Probability Distributions

And Expected Value….

Consider this frequency information:

x	2	3	4	5	6	7	8
Freq	1	6	15	20	15	6	1

What we used last unit to make histograms and to do standard deviation.

$$\bar{x} = \frac{\sum f \times x}{\sum f}$$

$$\sigma = \sqrt{\frac{\sum f\,(x - \bar{x})^2}{\sum f}}$$

Consider this frequency information:

x	2	3	4	5	6	7	8
Freq	1	6	15	20	15	6	1
P(x)	1/64	6/64	15/64	20/64	15/64	6/64	1/64

We can calculate the probability of each x value.

P(x=5)

$$\sum f = 1 + 6 + 15 + 20 + 15 + 6 + 1$$
$$= 64$$

Consider this frequency information:

x	2	3	4	5	6	7	8
Freq	1	6	15	20	15	6	1
P(x)	1/64	6/64	15/64	20/64	15/64	6/64	1/64
P(x)	0.01563	0.09375	0.23438	0.3125	0.23438	0.09375	0.01563

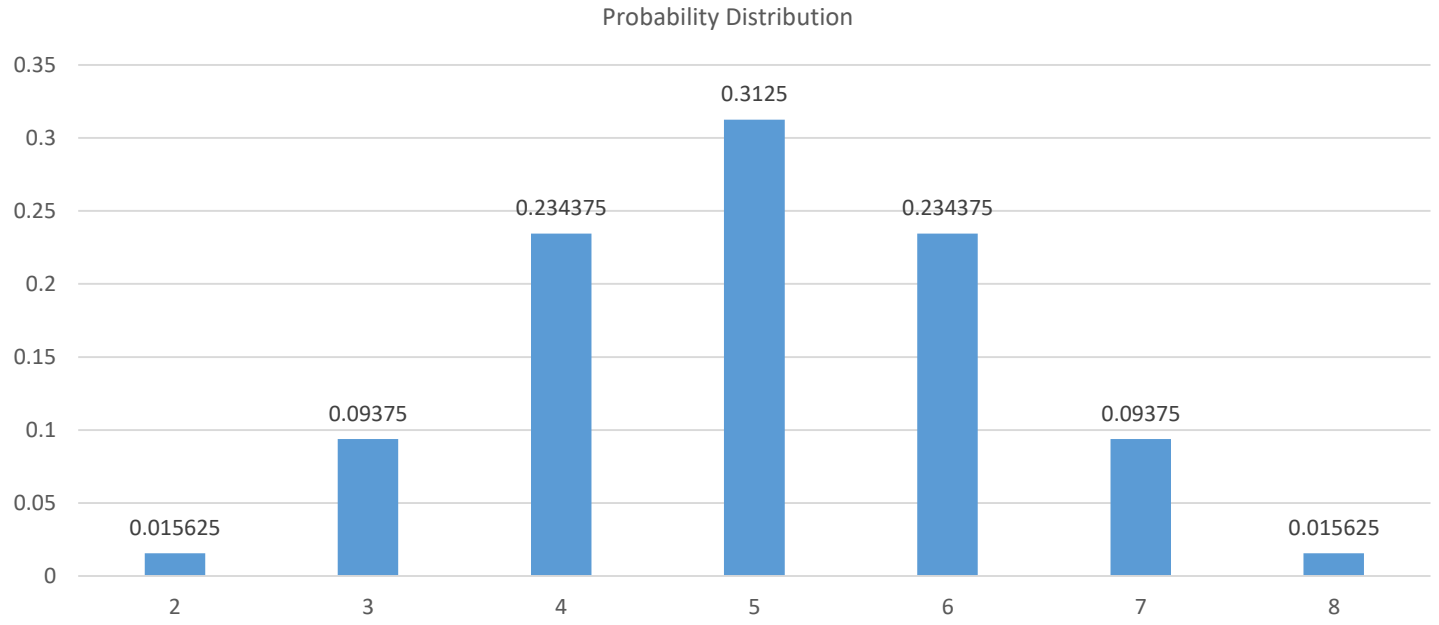
This can also be expressed as a decimal.

