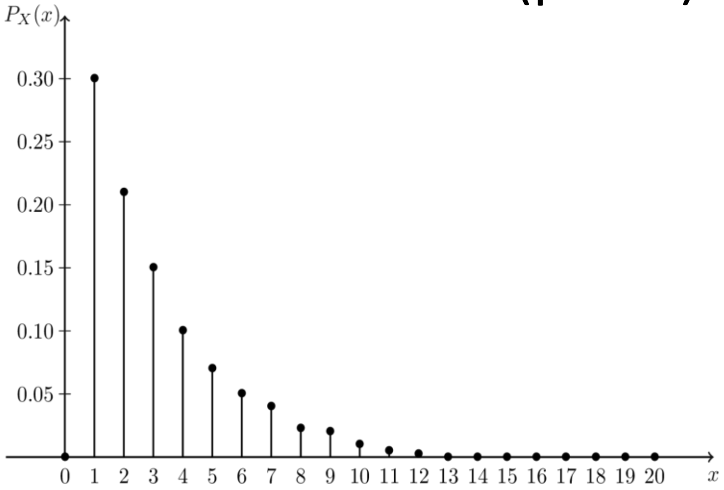
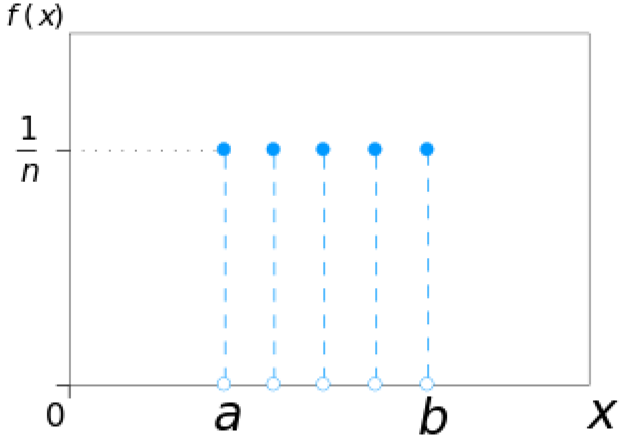
Geometric Distribution

Another probability distribution….

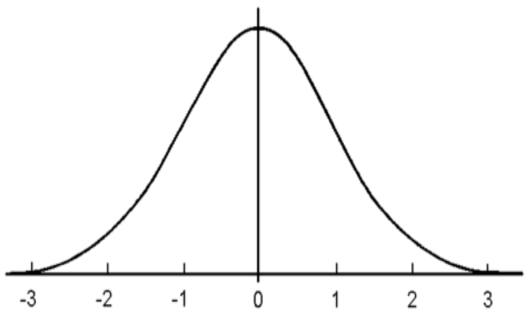
X ~ Geometric(p=0.3)



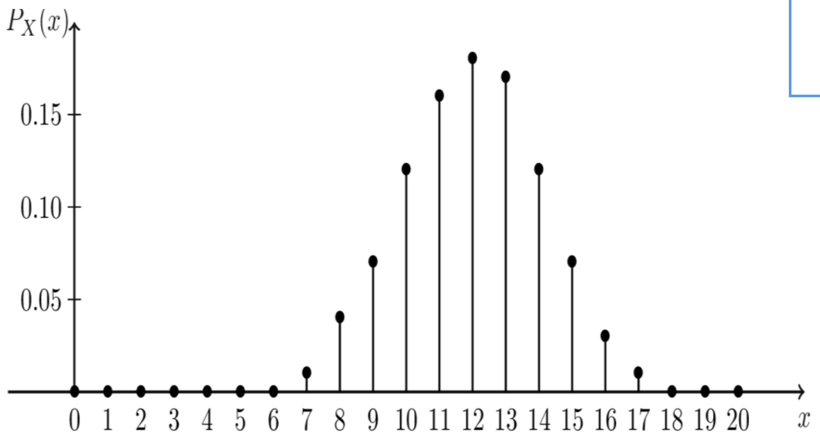
X ~ Uniform (n=5)



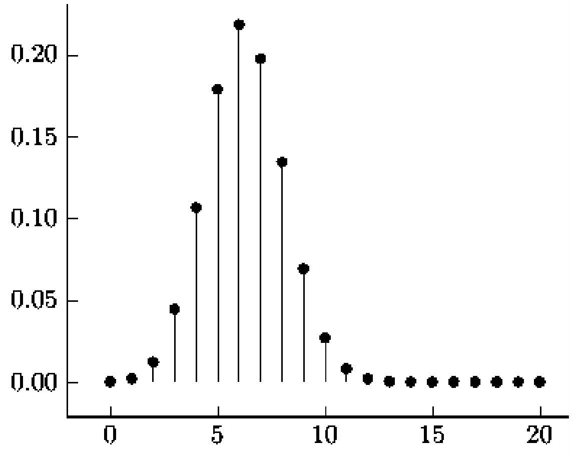
X ~ N(0, 1^2)



X ~ Binomial(n=20, p=0.6)



X ~ Hypergeometric (N=80, k=30, n=25)



Bionomic Distribution

- Bernoulli trials
- Independent
- Number of successes over a number of trials

Flipping a coin 6 times. Success is Head.

Geometric Distribution

- Bernoulli trials
- Independent
- Number of unsuccessful trials until success occurs

Flipping a coin until you get a head.

1. Which of the following situations is modelled by a geometric distribution? Explain your reasoning.
   a) rolling a die until a 6 shows
   b) counting the number of hearts when 13 cards are dealt from a deck
   c) predicting the waiting time when standing in line at a bank
   d) calculating the probability of a prize being won within the first 3 tries
   e) predicting the number of successful launches of satellites this year

Example 1 **Getting out of Jail in MONOPOLY®**

a) Calculate the probability distribution for getting out of jail in MONOPOLY® in x rolls of the dice.

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So, for each independent roll,

$p =$? and $q =$?

$=$? $=$?

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$$p = \frac{6}{36} \qquad \text{and} \qquad q = \text{?}$$

$$= \frac{1}{6} \qquad\qquad\qquad = \text{?}$$

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$$p = \frac{6}{36} = \frac{1}{6} \qquad \text{and} \qquad q = 1 - \frac{1}{6} = \frac{5}{6}$$

X	0	1	2	3	4	5	6	7
q^x	1	0.83333	0.69444	0.5787	0.48225	0.40188	0.3349	0.27908
p	0.16667	0.16667	0.16667	0.16667	0.16667	0.16667	0.16667	0.16667
P(x)=q^x*p	0.16667	0.13889	0.11574	0.09645	0.08038	0.06698	0.05582	0.04651

$$P(x) = q^x p$$

$$q = \frac{5}{6}$$

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$$q = \frac{5}{6}$$

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$$P(1) = \left(\frac{5}{6}\right)^1 \times \frac{1}{6} = \frac{5 \times 1}{6 \times 6}$$

$$P(2) = ?^{?} \times ?$$

X	0	1	2	3	4	5	6	7
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$$P(3) = \left(\frac{5}{6}\right)^3 \times \frac{1}{6} = \frac{5 \times 5 \times 5 \times 1}{6 \times 6 \times 6 \times 6}$$

