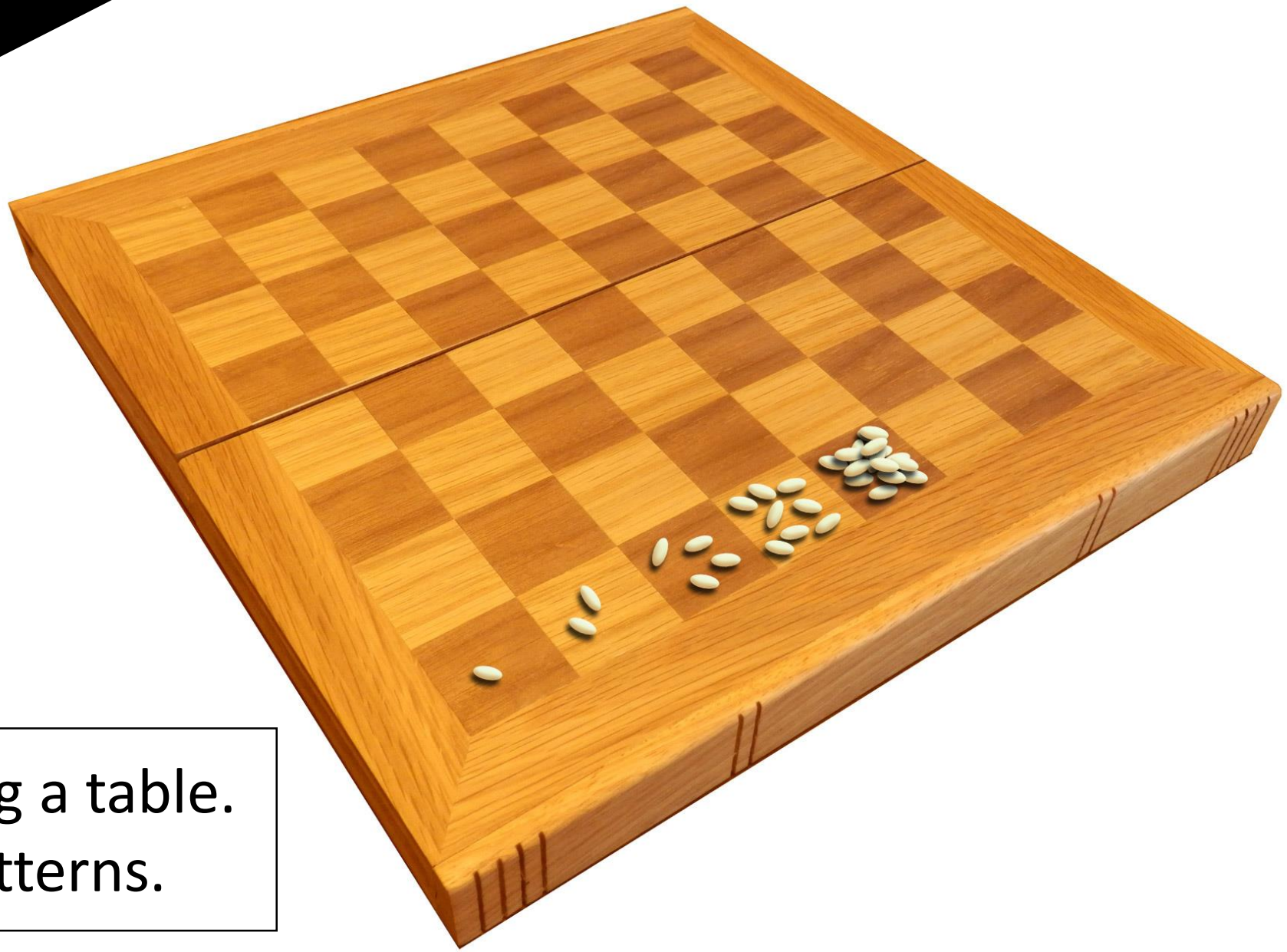


Before we get started, how many squares are on a chessboard?

Our problem: Calculate the total grains of wheat.

Problem solving technique: Consider a smaller subset of the problem – the first row.

Organize using a table. Analyse patterns.



How many grains are on each square in the first row?

Square	1	2	3	4	5	6	7	8
Wheat in that square?								
Power of 2?								
Total Wheat to that point?								