Consider this frequency information:

x	2	3	4	5	6	7	8
Freq	1	6	15	20	15	6	1
P(x)	1/64	6/64	15/64	20/64	15/64	6/64	1/64
P(x)	0.01563	0.09375	0.23438	0.3125	0.23438	0.09375	0.01563

Then, we can graph the probabilities.

Ta da! A probability distribution.



Create the probability distribution for rolling a dice.

x	1	2	3	4	5	6
P(x)						

In this case, we have no frequencies.

Just calculate the probabilities directly.

Create the probability distribution for rolling a dice.

x	1	2	3	4	5	6
P(x)	1/6	1/6	1/6	1/6	1/6	1/6

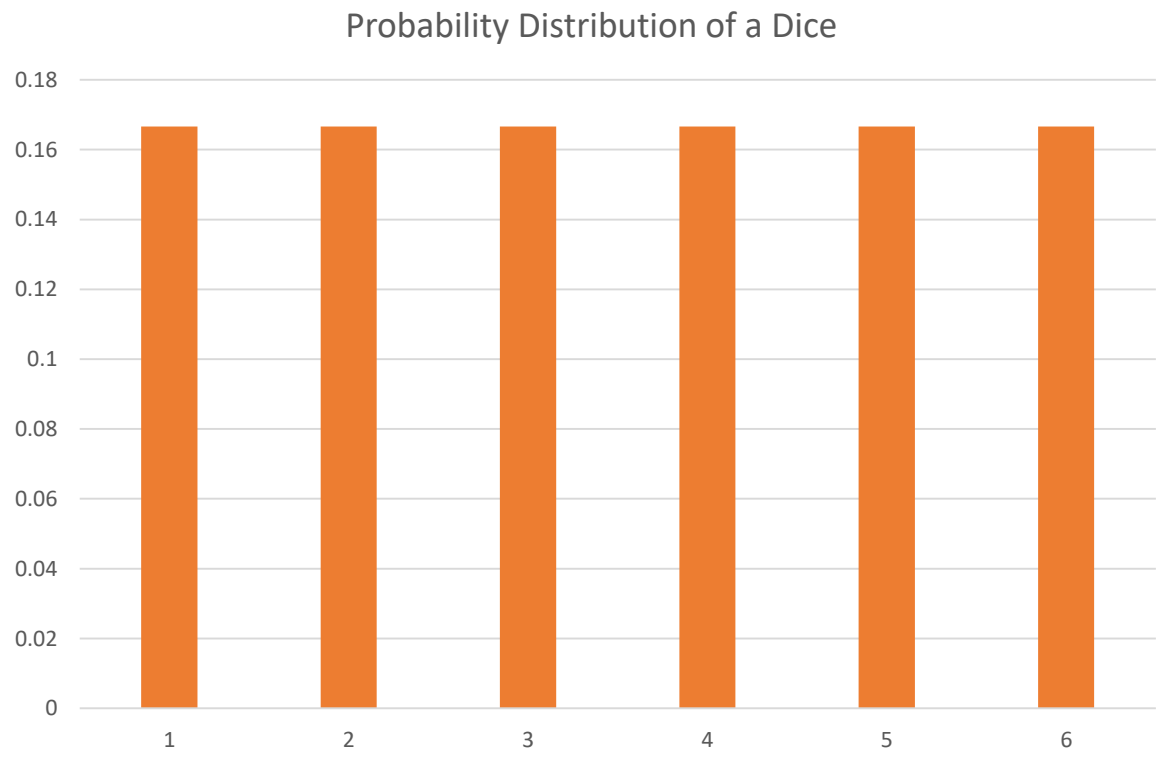
Create the probability distribution for rolling a dice.

x	1	2	3	4	5	6
P(x)	1/6	1/6	1/6	1/6	1/6	1/6
P(x)	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667

Create the probability distribution for rolling a dice.

x	1	2	3	4	5	6
P(x)	1/6	1/6	1/6	1/6	1/6	1/6
P(x)	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667

Then, we can graph the probabilities.