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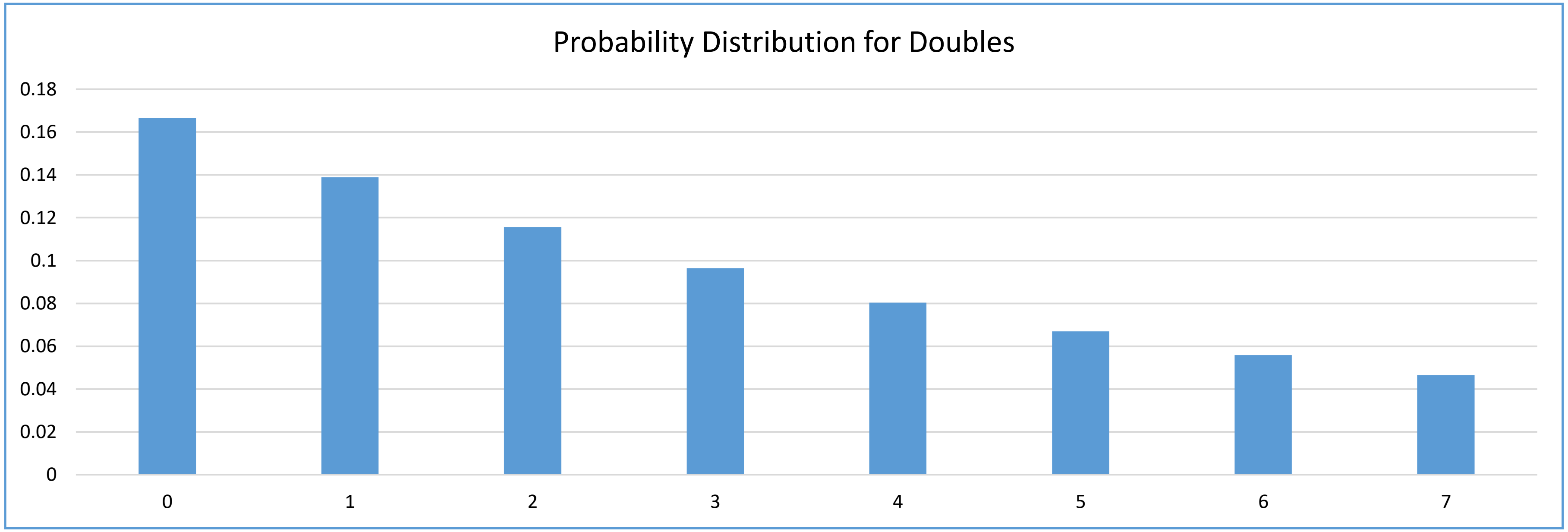

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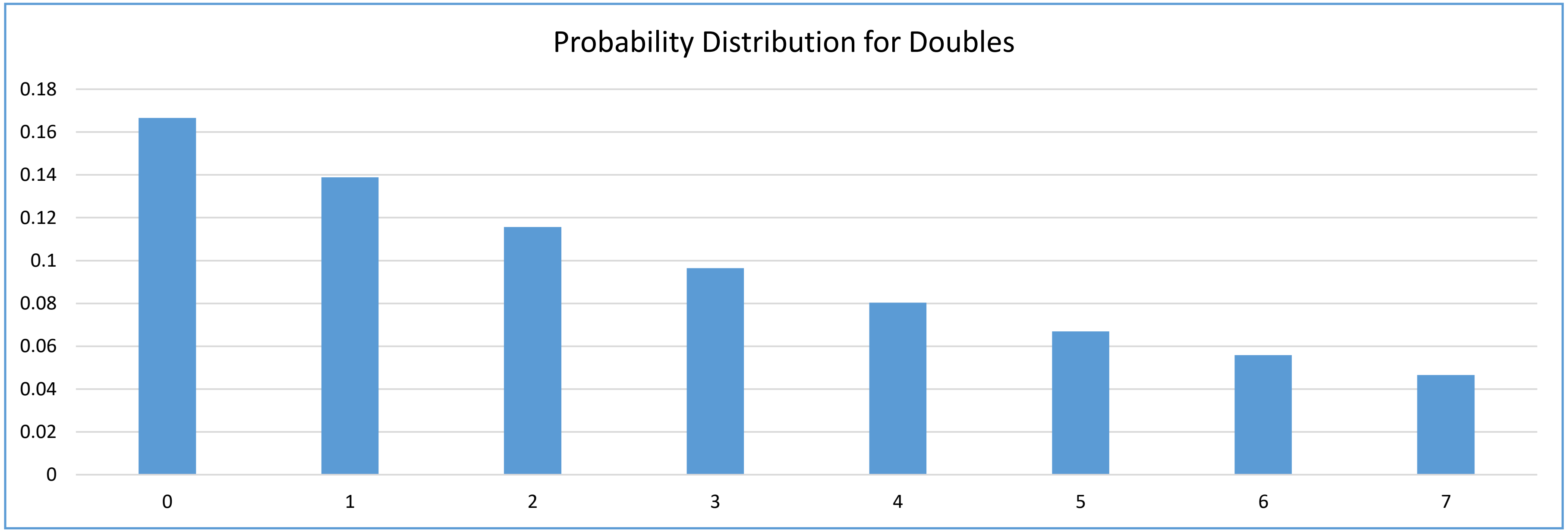


This distribution theoretically continues forever since one possible outcome is that the player never rolls doubles. However, the probability for a waiting time decreases markedly as the waiting time increases. Although this distribution is an infinite geometric series, its terms still sum to 1 since they represent the probabilities of all possible outcomes.

Example 2 Expectation of Geometric Distribution

Use the formula for the expectation of a geometric distribution to evaluate the accuracy of the estimates in Example 1.