Can you identify a pattern in the wheat on the first row?

$2^?$

Square	1	2	3	4	5	6	7	8
Wheat in that square?	1	2	4	8	16	32	64	128
Power of 2?								
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Square	1	2	3	4	5	6	7	8
Wheat in that square?	1	2	4	8	16	32	64	128
Power of 2?								
Total Wheat to that point?								

Use the pattern to fill in this column

Calculate the total grains of wheat to that point on the chessboard.

Square	1	2	3	4	5	6	7	8
Wheat in that square?	1	2	4	8	16	32	64	128
Power of 2?	2^0	2^1	2^2	2^3	2^4	2^5	2^6	2^7
Total Wheat to that point?								

Looking at our chart there is a brute force way to solve this problem.

Square	1	2	3	4	5	6	7	8
Wheat in that square?	1	2	4	8	16	32	64	128
Power of 2?	2^0	2^1	2^2	2^3	2^4	2^5	2^6	2^7
Total Wheat to that point?	1	1+2 3	1+2+4 7	1+2+4+8 15	1+2+4+8+16 31	Previous+32 63	Previous+64 127	Previous+128 255

The Brute-force method.

Calculate the wheat in each square. Add them up.

Total Grains of Sissa's Wheat

$$= 2^0 + 2^1 + 2^2 + 2^3 + \ldots + 2^{62} + 2^{63}$$

Why is this 63 and not 64?

The Brute-force method.

Calculate the wheat in each square. Add them up.

Total Grains of Sissa's Wheat

$$= 2^0 + 2^1 + 2^2 + 2^3 + \ldots + 2^{62} + 2^{63}$$

$$= 1 + 2 + 4 + 8 + \ldots$$

$$+ 4{,}611{,}686{,}018{,}427{,}387{,}904$$

$$+ 9{,}223{,}372{,}036{,}854{,}775{,}808$$

Identify the power of 2

Can you see an easier way?

Square	1	2	3	4	5	6	7	8
Wheat in that square?	1	2	4	8	16	32	64	128
Power of 2?	2^0	2^1	2^2	2^3	2^4	2^5	2^6	2^7
Total Wheat to that point?	1	1+2 3	1+2+4 7	1+2+4+8 15	1+2+4+8+16 31	Previous+32 63	Previous+64 127	Previous+128 255

What is the formula for the total rice on the 5th square?

Square	1	2	3	4	5	6	7	8
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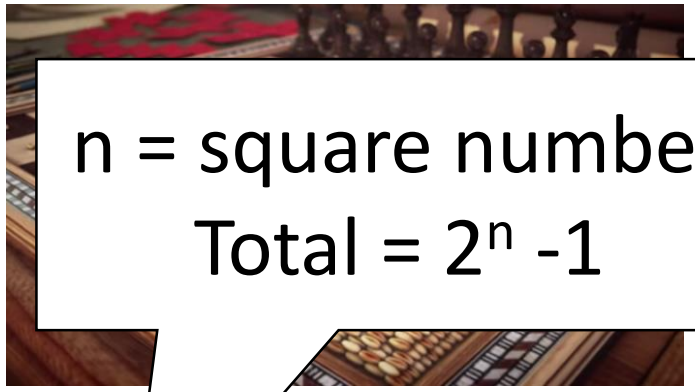

n = square number

Total = $2^n - 1$

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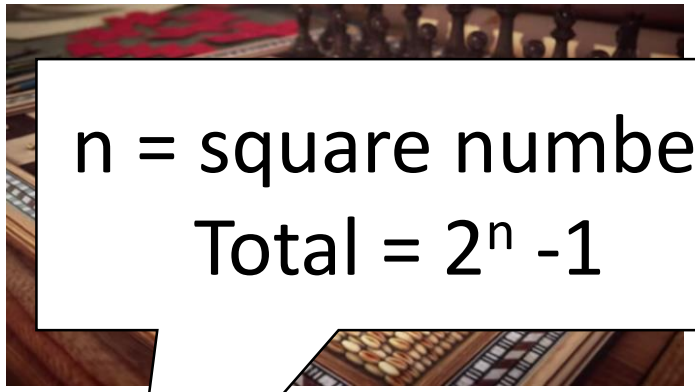

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Total

$= 2^n - 1$

$= 2^5 - 1$

$= 32 - 1$

$= 31$

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Wheat in that square?	1	2		8	16	32	64	128
Power of 2?	2^0	2^1			2^4	2^5	2^6	2^7
Total Wheat to that point?	1	1+2 3	1+2+4 7	1+2+4+8 15	1+2+4+8+16 31	Previous+32 63	Previous+64 127	Previous+128 255

The Clever Method

Total = $2^n - 1$,
n = square number

Total Grains of Sissa's Wheat
$= 2^{64} - 1$

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$= 18,446,744,073,709,551,616 - 1$

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= 18,446,744,073,709,551,615 grains

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This is a nice picture to explain it.