Ta da! Another probability distribution.

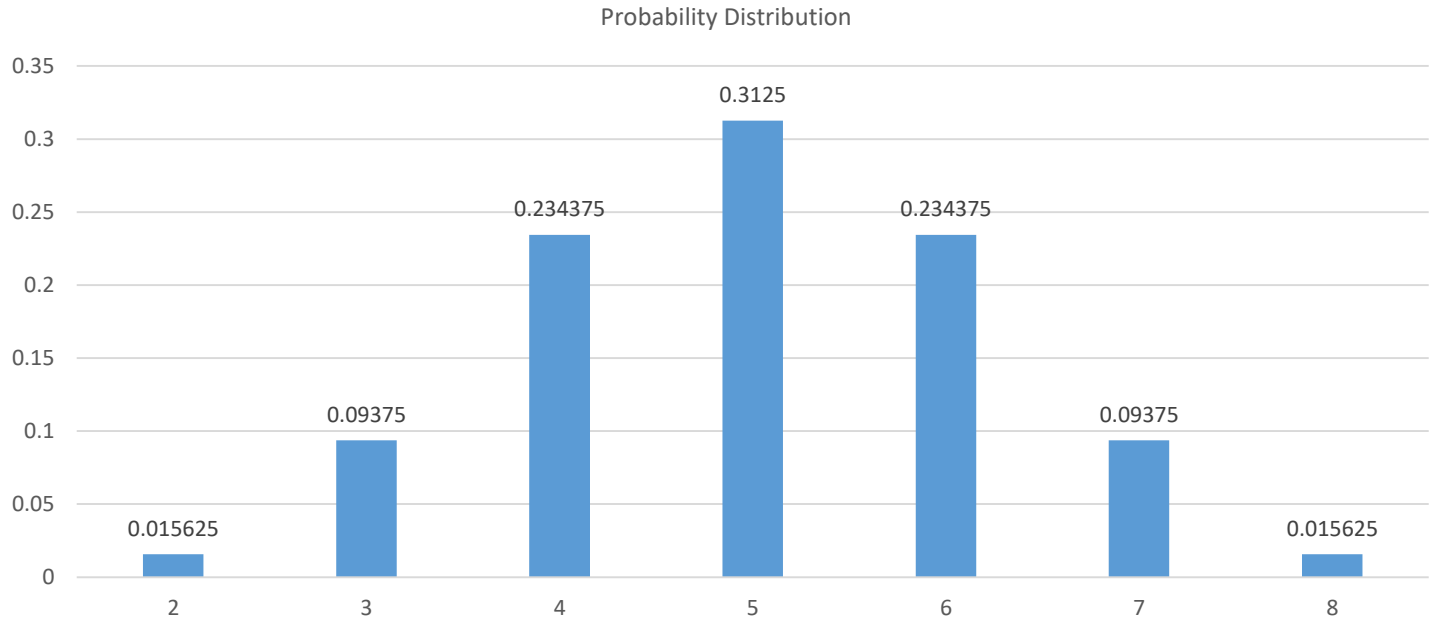
Compare the distribution shapes

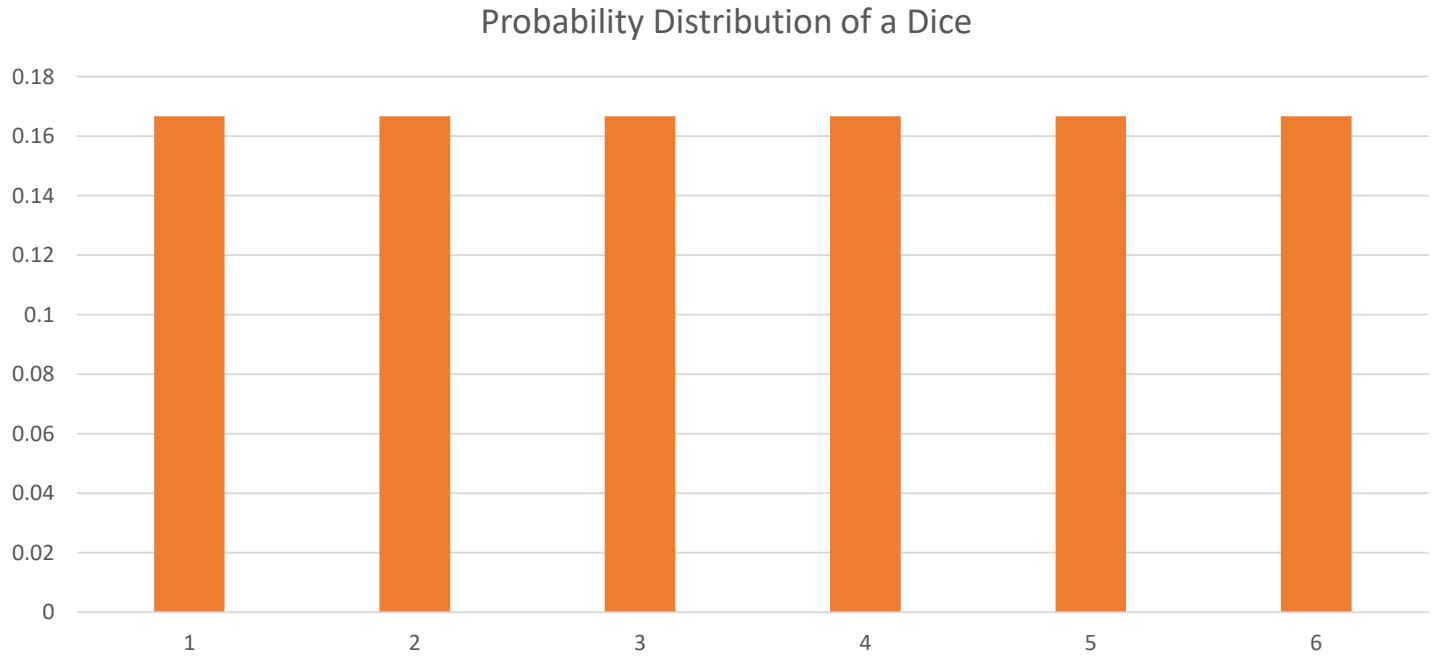
Symmetric?

Mode?

Median?

Standard Deviation Size?





Create the probability distribution for this situation.

x	1	2	3	4
Freq	1	3	3	1

Create the probability distribution for this situation.