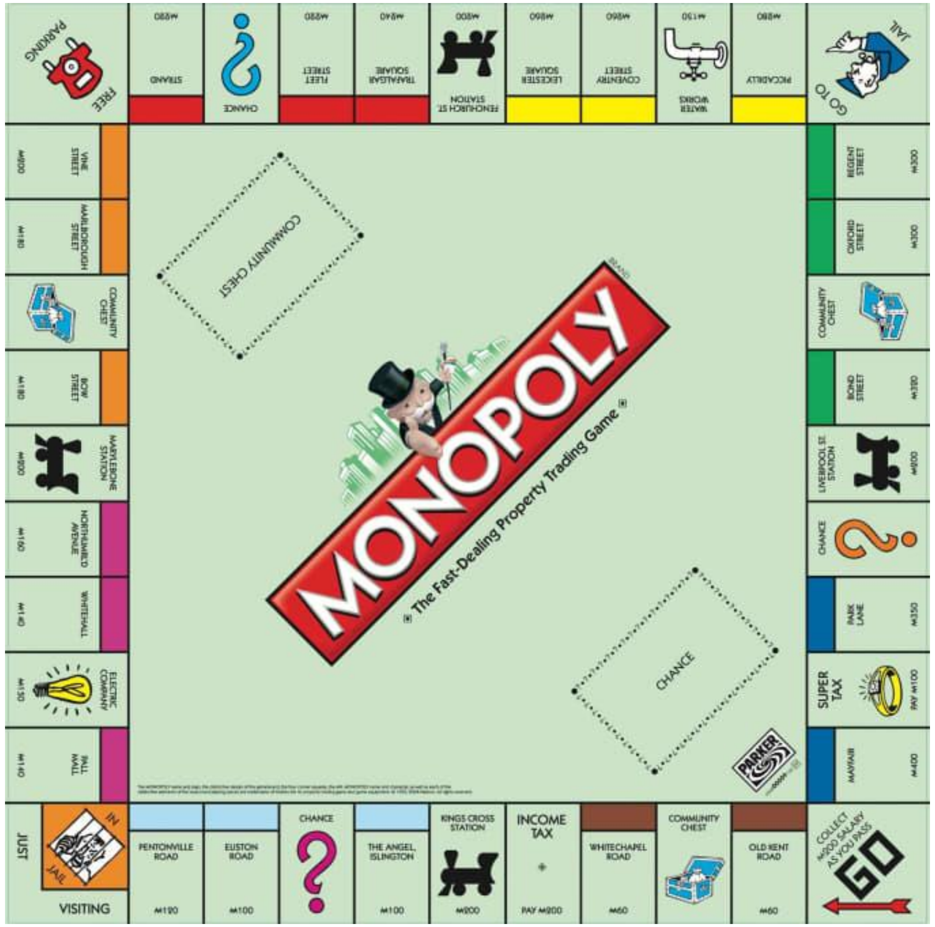
Solution

$$E(X) = \frac{q}{p}$$

$$= \frac{?}{?}$$

$$q = \frac{5}{6}$$

$$p = \frac{1}{6}$$



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$$E(X) = \frac{q}{p}$$

$$= \frac{\frac{5}{6}}{\frac{1}{6}}$$

$$q = \frac{5}{6}$$

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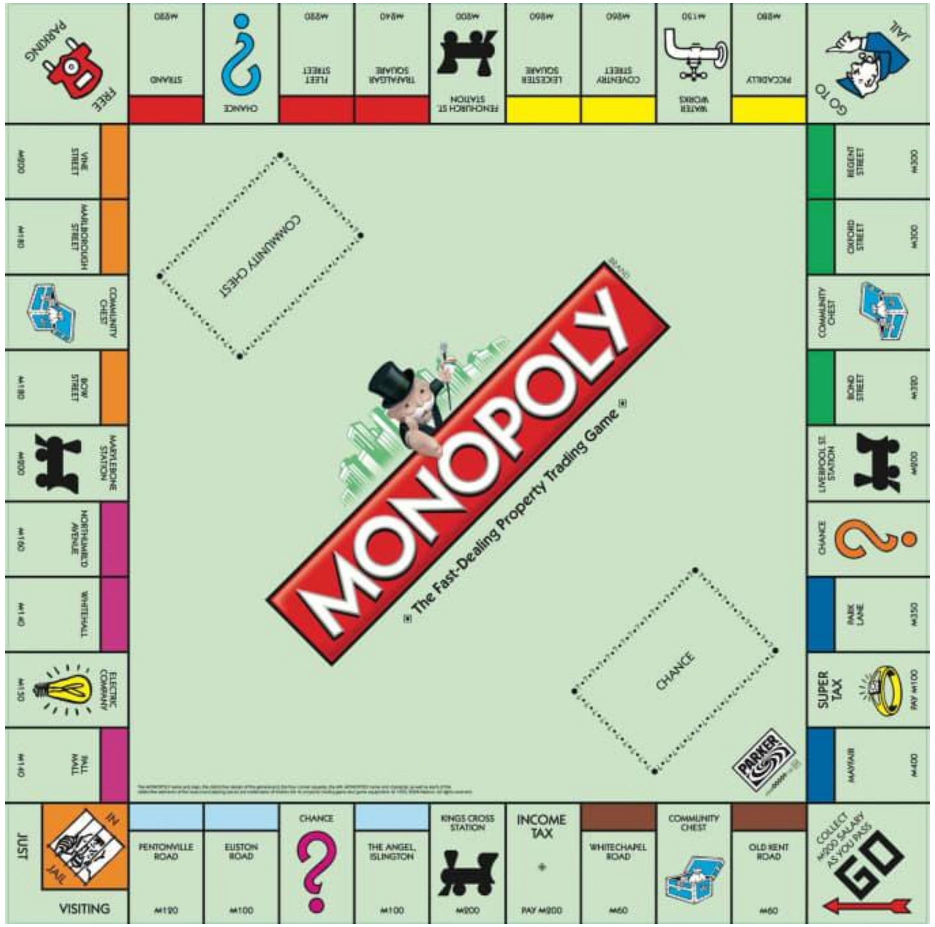


Example 2 Expectation of Geometric Distribution

Use the formula for the expectation of a geometric distribution to evaluate the accuracy of the estimates in Example 1.

