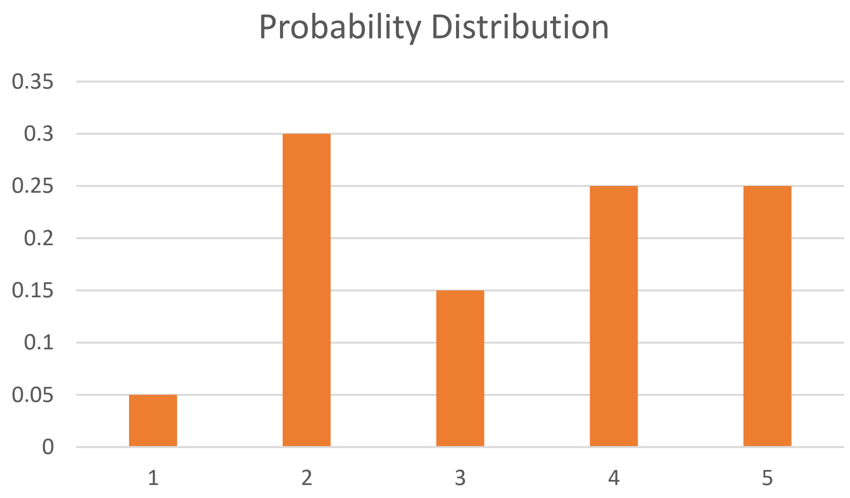
Expected Value

Playing the Lottery….hmm….

Calculate the utility function for this distribution:

x	12	14	16	18	20
$	7	1	3	4	6
P(x)	0.05	0.3	0.15	0.25	0.25
$*P(x)	0.35	0.3	0.45	1	1.5



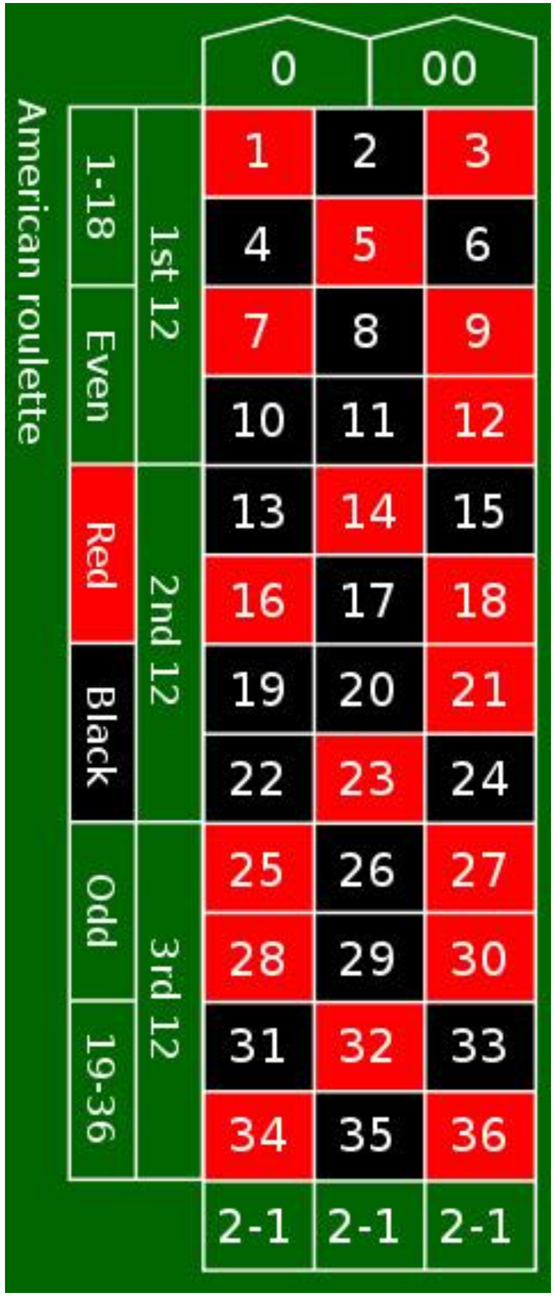
$$E(X) = \sum \$x \times P(x)$$

= $0.35 + $0.3 + $0.45 + $1 + $1.5

= $3.6

On average, you will expect to make $3.60 payout with this distribution.


