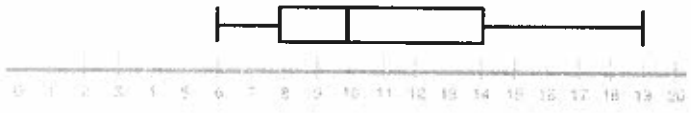
MDM4U – Sample Test 4 – One Variable Analysis – November 13, 2023

Name: Solutions

Knowledge	Application	Communication	Thinking	Total	Percent
26	28	21	18	93	%

Knowledge

1. Identify the following items on this box and whisker graph. /6



Min:	6	Max:	19
Q1:	8	Q3:	14
Median:	10	IQR:	6

2. Use the z-score table to fill in the probabilities of each z-score in the last column. /5

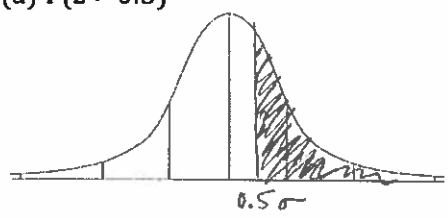
(a)	P(z < 1.0)	0.8413
(b)	P(z < -0.75)	0.2266
(c)	P(z < 0.56)	0.7123
(d)	P(z > 0.56)	0.2877
(e)	P(z > -0.75 and z < 0.56)	0.4857

1 - 0.7123

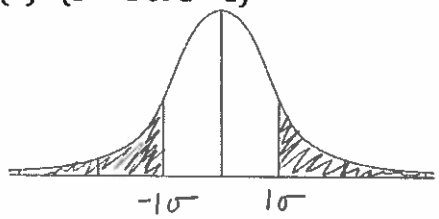
Big - Little
= 0.7123 - 0.2266

3. Shade in the area on the normal distribution indicated by the probability. /5

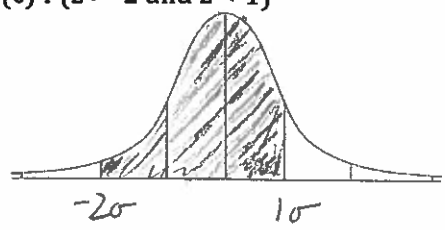
(a) P(z > 0.5)



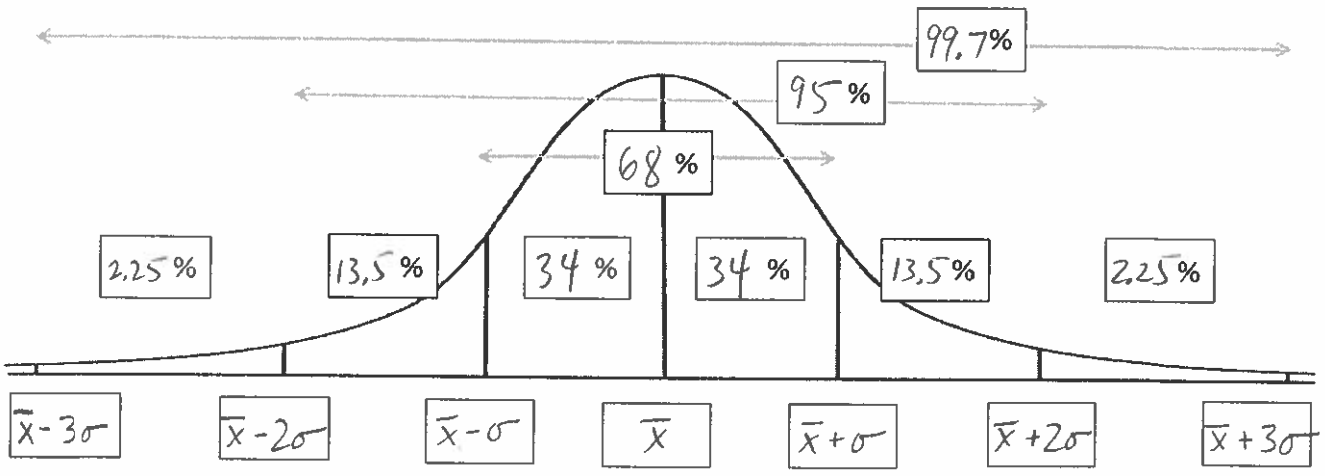
(b) P(z < -1 or z > 1)



(c) P(z > -2 and z < 1)



