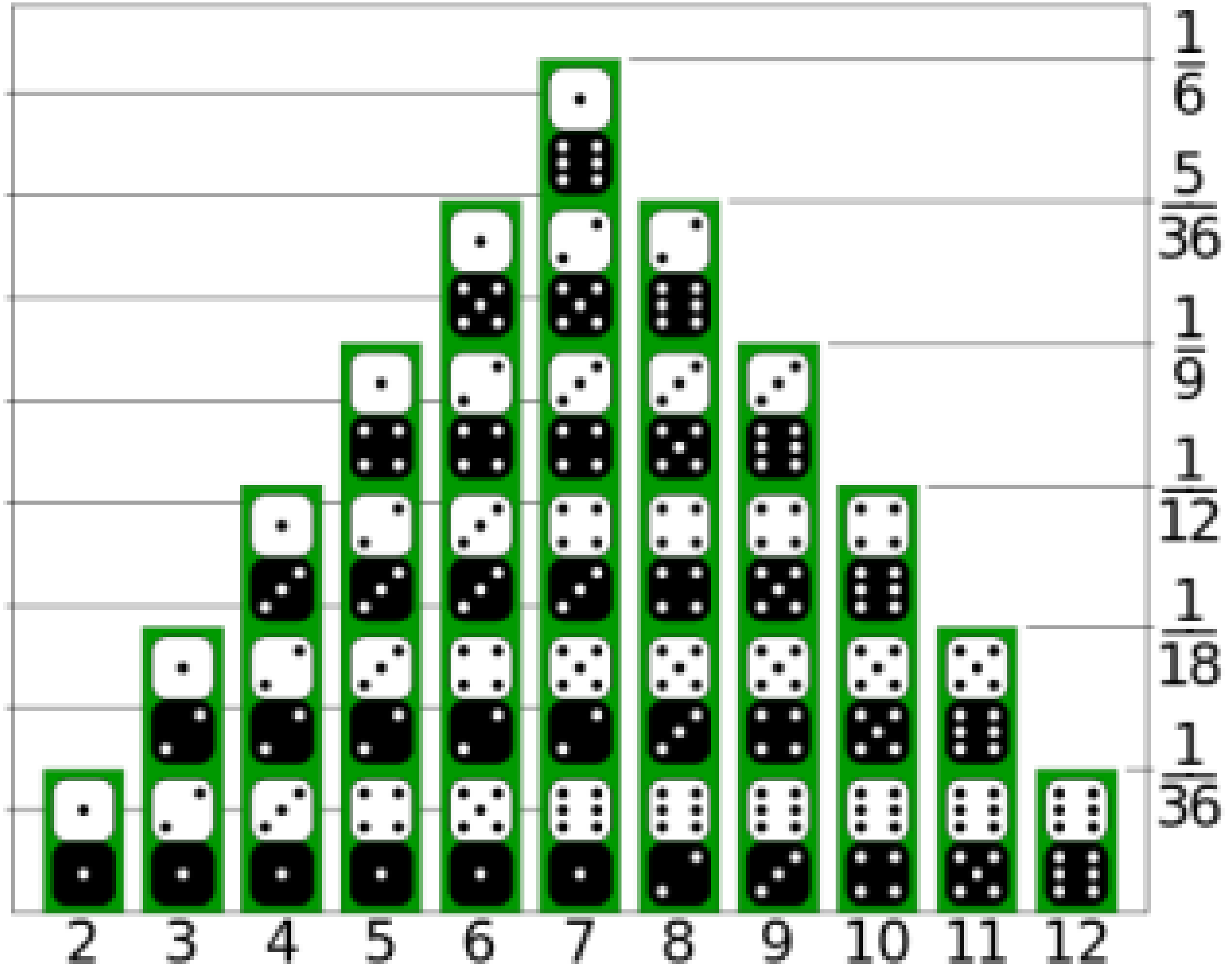
Hypergeometric Probability Distributions

If you loved combinations and probability, you are going to love these.
Sadly, the converse is also true.



1/6
5/36
1/9
1/12
1/18
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12

$$E(x) = \frac{ra}{n}$$

N = the total number of items to choose from
a = the total number of the “successful” item
r = the number of places to put them

Hypergeometric Distributions

Name: . . . . 5.8 K

1. Write out the formula for the expected value of a hypergeometric distribution 9 times.

$$P(x) = \frac{C(a, x) \times C(n - a, r - x)}{C(n, r)}$$

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2. Write out the formula for the probability of a hypergeometric event 9 times.