Why does Las Vegas look like this?



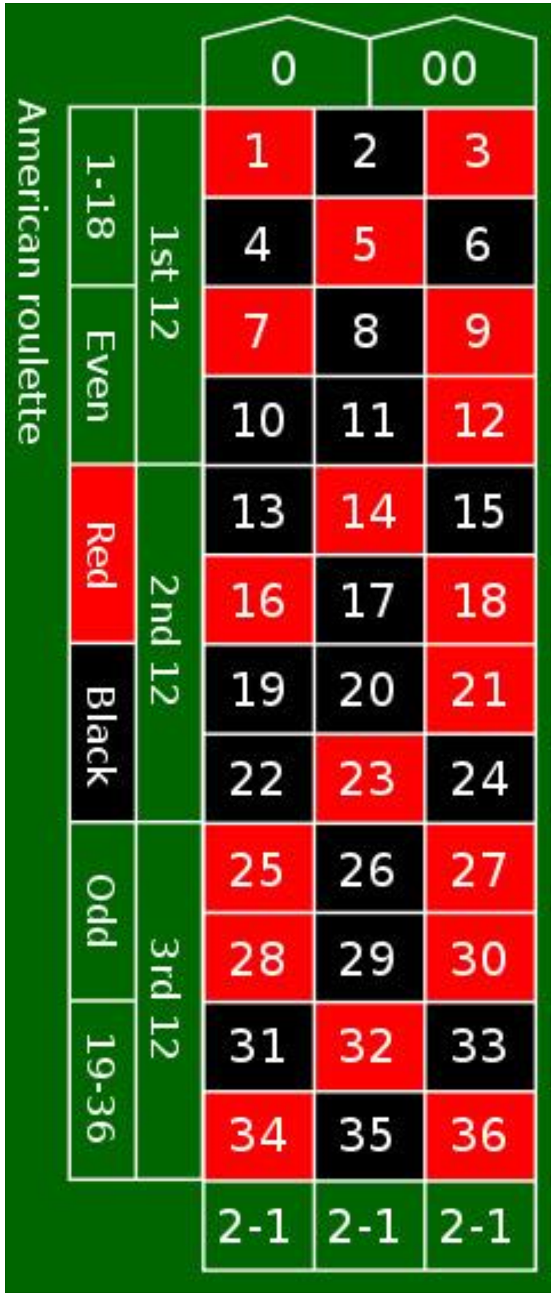
Bet common name	Winning spaces	Payout	Odds against winning
Straight up	Any single number including 0	35 to 1	36 to 1
Split	any two adjoining numbers vertical or horizontal	17 to 1	17.5 to 1
Basket	0, 1, 2 or 0, 2, 3	11 to 1	11.33 to 1
Street	any three numbers horizontal (1, 2, 3 or 4, 5, 6 etc.)	11 to 1	11.33 to 1
Corner	any four adjoining numbers in a block (eg 17, 18, 20, 21)	8 to 1	8.25 to 1
Six Line	any six numbers from two rows (eg 28, 29, 30, 31, 32, 33)	5 to 1	5.167 to 1
1st Column	1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34	2 to 1	2.083 to 1
2nd Column	2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35	2 to 1	2.083 to 1
3rd Column	3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36	2 to 1	2.083 to 1
1st Dozen	1 through 12	2 to 1	2.083 to 1
2nd Dozen	13 through 24	2 to 1	2.083 to 1
3rd Dozen	25 through 36	2 to 1	2.083 to 1
Odd	1, 3, 5, ..., 35	1 to 1	1.056 to 1
Even	2, 4, 6, ..., 36	1 to 1	1.056 to 1
Red	Red nos	1 to 1	1.056 to 1
Black	Black nos	1 to 1	1.056 to 1
1 to 18	1, 2, 3, ..., 18	1 to 1	1.056 to 1
19 to 36	19, 20, 21, ..., 36	1 to 1	1.056 to 1