4. Fill in the boxes to fully label the normal distribution. /10



Application

5. What are the formulas found in the indicated cells of this spreadsheet? /11

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	The Data:					The Analysis:							
2	2	3	4	7		Mean	4.08	Minimum	1	Q3	5.75	X	5
3	3	5	4	3		Standard Deviation	1.98	Q1	3	Maximum	8	Zscore	0.46
4	1	3	6	8		Mode	3	Median	3.5	IQR	2.75	P(x<5)	0.6789

G2	=average(A2:D4)
G3	=stdev.p(A2:D4)
G4	=mode(A2:D4)
I2	=min(A2:D4)
I3	=quartile.exc(A2:D4,1)
I4	=median(A2:D4)

K2	=quartile.exc(A2:D4,3)
K3	=max(A2:D4)
K4	=K2-I3
M3	=(M2-G2)/G3
M4	=norm.dist(M2,G2,G3,true)

6. Calculate the standard deviation of the following values. /5

Mean

$$\bar{x} = \frac{\sum x}{n}$$

$= \frac{90}{6}$

$= 15$

Standard Deviation Pieces

x	$\bar{x} - x$	$(\bar{x} - x)^2$
11	4	16
13	2	4
14	1	1
16	-1	1
17	-2	4
19	-4	16
Σ= 90		Σ= 42

Standard Deviation

$$\sigma = \sqrt{\frac{\sum(\bar{x} - x)^2}{n}}$$

$= \sqrt{\frac{42}{6}}$

$= \sqrt{7}$

$= 2.6458$

7. Calculate the standard deviation of the following frequency values. /7

Mean

$$\bar{x} = \frac{\sum x \times f}{\sum f}$$

$= \frac{210}{21}$

$= 10$

Standard Deviation Pieces

x	freq	$x \times f$	$\bar{x} - x$	$(\bar{x} - x)^2$	$f(\bar{x} - x)^2$
8	3	24	2	4	12
9	8	72	1	1	8
11	6	66	-1	1	6
12	4	48	-2	4	16
	Σ= 21	Σ=210			Σ= 42

Standard Deviation

$$\sigma = \sqrt{\frac{\sum f(\bar{x} - x)^2}{\sum f}}$$

$= \sqrt{\frac{42}{21}}$

$= \sqrt{2}$

$= 1.414214$

8. If the mean height of a newborn kitten is 12 cm with a standard deviation of 1.5 cm, then what percentage of kittens are born smaller than 10 cm? /5

Given

$x = 10$

$\bar{x} = 12$

$\sigma = 1.5$

$$z = \frac{x - \bar{x}}{\sigma}$$

$= \frac{10 - 12}{1.5}$

$= -1.33$

From z-score table:

$P(x < 10) = 0.0918$

∴ 9.18% of kittens are smaller than 10cm.

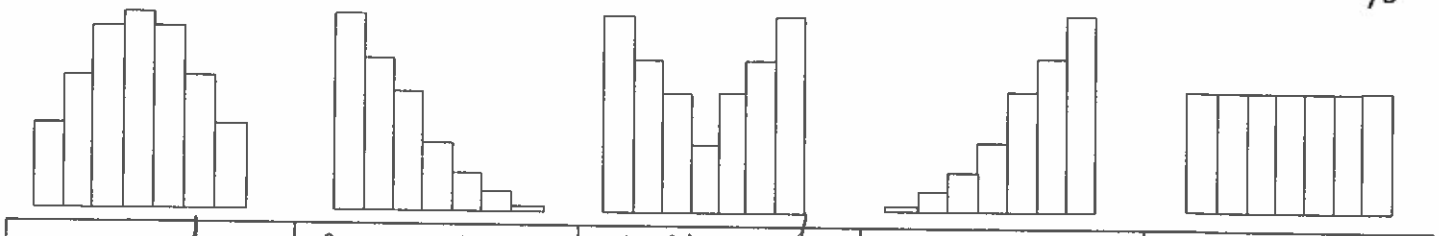
Communication

9. Write the terms indicated in the last columns. /2

the range isn't anyway near as good as σ.