Solution

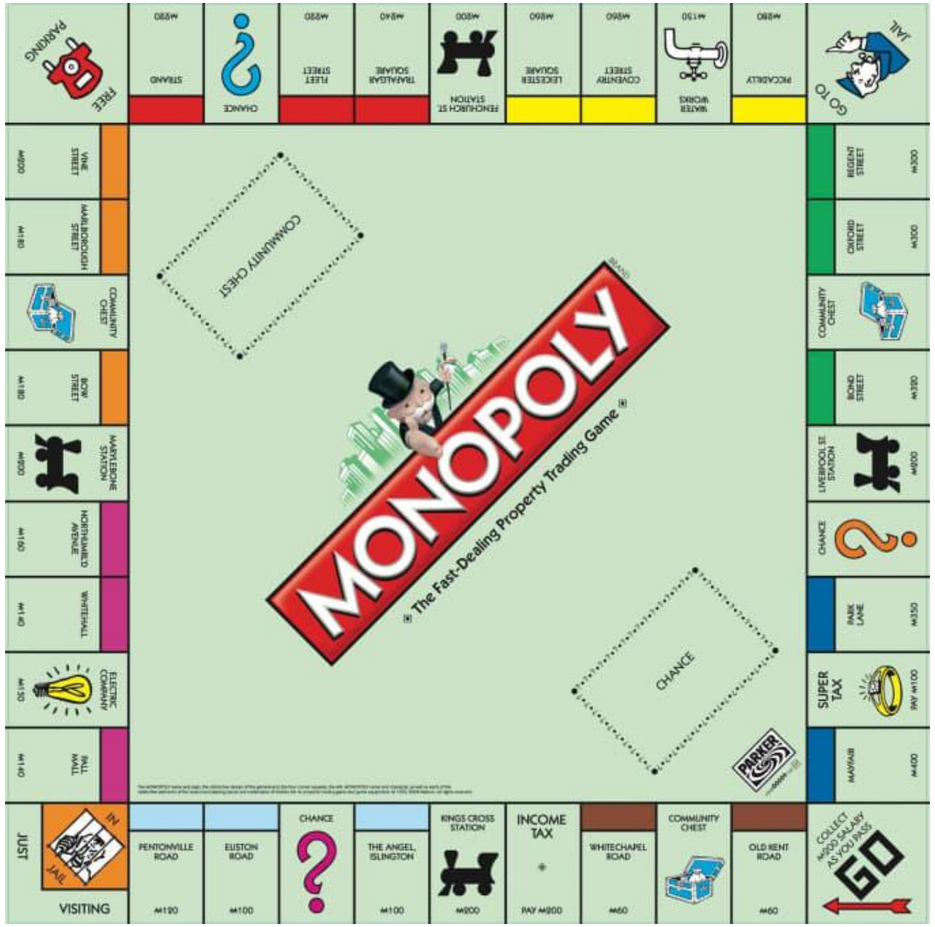
$$E(X) = \frac{q}{p}$$

$$= \frac{\frac{5}{6}}{\frac{1}{6}}$$

$$= 5$$

$$q = \frac{5}{6}$$

$$p = \frac{1}{6}$$



Related Problems….

And their uses….

Gambler's Ruin

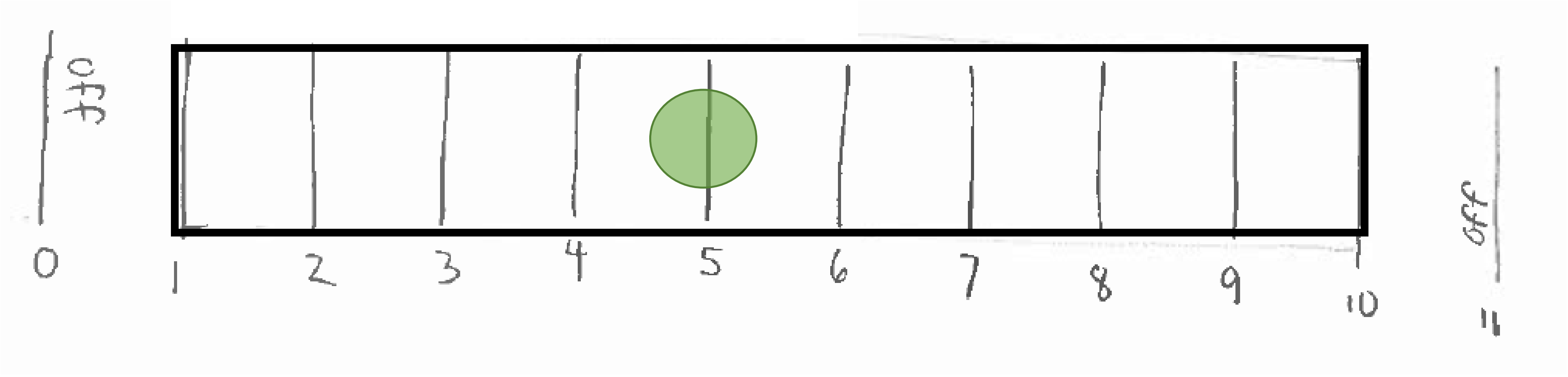
Two gamblers, A and B, with a sequence of rounds bet $1 each time.

p = probability A wins; $q = 1 - p$

They repeat, either forever, or until one wins the entire game.

Random walk:

A frog is in the middle of a 10 m bridge. He is equally likely to jump forward as backward. Each hop is 1 meter long. How many hops does he take to get off the bridge?



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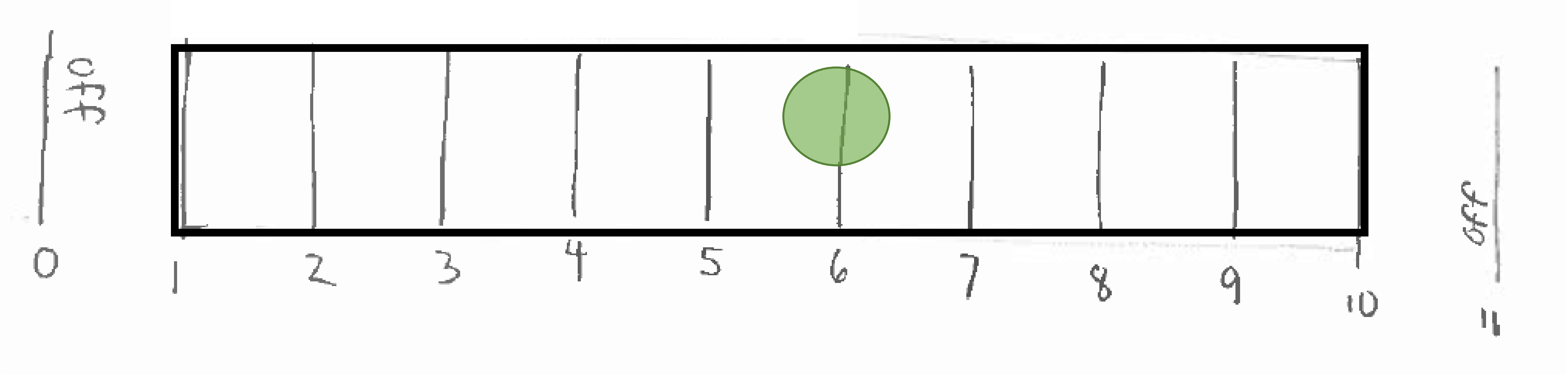

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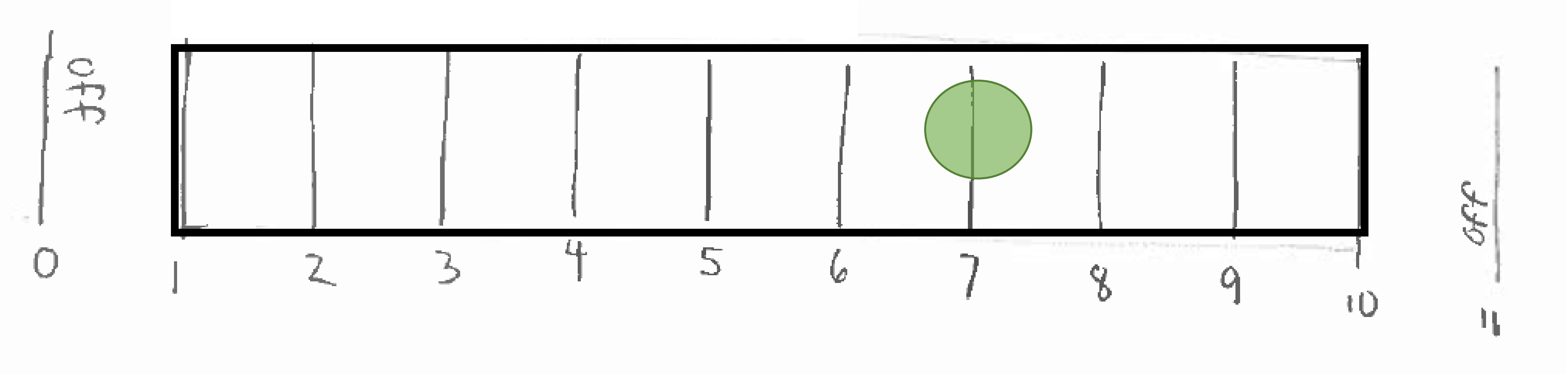