$10 roulette bet on a single space

x	Lose	Win
$	0	350
P(x)	0.9737	0.0263
$*P(x)	0	9.21

$$E(X) = \sum \$x \times P(x)$$

= $9.21

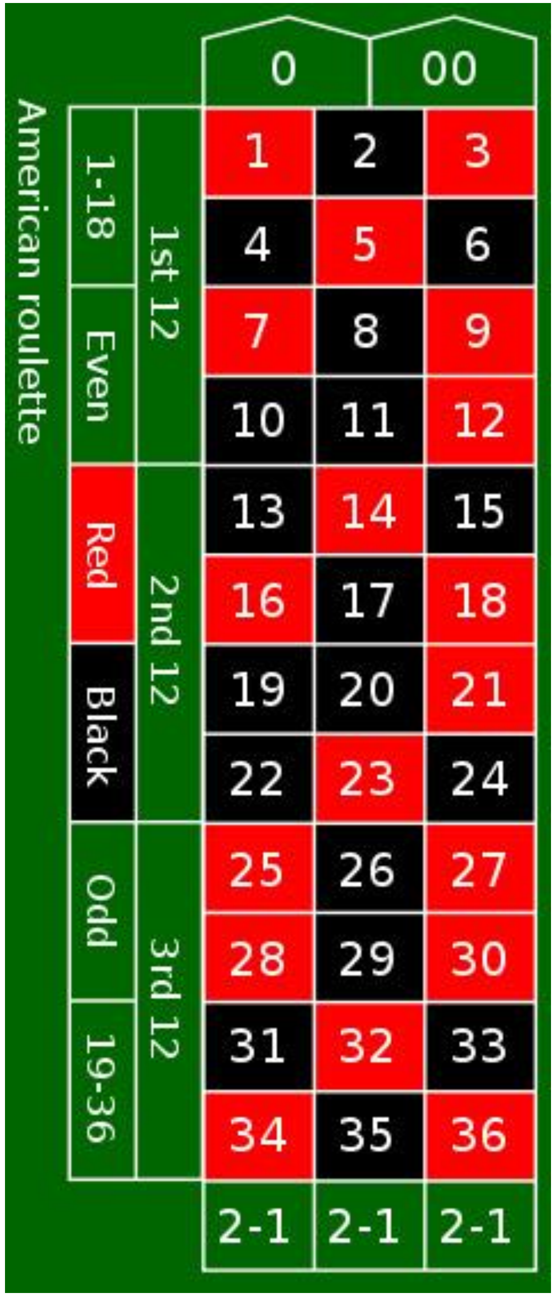


$10 roulette bet on a red/black/even/odd

x	Lose	Win
$	0	10
P(x)	0.5264	0.4736
$*P(x)	0	4.74

$$E(X) = \sum \$x \times P(x)$$

$$= \$4.74$$

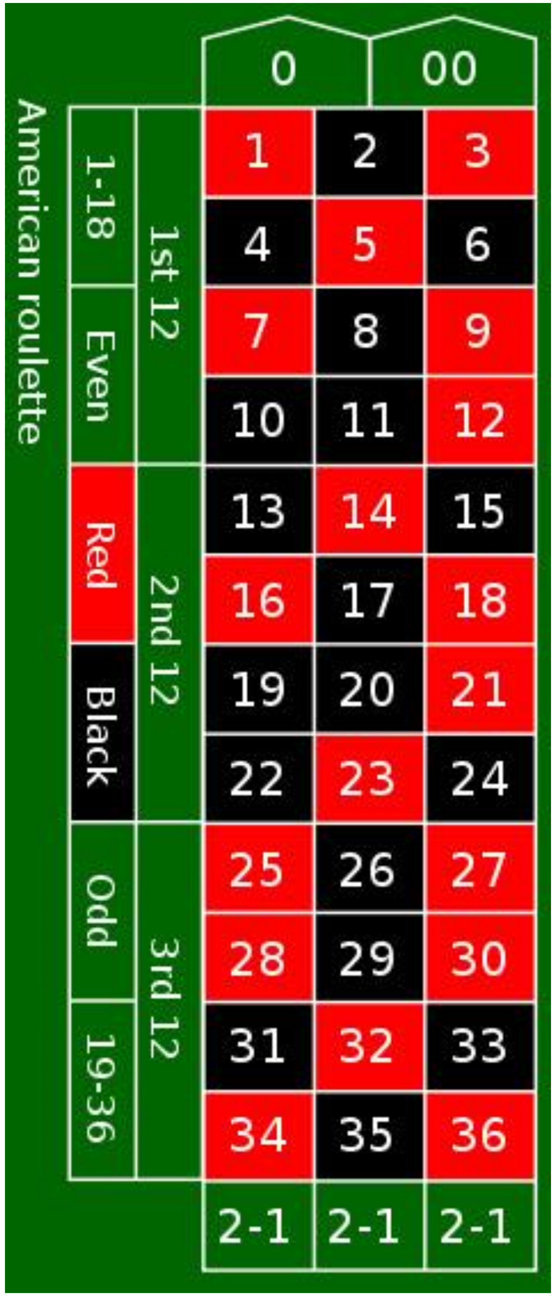


$10 roulette bet on a dozen

x	Lose	Win
$	0	20
P(x)	0.6842	0.3158
$*P(x)	0	6.32

$$E(X) = \sum \$x \times P(x)$$

= $6.32



Why do governments run lotteries?

(5 20) 100

#1745 $500,000 MONEY MANIA

At Start of Game:
Overall odds of winning any prize including prizes of less than $20: 1 in 1.00
Odds of winning a prize of $20 or more: 1 in 3.13
Top Prize odds: 1 in 2,000,000.00
(Top prize odds may vary +/- 2%)

- Sign your ticket upon receipt.
- Prize amounts for this game are $5, $10, $20, $25, $50, $100, $250, $500, $1,000, $10,000 and $500,000.

X	$5 (Under $20)	Over $20
P(x)	1	0.319
x * P(x)	$5	$6.38

claimed, se ... 886.

E(x) = $11.38

- Game closi ... documented business reasons. These games may have prizes unclaimed, including top prizes. Game closing procedures will be

“People play the lottery all the time unaware of how mind-bogglingly difficult it is to win. It seems like they take a different approach to probabilities. Their rationale must be, “Well, I can either win it or not win it, so my odds of winning are 50/50.”