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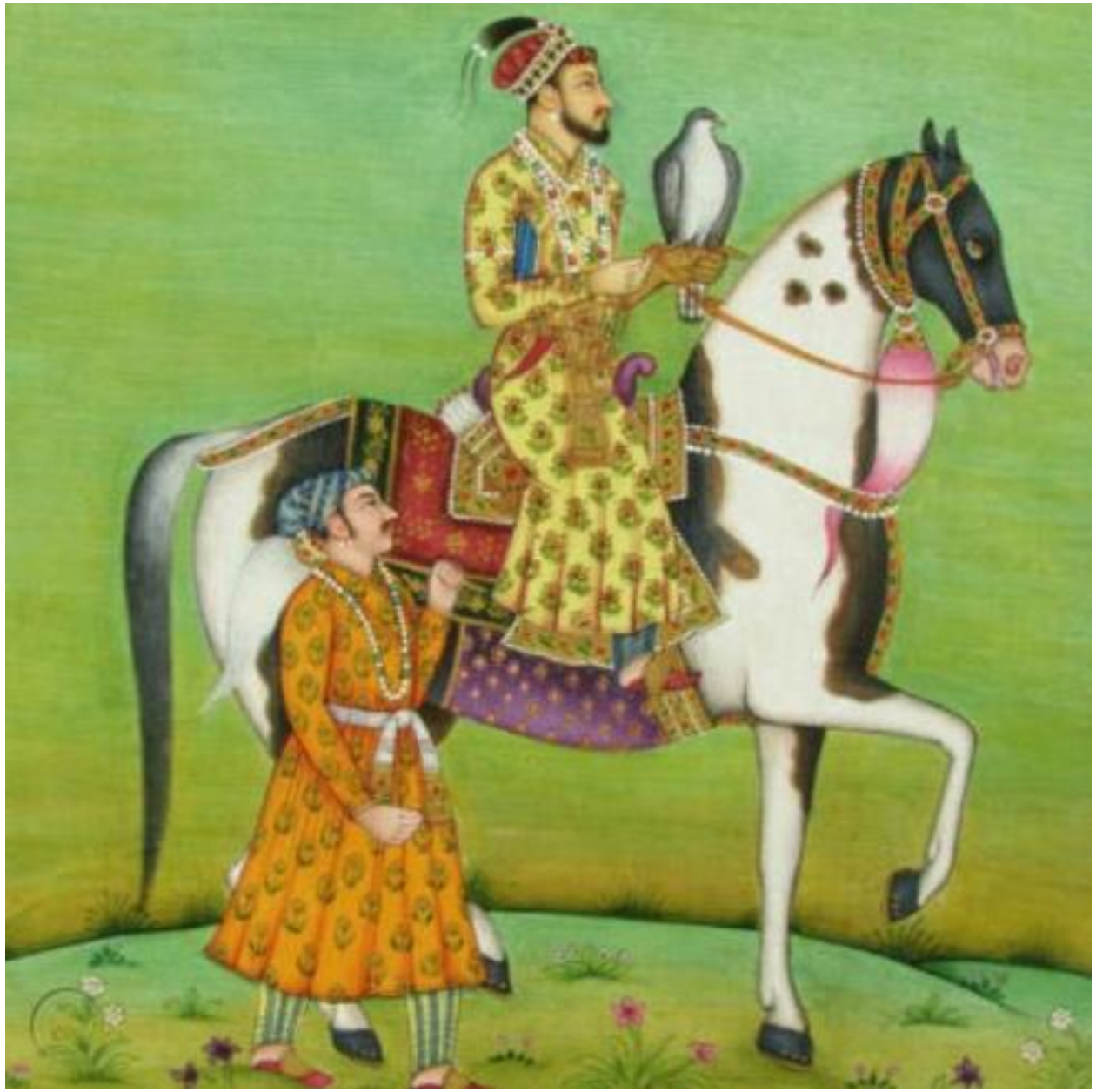
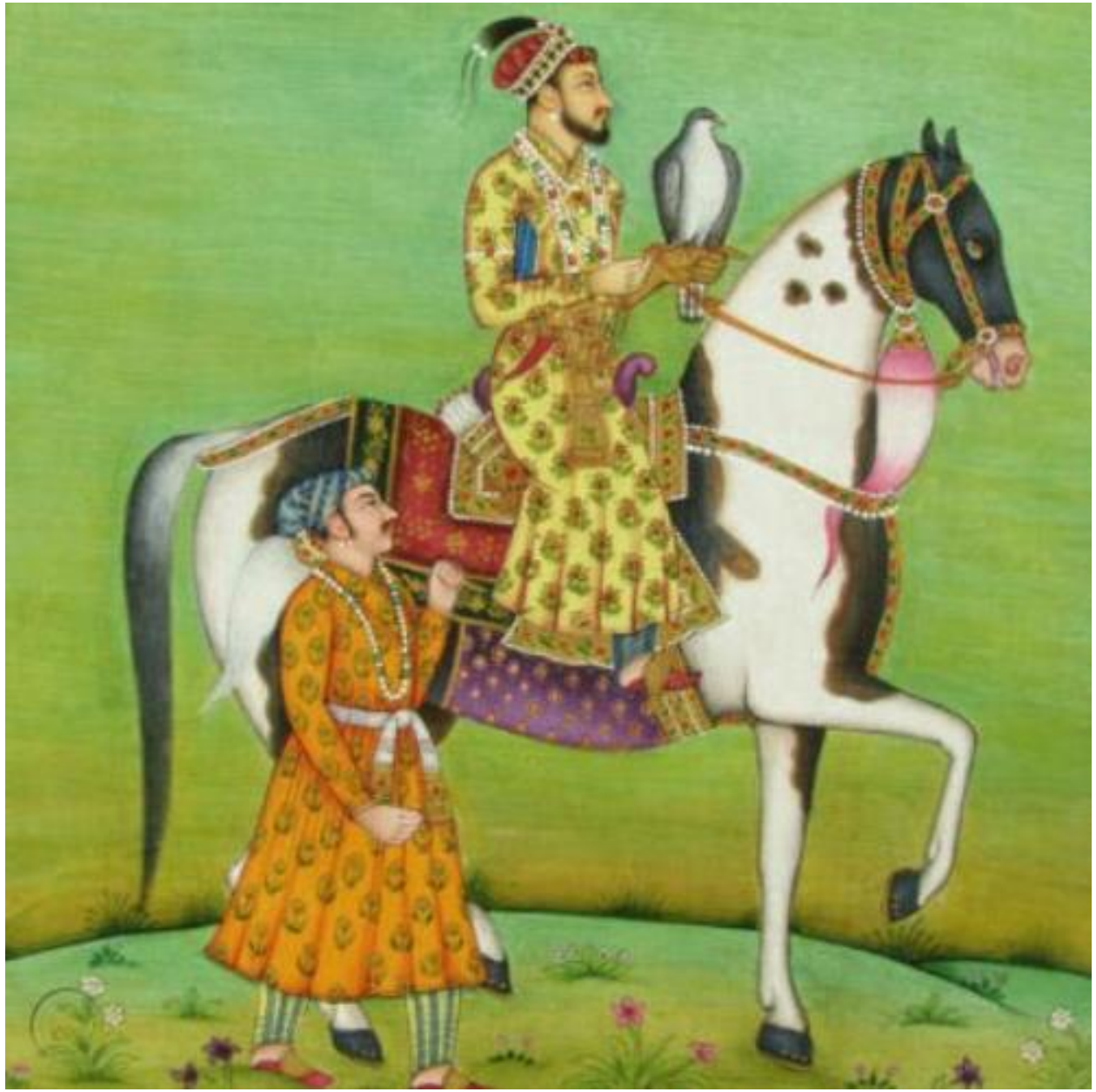
With about 100 grams to a cubic centimeter, the total volume of Sissa’s wheat would be nearly 200 cubic kilometers, to be loaded on two thousand million railway wagons, which would make up a train reaching a thousand times around the Earth.