3. A mini-van has 6 seats. There are 18 people at a family picnic, 8 adults and 10 children. Six people are selected at random to go for ice cream. Calculate the probability that there are exactly 2 adults in the van.

n = ________; a = ________; r = ________; x = ________

$$P(x = \_\_\_\_) = \frac{C(\_\_\_\_, \_\_\_\_) \times C(\_\_\_\_, \_\_\_\_)}{C(\_\_\_\_, \_\_\_\_)}$$

$$P(x) = \frac{C(a,x) \times C(n-a, r-x)}{C(n,r)}$$

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4. A track team has 25 members. 10 are sprinters and 15 are long distance runners. If 5 people are randomly selected to be in a picture, what is the probability that exactly 3 of them are sprinters?

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7. You have a bag of 20 marbles, 4 are red and 16 are blue. You draw out 2 marbles without replacement. Make the probability distribution for the number of red marbles in the 2 marble selection.

X~ Hypergeometric (n=20, a=4, r=2). Thus, E(x) = ra/n = ____________

x	0 red marbles	1 red marble	2 red marbles
C(a, x)	C(__, __)=	C(__, __)=	C(__, __)=
C(n-a, r-x)	C(__, __)=	C(__, __)=	C(__, __)=
C(n,r)			
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P(x)			

x	0	1	2
C(a,x)	C(4,0) = 1	C(4,1) = 4	C(4,2) = 6
C(n-a, r-x)	C(16,2) = 120	C(16,1) = 16	C(16,0) = 1
Numerator	120	64	6
C(n,r)	C(20,2) = 190	C(20,2) = 190	C(20,2) = 190
P(x)	0.6316	0.3368	0.0316

Insulators for transformers are purchased in cases of 10. From the case, 4 insulators are sampled and inspected. If the sample contains 1 or more defective insulators, the whole case is sent back to the supplier. Suppose the case contains 3 defective insulators. What is the probability that the case will be returned?

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x	0	1	2	3
C(a,x)	C(3,0) = 1	C(3,1) = 3	C(3,2) = 3	C(3,3) = 1
C(n-a, r-x)	C(7,4) = 35	C(7,3) = 35	C(7,2) = 21	C(7,1) = 7
Numerator	35	105	63	7
C(n,r)	C(10,4) = 210	C(10,4) = 210	C(10,4) = 210	C(10,4) = 210
P(x)	0.1667	0.5	0.3	0.0333

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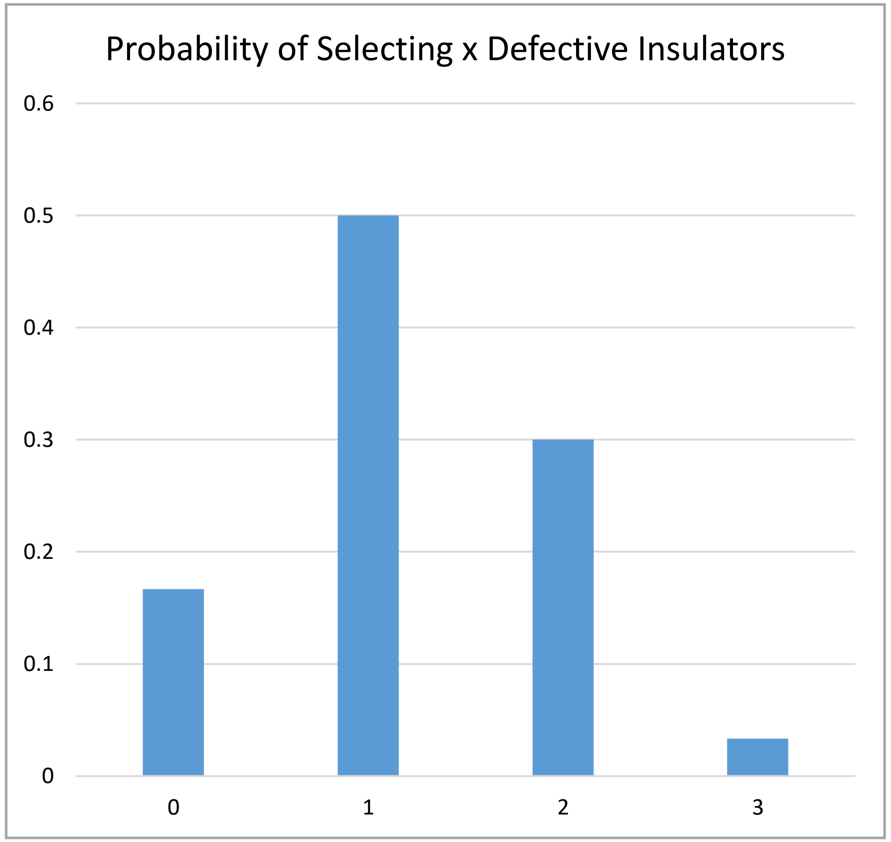

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P(x>=1)	0.8333