POWERBALL EXPECTED VALUE

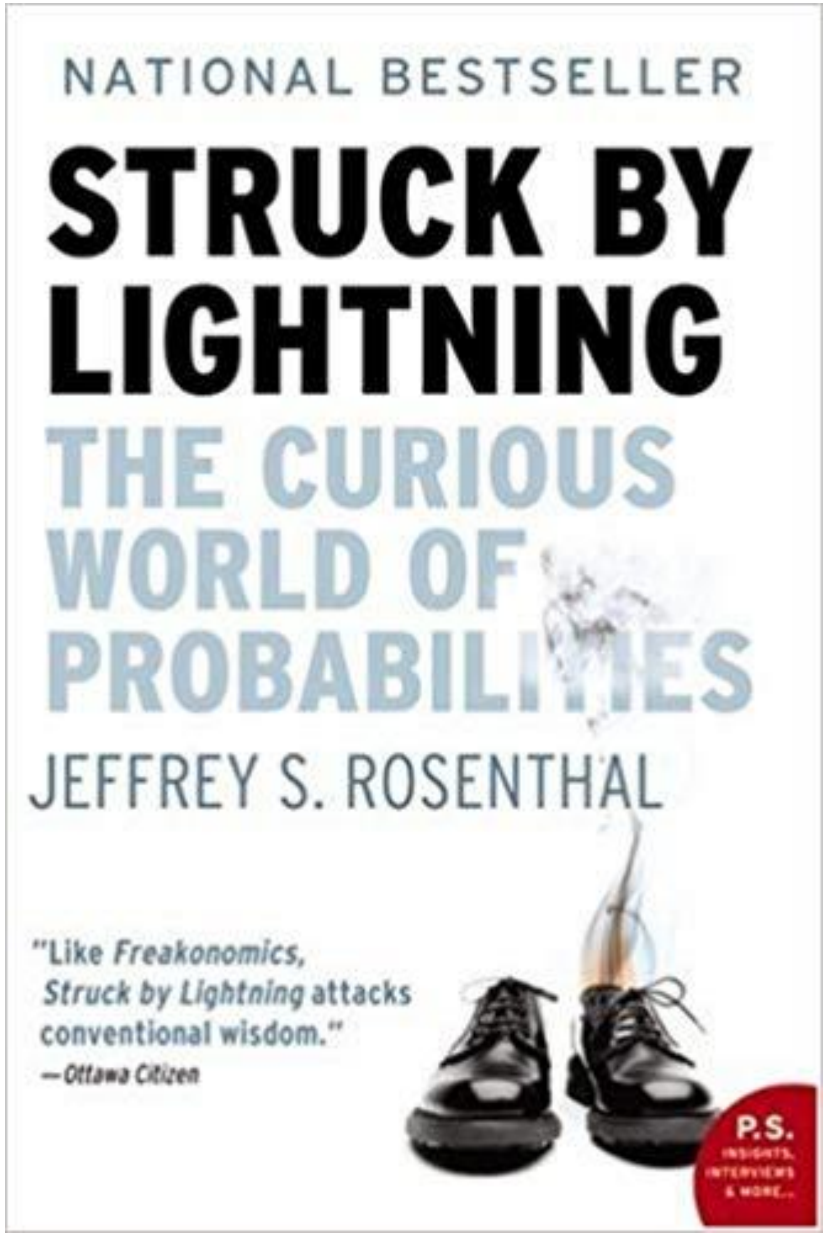
NUMBERS MATCHED	PRIZE	PRIZE - COST	ODDS	PROBABILITY	(PRIZE - COST) x PROBABILITY
5 white + 1 red	$700,000,000	$699,999,998	1 in 292,201,338	0.00000034%	$2.40
5 white	$1,000,000	$999,998	1 in 11,688,054	0.00000856%	$0.09
4 white + 1 red	$50,000	$49,998	1 in 913,129	0.00010951%	$0.05
4 white	$100	$98	1 in 36,525	0.00273784%	$0.00
3 white + 1 red	$100	$98	1 in 14,494	0.00689935%	$0.01
3 white	$7	$5	1 in 580	0.17248517%	$0.01
2 white + 1 red	$7	$5	1 in 701	0.14258623%	$0.01
1 white + 1 red	$4	$2	1 in 92	1.08719287%	$0.02
0 white + 1 red	$4	$2	1 in 38	2.60552371%	$0.05
Nothing	$0	-$2	1 in 1.04	95.98245642%	-$1.92

EXPECTED VALUE: $0.72

SOURCE: Business Insider calculations with odds and prizes from Powerball

BUSINESS INSIDER

If the Expected Value is $0.72… then…. hmmmm…….