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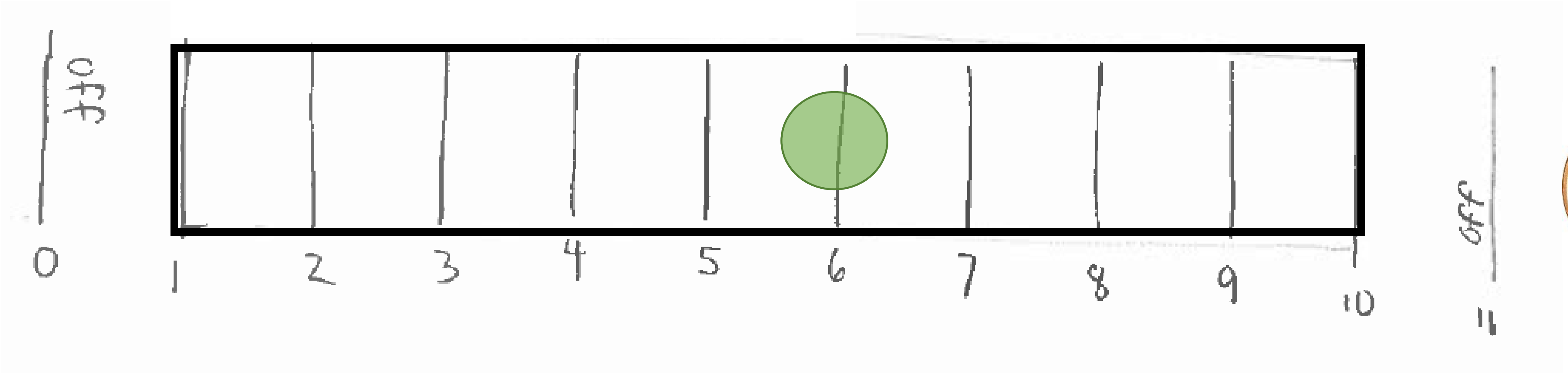

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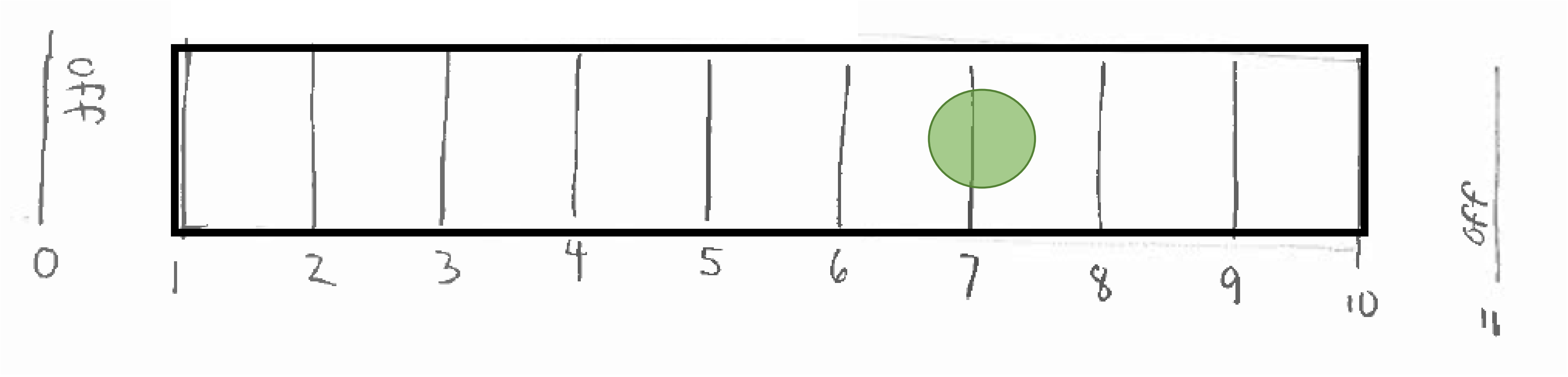

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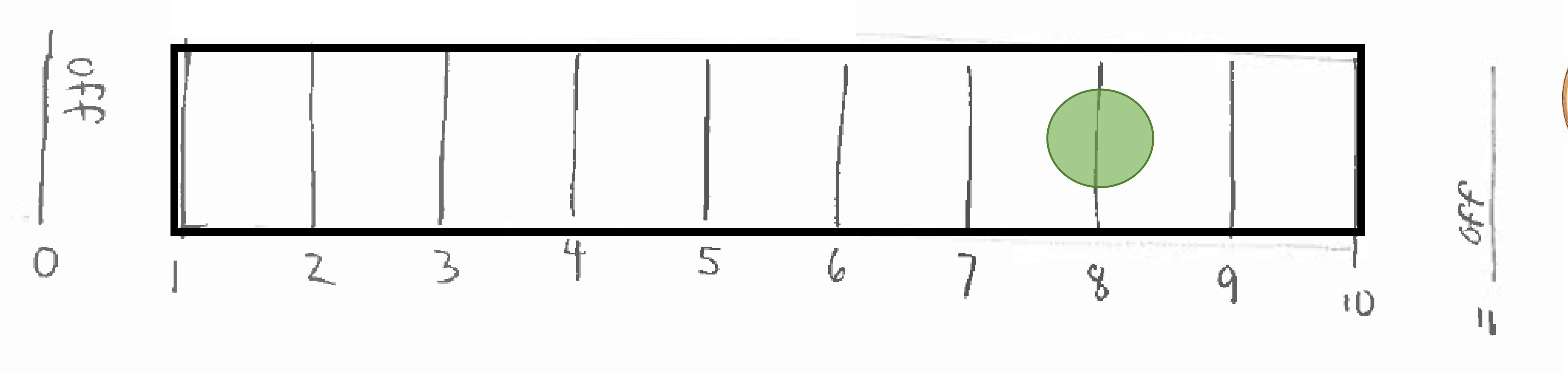