So why are we talking about this?



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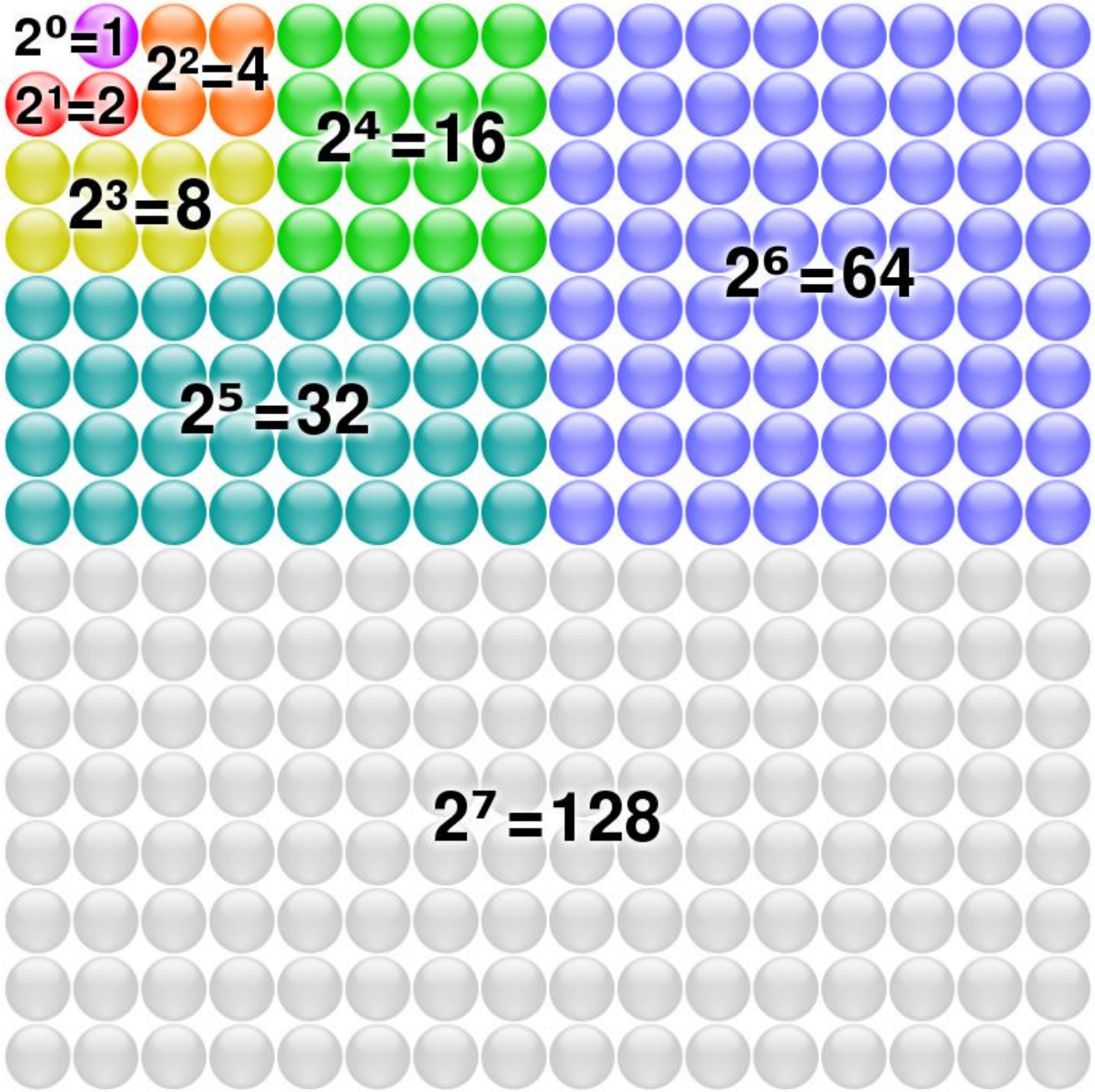
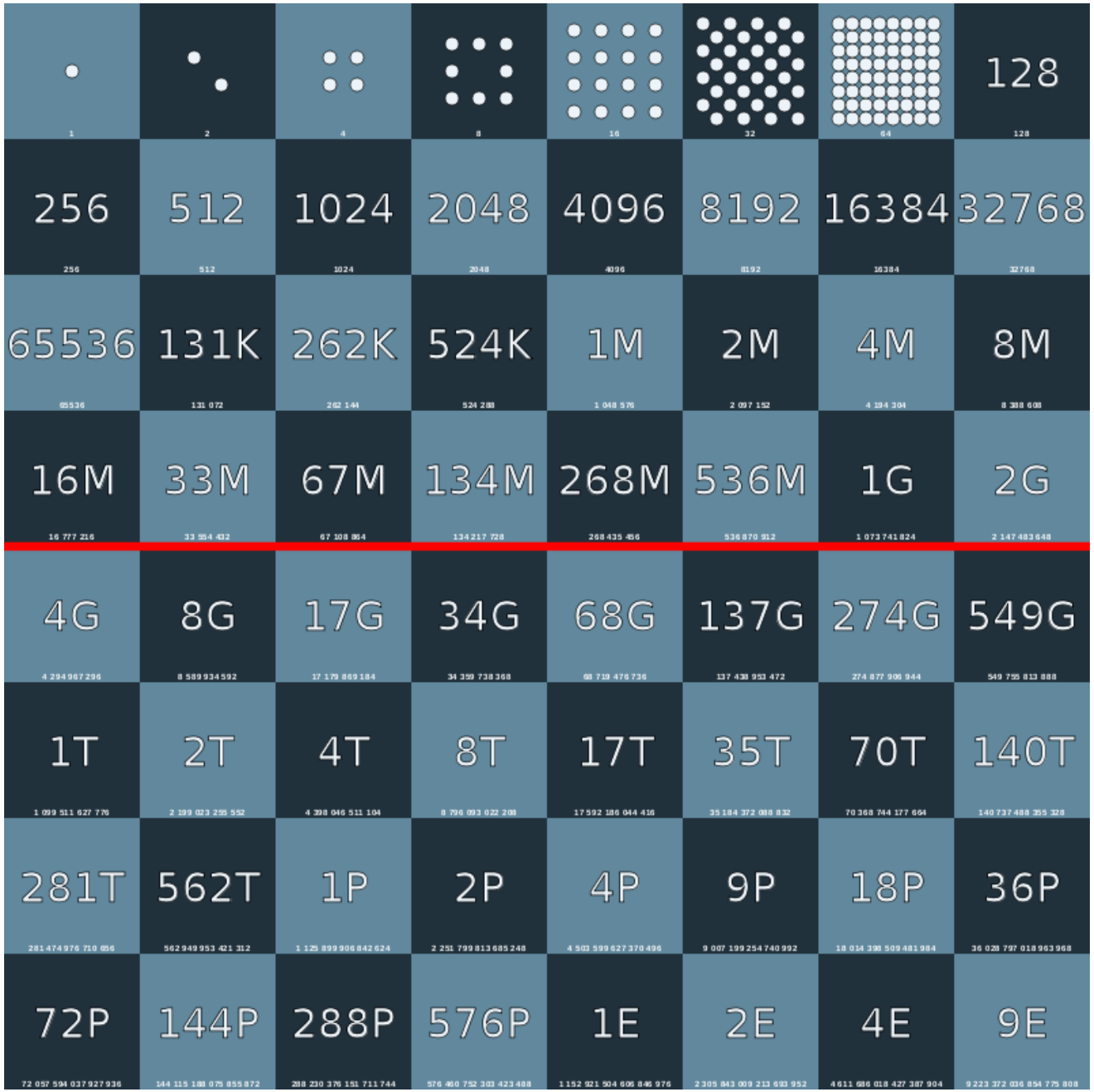
Turns out, like the Indian King Shirham, humans are terrible at conceiving how fast exponents grow.

With exponential growth, humans underestimate how quickly things will grow.





This diagram is called "The second half of the chessboard"

Prefix	Symbol	Multiplier	
exa	E	10^{18}	1,000,000,000,000,000,000
peta	P	10^{15}	1,000,000,000,000,000
tera	T	10^{12}	1,000,000,000,000
giga	G	10^{9}	1,000,000,000
mega	M	10^{6}	1,000,000
kilo	k	10^{3}	1,000

It uses metric symbols (above) to show how fast the board's wheat grains accumulate.

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There is evidence that they are making the same error as the Indian King Shirham.

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Understanding exponential growth helps us to identify important trends to watch.