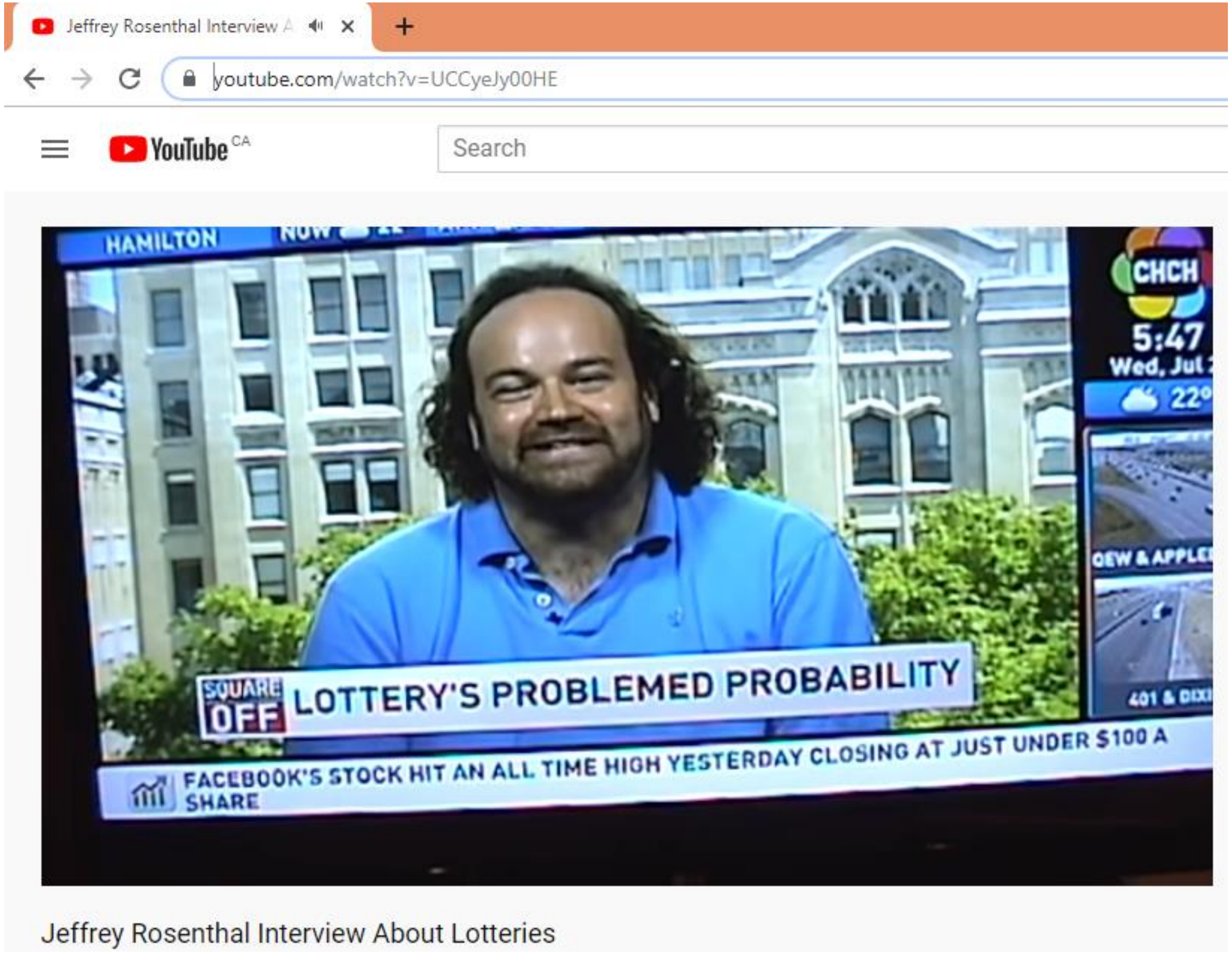
Pg 79 "The first rule when making decisions about randomness is that events of extremely small probability should generally be ignored. This is a very simple rule that most people do not follow."

"To put it in context, you are over 1,000 times more likely to die in a car crash in the next year. In fact, you are more likely to die in a car crash on the way tot eh store to buy your lottery ticket, than you are to win the lottery. Indeed, if you bought one lottery ticket a week, on average you would win the jackpot less than once every 250,000 years."