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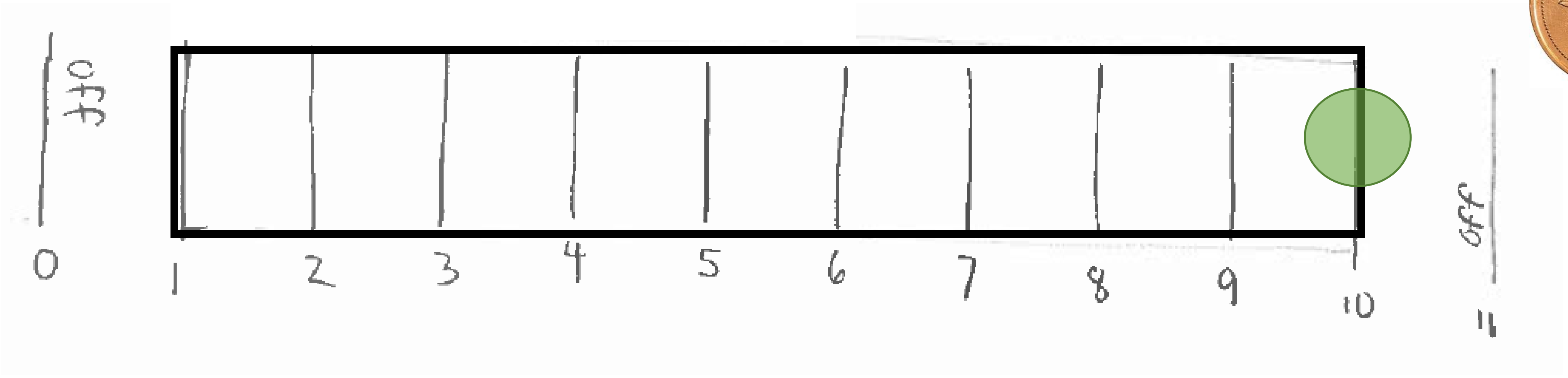

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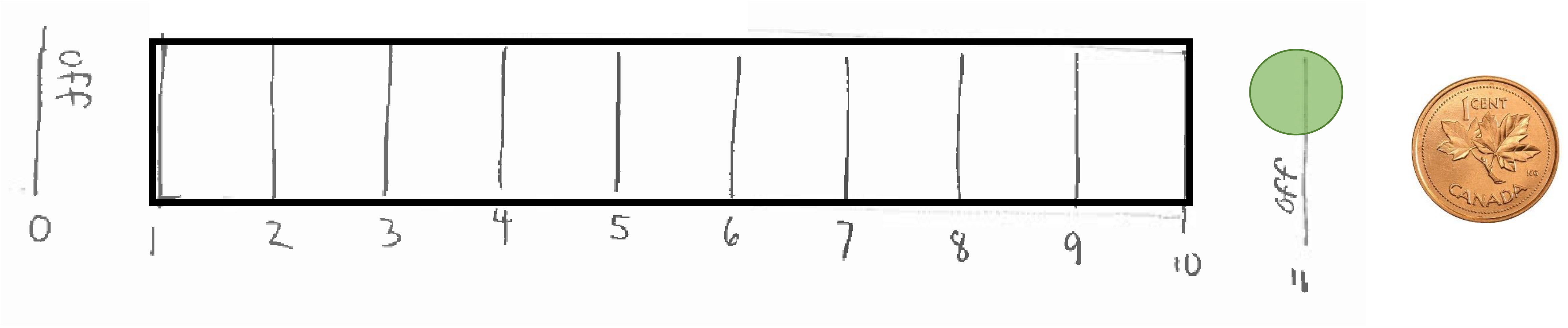
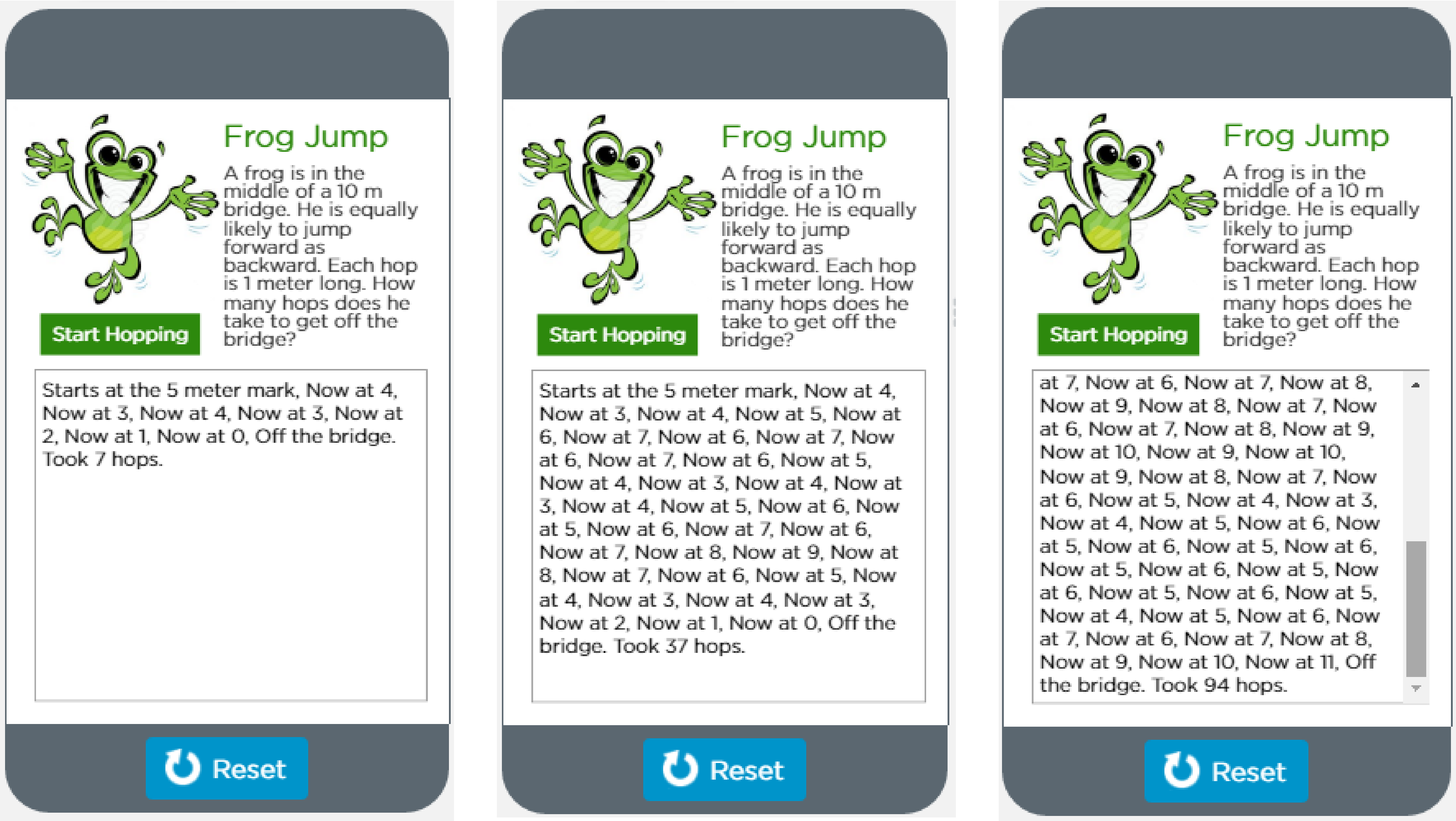
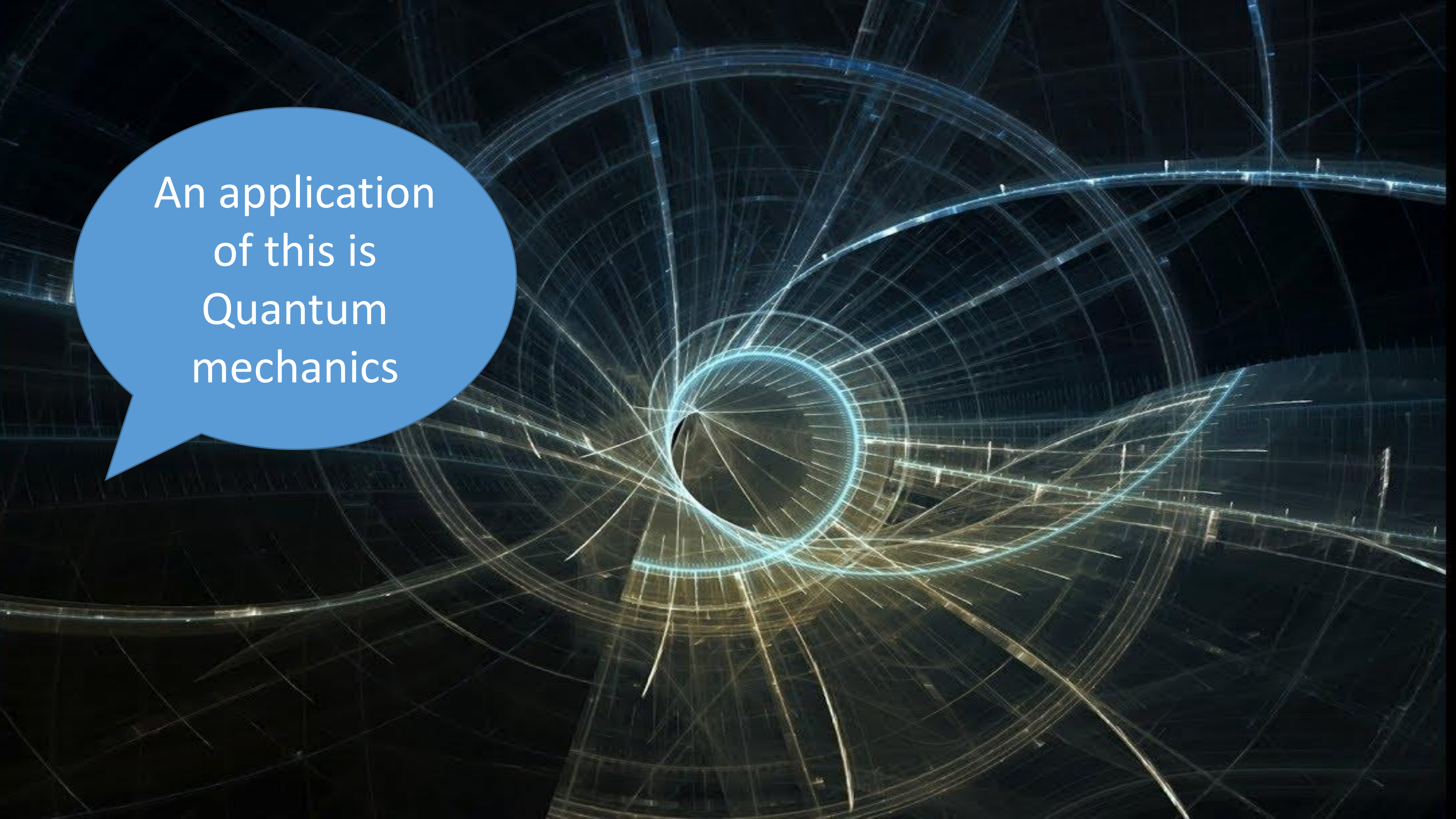