(a) Measures of spread.	σ	IQR
(b) Measure of central tendency.	mean	median	mode

10. Classify each histogram's shape. /5



Mound	Right Skew	U-Shaped Bi-Modal	Left Skew	Uniform

11. Fill in the final column with the term or number indicated. /10

(a) The standard deviation for: $X \sim N(34, 25)$ $X \sim N(\bar{x}, \sigma^2)$	5
(b) The distribution shape where mode > median > mean.	Left Skew
(c) A distribution shape where mode = median = mean.	Mound [Uniform has many modes]
(d) A distribution shape with two modes.	UShaped or BiModal
(e) The measure of spread that goes with a median.	IQR
(f) The number of standard deviations something is from the mean.	z-score
(g) A measure of how tightly grouped data is around the mean.	σ
(h) A measure of central tendency not effected by outliers.	median [mean is effected]
(i) The term for the most frequently occurring value.	Mode
(j) The top percentile on the SAT test (or any test, for that matter)	99th

12. Define and explain the importance of the term "Normal Distribution". /4

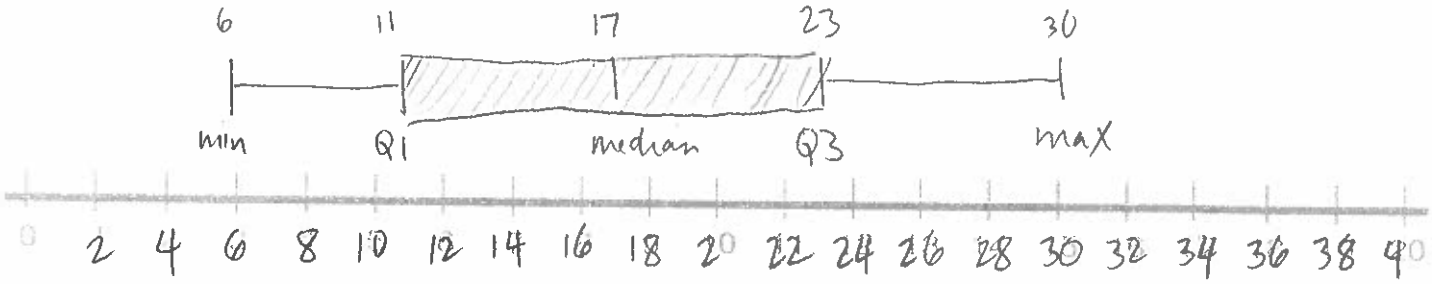
The normal distribution is a shape commonly found in one variable data. It has many characteristics including: ① symmetry about the mean ② bell shape, ③ standard % of data falling in set distances from the mean (eg. 95% in 2σ). Its importance is found in it's almost universal presence in measured (continuous) biologically-base variables eg. height of trees, weight of babies, IQ test results. Because it's probabilities (%) are so well studied, we can use them to make accurate predictions about a normally distributed population using the zscore table.

Thinking

13. Draw a box and whisker graph for this data. /5

Position:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Value:	6	8	9	11	12	14	15	17	19	20	21	23	24	26	30

Q1 = 11, median = 17, Q3 = 23



14. The weights of babies born at Prince Louis Hospital last year averaged 3.0 kg with a standard deviation of 0.2 kg. If there were 545 babies born at the hospital last year, how many weighed less than 3.3 kg?
Write down any formulas you use. Be careful to use titles. /6

Given values

$\bar{x} = 3.0$

$\sigma = 0.2$

$x = 3.3$

$n = 545$

Zscore

$z = \frac{x - \bar{x}}{\sigma}$

$= \frac{3.3 - 3}{0.2}$

$= 1.5$

From table:

$P(x < 3.3)$

$= 0.9332$

Number of babies

$= 545 \times 0.9332$

$= 508.5921$

$= 509$

∴ 509 of the babies should weigh less than 3.3 kg.

15. The weights of Florida's oranges are normally distributed. 84% of the crop weighs more than 152 grams and 16% weigh more than 200 g. What is the mean and standard deviation of the crop?
Write down any formulas you use. Be careful to use titles. /7

Equation 1

$P(x > 152) = 0.84$

so, $P(x < 152) = 0.16$

from table, $z = -1.0$

$z = \frac{x - \bar{x}}{\sigma}$

$-1.0 = \frac{152 - \bar{x}}{\sigma}$

$\sigma = \frac{152 - \bar{x}}{-1.0}$

$\sigma = -152 + \bar{x}$

Equation 2

$P(x > 200) = 0.16$

so, $P(x < 200) = 0.84$

from table, $z = 1.0$

$z = \frac{x - \bar{x}}{\sigma}$

$1.0 = \frac{200 - \bar{x}}{\sigma}$

$\sigma = \frac{200 - \bar{x}}{1.0}$

$\sigma = 200 - \bar{x}$

sub 1 into 2, solve for $\bar{x}$

$-152 + \bar{x} = 200 - \bar{x}$

$2\bar{x} = 352$

$\bar{x} = 176$

Sub $\bar{x} = 176$ in Eqn 2

$\sigma = 200 - (176)$

$= 24$

∴ The mean is 176 g and the σ is 24 g