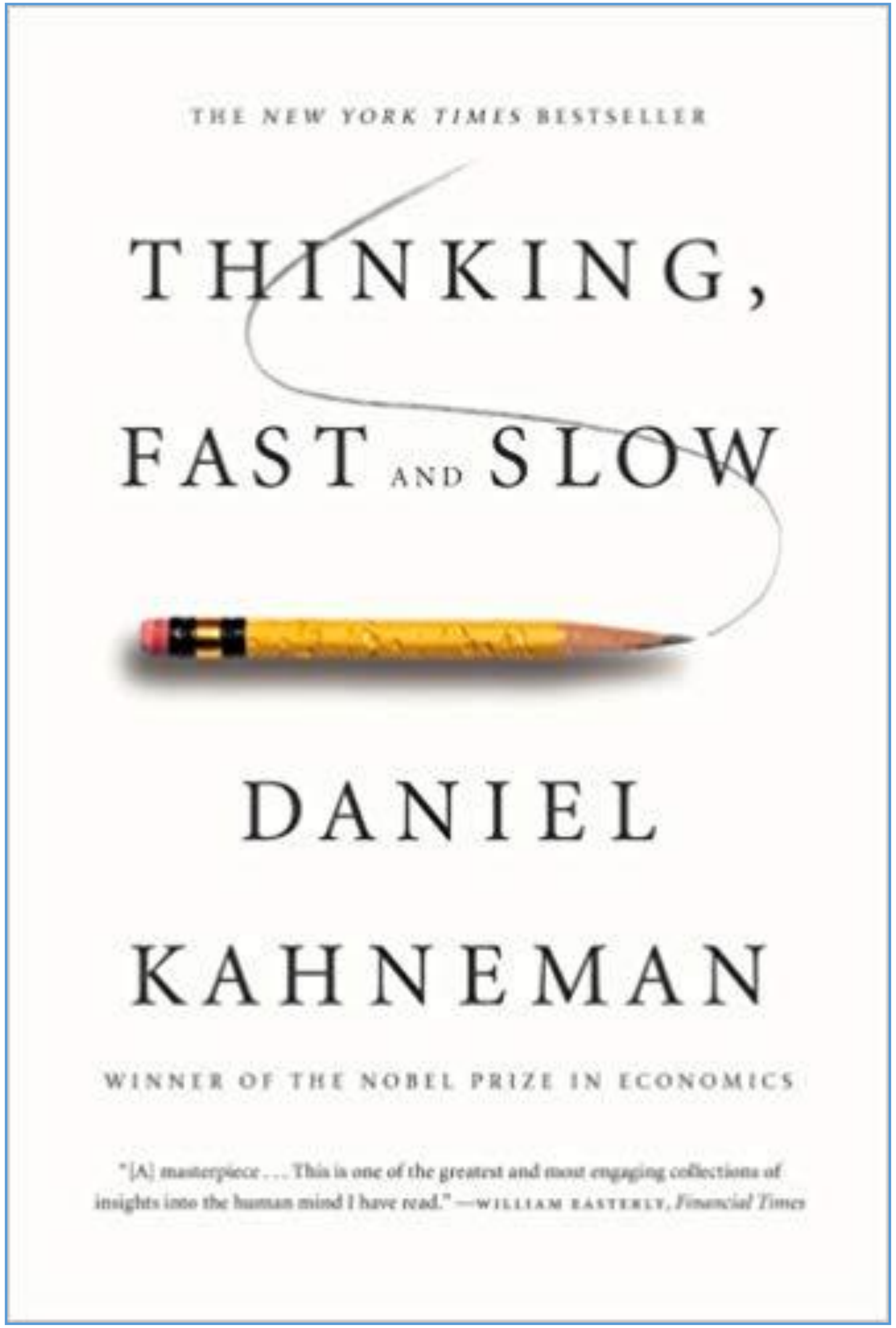
"It may be true that *someone* is going to win the lottery jackpot this week, but let me assure you: that someone will not be you."

I guess I think of lotteries as a tax on the mathematically challenged.

Lottery: A tax on people who are bad at math.



https://www.youtube.com/watch?v=UCCyeJy00HE



Chapter 21: Intuitions vs. Formulas

Humans, especially experts, believe that they can overrule the formula because they have additional information.

They are wrong. Formulas are more accurate.

Problem 1: Which do you choose?

a) Get $900 for sure

b) 90% chance to get $1,000

Problem 2: Which do you choose?

a) Lose $900 for sure

b) 90% chance to lose $1,000

Problem 1: Which do you choose?

a) Get $900 for sure ← Majority

b) 90% chance to get $1,000

Problem 2: Which do you choose?

a) Lose $900 for sure

b) 90% chance to lose $1,000 ← Majority

	GAINS	LOSSES
HIGH PROBABILITY Certainty Effect	95% chance to win $10,000 Fear of disappointment RISK AVERSE Accept unfavorable settlement	95% chance to lose $10,000 Hope to avoid loss RISK SEEKING Reject favorable settlement
LOW PROBABILITY Possibility Effect	5% chance to win $10,000 Hope of large gain RISK SEEKING Reject favorable settlement	5% chance to lose $10,000 Fear of large loss RISK AVERSE Accept unfavorable settlement