Frog Jump
A frog is in the middle of a 10 m bridge. He is equally likely to jump forward as backward. Each hop is 1 meter long. How many hops does he take to get off the bridge?
Start Hopping
Starts at the 5 meter mark, Now at 4, Now at 3, Now at 4, Now at 3, Now at 2, Now at 1, Now at 0, Off the bridge. Took 7 hops.
Reset
Frog Jump
A frog is in the middle of a 10 m bridge. He is equally likely to jump forward as backward. Each hop is 1 meter long. How many hops does he take to get off the bridge?
Start Hopping
Starts at the 5 meter mark, Now at 4, Now at 3, Now at 4, Now at 5, Now at 6, Now at 7, Now at 6, Now at 7, Now at 6, Now at 7, Now at 6, Now at 5, Now at 4, Now at 3, Now at 4, Now at 3, Now at 4, Now at 5, Now at 6, Now at 5, Now at 6, Now at 7, Now at 6, Now at 7, Now at 8, Now at 9, Now at 8, Now at 7, Now at 6, Now at 5, Now at 4, Now at 3, Now at 4, Now at 3, Now at 2, Now at 1, Now at 0, Off the bridge. Took 37 hops.
Reset
Frog Jump
A frog is in the middle of a 10 m bridge. He is equally likely to jump forward as backward. Each hop is 1 meter long. How many hops does he take to get off the bridge?
Start Hopping
at 7, Now at 6, Now at 7, Now at 8, Now at 9, Now at 8, Now at 7, Now at 6, Now at 7, Now at 8, Now at 9, Now at 10, Now at 9, Now at 10, Now at 9, Now at 8, Now at 7, Now at 6, Now at 5, Now at 4, Now at 3, Now at 4, Now at 5, Now at 6, Now at 5, Now at 6, Now at 5, Now at 6, Now at 5, Now at 6, Now at 5, Now at 6, Now at 5, Now at 6, Now at 5, Now at 4, Now at 5, Now at 6, Now at 7, Now at 6, Now at 7, Now at 8, Now at 9, Now at 10, Now at 11, Off the bridge. Took 94 hops.
Reset


An application of this is Quantum mechanics

Example 3 Basketball Free Throws

Jamaal has a success rate of 68% for scoring on free throws in basketball. What is the expected waiting time before he misses the basket on a free throw?

Solution

Here, the random variable is the number of trials before Jamaal misses on a free throw. For calculating the waiting time, a success is Jamaal *failing* to score.

Thus,

$$q = 0.68 \quad \text{and} \quad p = 1 - 0.68 = 0.32$$

Using the expectation formula for the geometric distribution,

$$E(X) = \frac{q}{p} = \frac{0.68}{0.32} = 2.1$$

The expectation is that Jamaal will score on 2.1 free throws before missing.

Suppose that an intersection you pass on your way to school has a traffic light that is green for 40 s and then amber or red for a total of 60 s.

a) What is the probability that the light will be green when you reach the intersection at least once a week?

b) What is the expected number of days before the light is green when you reach the intersection?

Each trial is independent with

$p =$? and $q =$?

$=$?

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a) What is the probability that the light will be green when you reach the intersection at least once a week?

b) What is the expected number of days before the light is green when you reach the intersection?

