## Binomial Distribution

Combinations, Probability and Distributions. Oh my!

Pg 375, \#8. A lottery has a \$1,000,000 first prize, a \$25,000 second prize and five $\$ 1,000$ third prizes. A total of $2,000,000$ tickets are sold. If a ticket costs $\$ 2.00$ what is the expected profit per ticket?

| - | A | B | C | D | E | F |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Prize | First | Second | Third | No win | Sum |
| 2 | \$ | \$ 1,000,000.00 | \$ 25,000.00 | \$1,000.00 | \$ | \$ 1,026,000.00 |
| 3 | Freq | 1 | 1 | 5 | 1999993 | 2000000 |
| 4 | $\mathrm{P}(\mathrm{x})$ | 0.0000005 | 0.0000005 | 0.0000025 | 0.999997 | 1 |
| 5 | \$*P(x) | \$ 0.50 | \$ 0.01 | \$ 0.00 | \$ | \$ 0.52 |
| 6 |  |  |  |  |  |  |
| 7 | (b) | One Ticket | 2 |  | One Ticket | 2 |
| 8 |  | All Tickets | \$ 4,000,000.00 |  | E(x) | \$ 0.52 |
| 9 |  | Costs | \$ 1,026,000.00 |  | Profit | \$ 1.49 |
| 10 |  | Profit | \$ 2,974,000.00 |  |  |  |
| 11 |  | Profit per ticket | \$ 1.49 |  |  |  |

Bernoulli Trial

- Two outcomes: success/failure; boy/girls; true/false
- Independent
- The probability of success is the same at every trial
- The trial happens a series of times
- Flipping a coin
- Rolling a six
- Opinion poll; voter that will vote "yes"
- Is the top card of a shuffled deck an ace
- Was the new born child a girl?



## Bernoulli Trial

- Ball hits a peg.
- It can go right (success) or left, with a probability of 0.5 in this model.
https://phet.colorado.edu/sims/html/ plinko-probability/latest/plinkoprobability en.html




## Genuine not simulated

## Probability of an Event in a Binomial Distribution

$$
\mathrm{P}(\mathrm{X})=C(n, x) p^{x} q^{n-x} \quad \begin{aligned}
& \mathrm{n}=\text { number of trials } \\
& \mathrm{x}=\text { number of successes } \\
& \mathrm{q}=\text { prob of success }
\end{aligned}
$$

What is the Probability of flipping a coin 6 times and getting 5 heads and 1 tail?
success is flip heads

$$
\begin{aligned}
& n=6 \\
& x=5 \\
& p=0.5 \\
& q=0.5
\end{aligned}
$$

$$
\mathrm{P}(\mathrm{x}=5)=C(6,5) 0.5^{5} 0.5^{1}
$$

$$
\begin{aligned}
& =6 \times 0.01563 \\
& =0.094
\end{aligned}
$$

## Binomial Distributions

Name:
5.6 K

1. Write out the formula for the probability of a binomial event 9 times.

2. You are flipping a weighted coin twice. It lands on heads with a probability of 0.6. What is the probability distribution for heads?

$$
x^{\sim} \operatorname{Bin}(n=2, p=0.6) \text {. Thus, } q=
$$

$\qquad$

| x | 0 heads | 1 head | 2 heads |
| :---: | :---: | :---: | :---: |
| $C(n, x)$ | $C(\ldots, \ldots)=$ | $\mathrm{C}(\ldots, \ldots)=$ | $C(\ldots, \ldots)=$ |
| $\mathbf{p}^{\text {x }}$ | $\wedge \ldots$ | $\ldots$ ___ $=$ | $\ldots$ ___ $=$ |
| $q^{n+x}$ | $\wedge \ldots$ | $\wedge \ldots$ | $\wedge \ldots$ |
| $\mathrm{P}(\mathrm{x})$ |  |  |  |

6. For each question, identify the "success", $n, p$ and $q$.

|  | Success | n | p | q |
| :--- | :---: | :---: | :---: | :---: |
| a. You are writing a multiple choice test and <br> have 0.9 probability of getting a question <br> correct. There are 10 questions. |  |  |  |  |
| b. The probability of getting a red light is 0.4. <br> On your way to school there are 19 lights. |  |  |  |  |

7. Write the equation in the form: $X^{\sim} \operatorname{Bin}(n=$ $\qquad$ $p=$ $\qquad$ ).
Then write the formula for the probability of event $x$, with $n, p$ and $q$ filled in.

|  | Equation | Probability of X |
| :--- | :---: | :---: |
| a. You are writing a multiple choice test and <br> have 0.9 probability of getting a question <br> correct. There are 10 questions. |  |  |
| b. The probability of getting a red light is 0.4. <br> On your way to school there are 19 lights. |  |  |

## $\mathrm{P}(\mathrm{x})=C(n, x) \times p^{x} q^{n-x}$

## $=\mathrm{COMBIN}(\$ F \$ 3, B 5)^{*}\left(\$ \mathrm{~B} \$ 3^{\wedge} \mathrm{B} 5\right)^{*}\left(\$ \mathrm{D} \$ 3^{\wedge}(\$ \mathrm{~F} \$ 3-\mathrm{B} 5)\right)$

|  | A | B | C | D | E | F | G | H |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | Binomial Distribution |  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |  |  |
| 3 | p | 0.2 | q | 0.8 | n | 5 |  |  |
| 4 |  |  |  |  |  |  |  |  |
| 5 | x | O | 1 | 2 | 3 | 4 | 5 | Sum |
| 6 | $\mathrm{p}(\mathrm{x})$ | $=$ COMBIN | 0.41 | 0.205 | 0.051 | 0.006 | 0.0003 | 1 |

## Discretetown






Treatment works 60\%
of the time


We expect to be able to cure 6 patients per day.

However, on a particular day, we could easily have more than 6 or fewer than 6.

$\operatorname{Bin}(10,0.6)$




1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
$90 \%$ chance it is between 4 and 8.
$\operatorname{Bin}(n, p)$
\# of successes

$$
n=
$$

Number of Trials

$$
p=
$$

Probability of Success

| Trial |  |  | Result |
| :---: | :---: | :---: | :---: |
| 1 | $\checkmark$ |  |  |
| 2 | $\times$ |  |  |
| 3 | $\times$ |  |  |
| 4 | $\checkmark$ |  |  |
| $\cdots$ | $\cdots$ |  |  |
| $n$ | $\nearrow$ |  |  |
|  | $H \mathbf{X}$ |  |  |