A: 61% chance to win $520,000

or

B: 63% chance to win $500,000

C: 98% chance to win $520,000

or

D: 100% chance to win $500,000

A: 61% chance to win $520,000 ← Majority

or

B: 63% chance to win $500,000

C: 98% chance to win $520,000

or

D: 100% chance to win $500,000 ← Majority

61% chance to win $520,000 ← $317,200

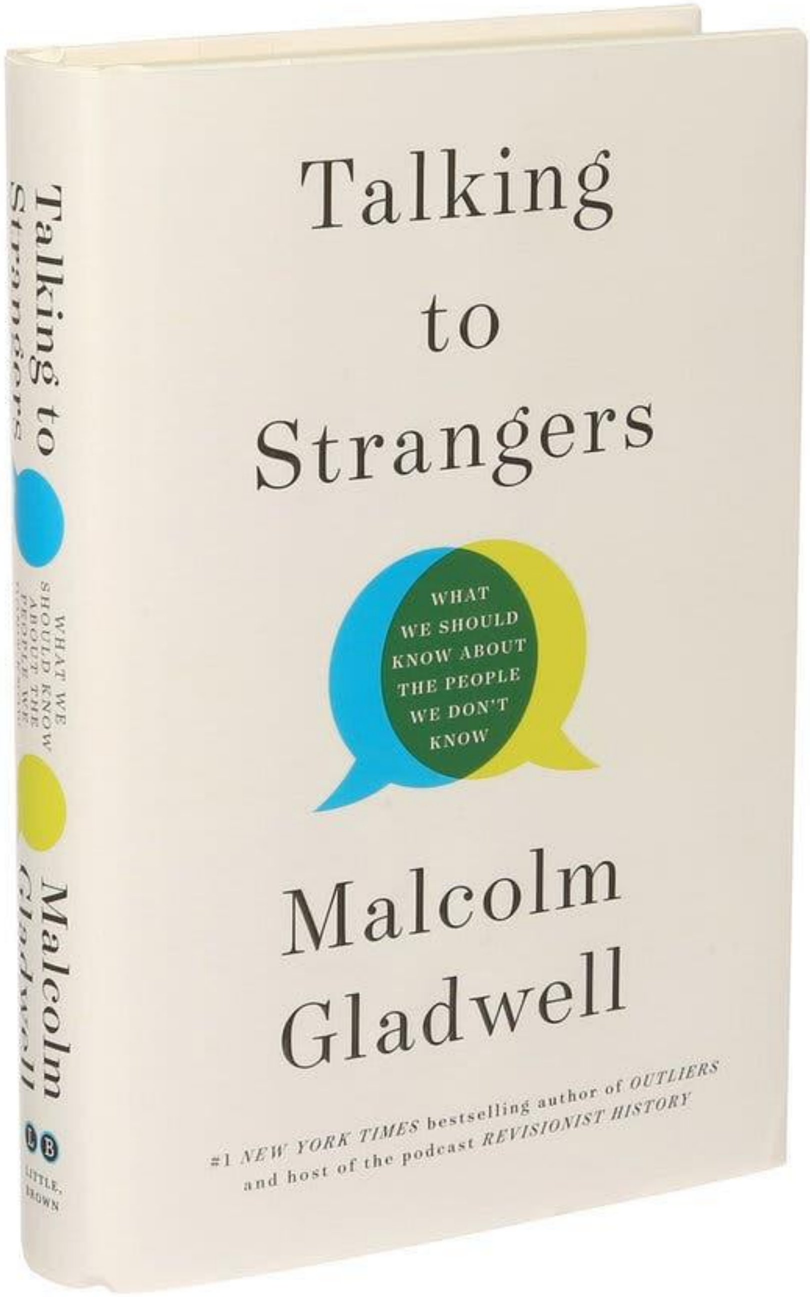
63% chance to win $500,000 ← $315,000

98% chance to win $520,000

$509,600

100% chance to win $500,000

$500,000



The problem occurs when we try to determine who is lying and who is telling the truth.