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$$p = \frac{40}{100} = 0.40 \quad \text{and} \quad q = \ ?$$

Suppose that an intersection you pass on your way to school has a traffic light that is green for 40 s and then amber or red for a total of 60 s.

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$$= 0.40$$

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Each trial is independent with

$$p = \frac{40}{100} = 0.40 \quad \text{and} \quad q = 0.60$$

$$P(x) = q^x p$$

There are five school days in a week. To get a green light on one of those five days, your waiting time must be four days or less.

$$P(0 \le x \le 4) = ? + ? + ? + ? + ?$$
$$= ?$$

Suppose that an intersection you pass on your way to school has a traffic light that is green for 40 s and then amber or red for a total of 60 s.

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$$= \boxed{?}$$

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There are five school days in a week. To get a green light on one of those five days, your waiting time must be four days or less.

$$P(0 \le x \le 4) = 0.40 + (0.60)(0.40) + \text{?} + \text{?} + \text{?}$$
$$= \text{?}$$

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$$P(0 \le x \le 4) = 0.40 + (0.60)(0.40) + (0.60)^2(0.40) + \text{?} + \text{?})$$

$$= \text{?}$$

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$$P(0 \le x \le 4) = 0.40 + (0.60)(0.40) + (0.60)^2(0.40) + (0.60)^3(0.40) + (0.60)^4(0.40)$$

$= $?

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$$P(x) = q^x p$$

There are five school days in a week. To get a green light on one of those five days, your waiting time must be four days or less.

$$P(0 \le x \le 4) = 0.40 + (0.60)(0.40) + (0.60)^2(0.40) + (0.60)^3(0.40) + (0.60)^4(0.40)$$
$$= 0.92$$

The probability of the light being green when you reach the intersection at least once a week is 0.92.

Each trial is independent with

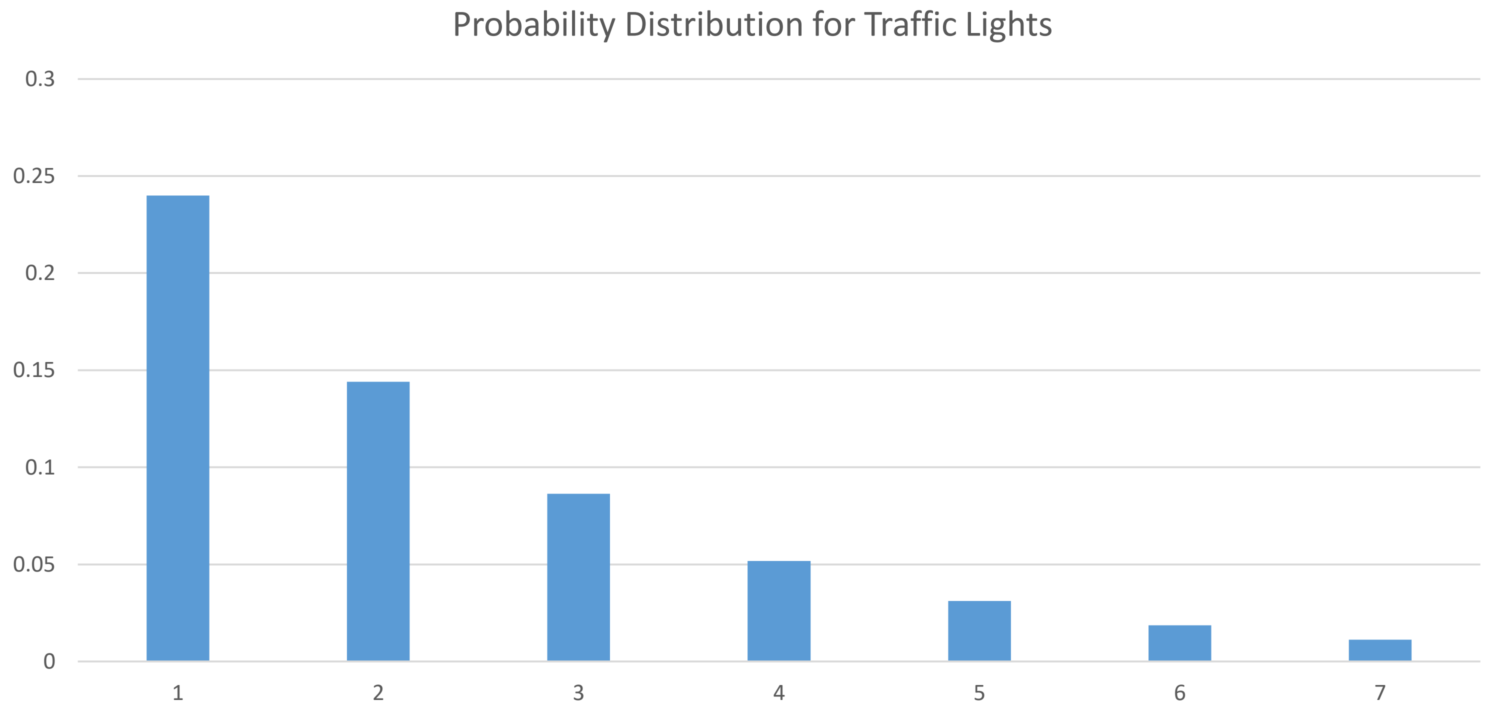
$$p = \frac{40}{100} = 0.40 \quad \text{and} \quad q = 0.60$$

There are five school days in a week. To get a green light on one of those five days, your waiting time must be four days or less.

$$P(0 \le x \le 4) = 0.40 + (0.60)(0.40) + (0.60)^2(0.40) + (0.60)^3(0.40) + (0.60)^4(0.40) = 0.92$$

	A	B	C	D	E	F	G	H	I
1	X	0	1	2	3	4	5	6	7
2	q^x	1	0.6	0.36	0.216	0.1296	0.07776	0.046656	0.027994
3	p	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4
4	P(X)=q^x*p	0.4	0.24	0.144	0.0864	0.05184	0.031104	0.018662	0.011197
5									
6	P(X<=4)	0.92224							

	A	B	C	D	E	F	G	H	I
1	X	0	1	2	3	4	5	6	7
2	q^x	1	0.6	0.36	0.216	0.1296	0.07776	0.046656	0.027994
3	p	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4
4	P(X)=q^x*p	0.4	0.24	0.144	0.0864	0.05184	0.031104	0.018662	0.011197
5									
6	P(X<=4)	0.92224							



Suppose that an intersection you pass on your way to school has a traffic light that is green for 40 s and then amber or red for a total of 60 s.

a) What is the probability that the light will be green when you reach the intersection at least once a week?

b) What is the expected number of days before the light is green when you reach the intersection?

b) $E(X) = \frac{q}{p}$

$= \frac{?}{?}$

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b)
$$E(X) = \frac{q}{p}$$
$$= \frac{0.60}{0.40}$$
$$= \boxed{?}$$

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b)
$$\begin{aligned} E(X) &= \frac{q}{p} \\ &= \frac{0.60}{0.40} \\ &= 1.5 \end{aligned}$$

The expected waiting time before catching a green light is 1.5 days.