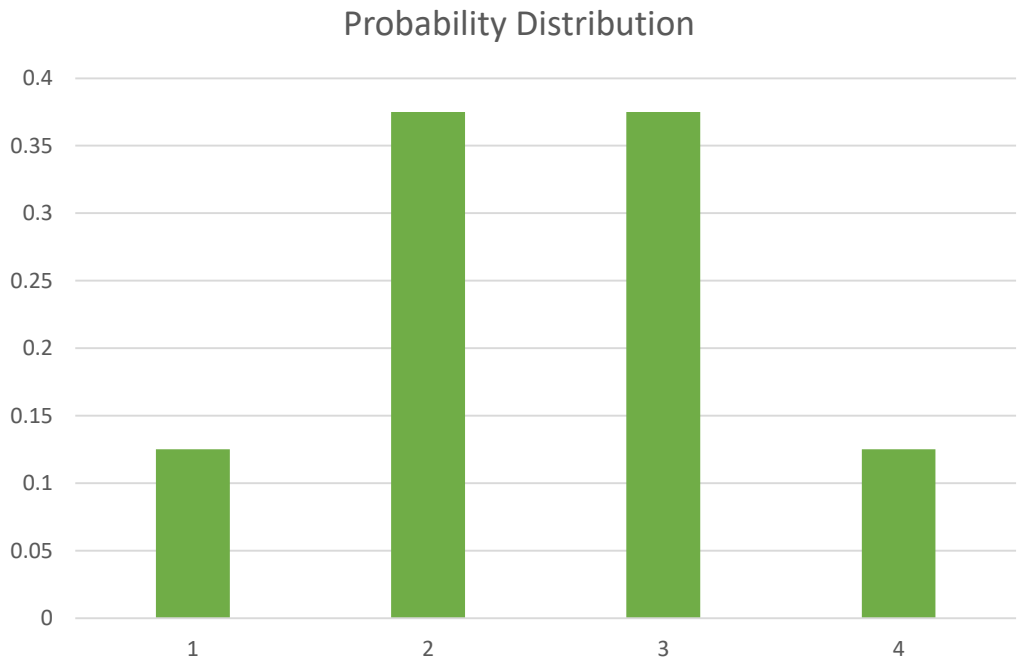
x	1	2	3	4
Freq	1	3	3	1
P(x)	1/8	3/8	3/8	1/8

Create the probability distribution for this situation.

x	1	2	3	4
Freq	1	3	3	1
P(x)	1/8	3/8	3/8	1/8
P(x)	0.125	0.375	0.375	0.125

Create the probability distribution for this situation.

x	1	2	3	4
Freq	1	3	3	1
P(x)	1/8	3/8	3/8	1/8
P(x)	0.125	0.375	0.375	0.125

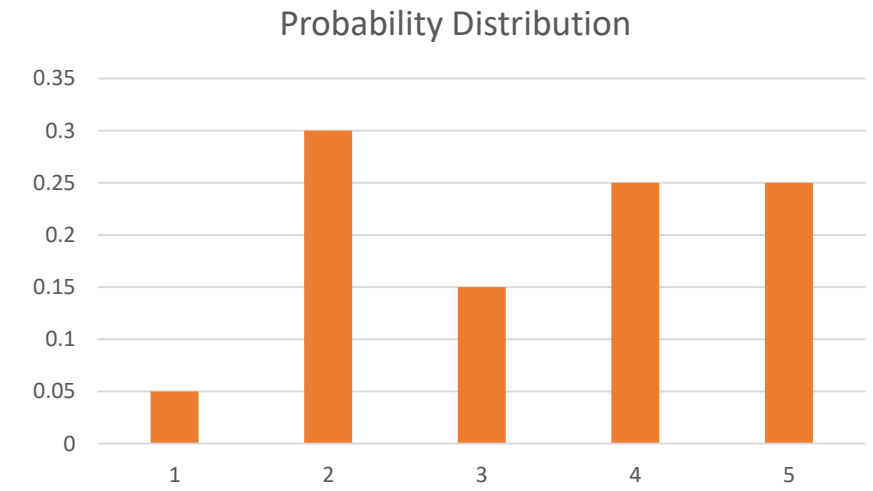


Expected Value

$$E(X) = \sum x \times P(x)$$

Calculate the expected value for this distribution:

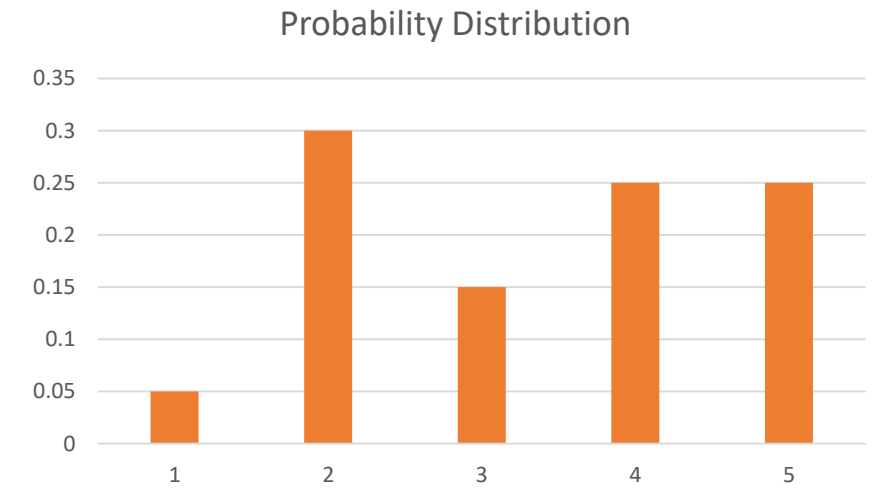
x	12	14	16	18	20
P(x)	0.05	0.3	0.15	0.25	0.25



$$E(X) = \sum x \times P(x)$$

Calculate the expected value for this distribution:

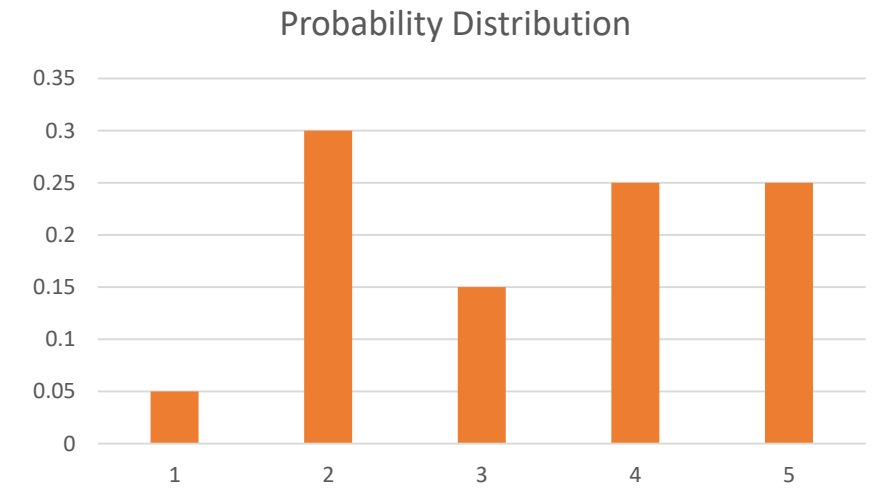
x	12	14	16	18	20
P(x)	0.05	0.3	0.15	0.25	0.25
X*P(x)					



$$E(X) = \sum x \times P(x)$$

Calculate the expected value for this distribution:

x	12	14	16	18	20
P(x)	0.05	0.3	0.15	0.25	0.25
X*P(x)	0.6	4.2	2.4	4.5	5

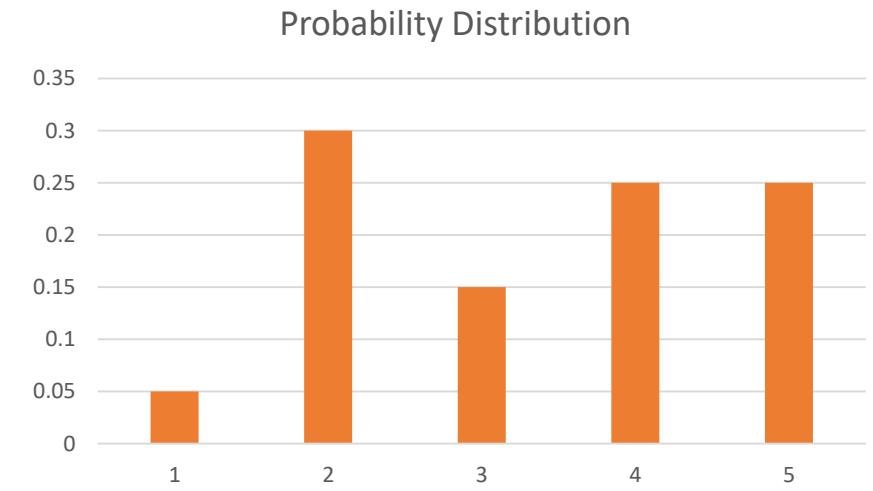


$$E(X) = \sum x \times P(x)$$

$$= 0.6 + 4.2 + 2.4 + 4.5 + 5$$

Calculate the expected value for this distribution:

x	12	14	16	18	20
P(x)	0.05	0.3	0.15	0.25	0.25
X*P(x)	0.6	4.2	2.4	4.5	5



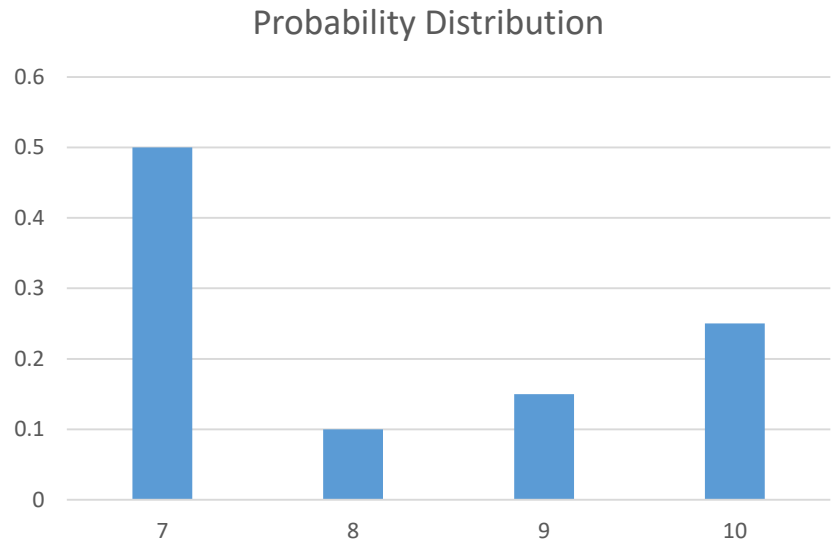
$$E(X) = \sum x \times P(x)$$

$$= 0.6 + 4.2 + 2.4 + 4.5 + 5$$

$$= 16.7$$

Calculate the expected value for this distribution:

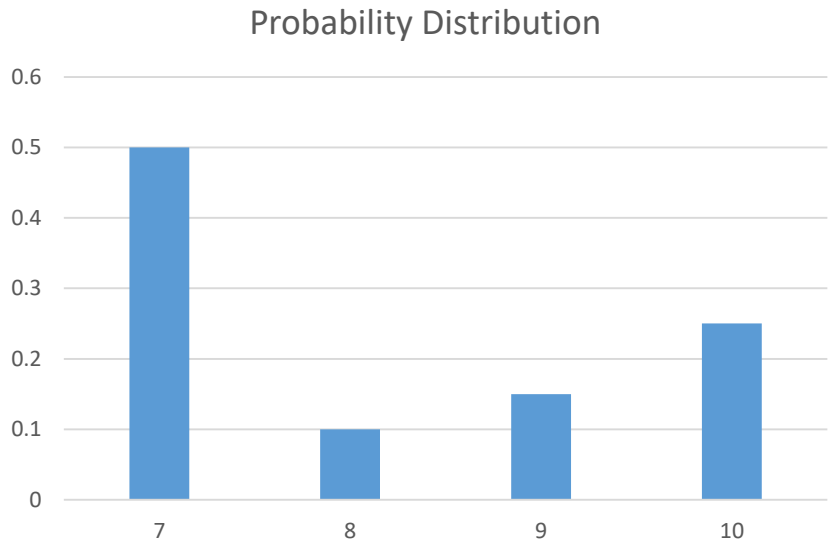
x	7	8	9	10
P(x)	0.5	0.1	0.15	0.25



$$E(X) = \sum x \times P(x)$$

Calculate the expected value for this distribution:

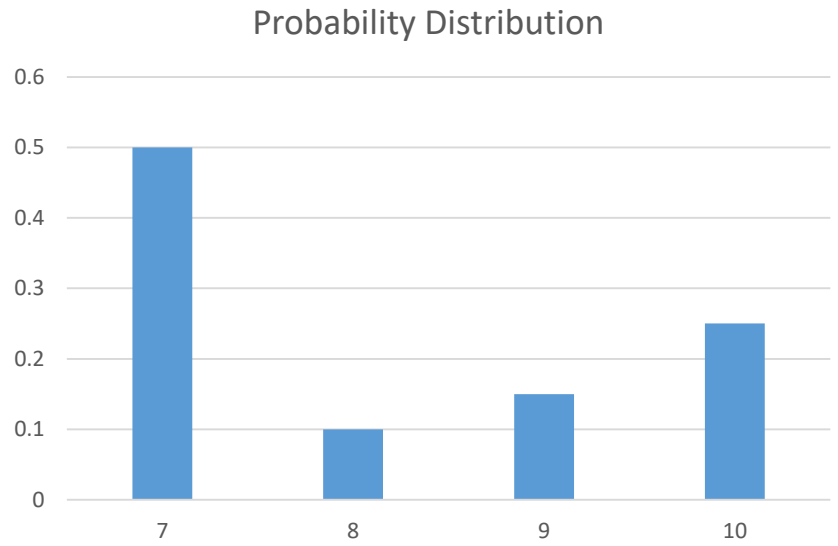
x	7	8	9	10
P(x)	0.5	0.1	0.15	0.25
X*P(x)				



$$E(X) = \sum x \times P(x)$$

Calculate the expected value for this distribution:

x	7	8	9	10
P(x)	0.5	0.1	0.15	0.25
X*P(x)	3.5	0.8	1.35	2.5

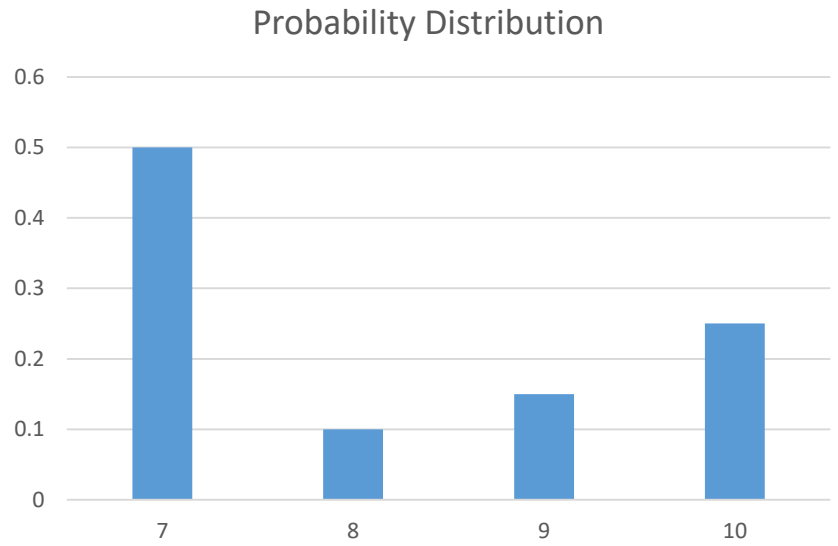


$$E(X) = \sum x \times P(x)$$

$$= 3.5 + 0.8 + 1.35 + 2.5$$

Calculate the expected value for this distribution:

x	7	8	9	10
P(x)	0.5	0.1	0.15	0.25
X*P(x)	3.5	0.8	1.35	2.5



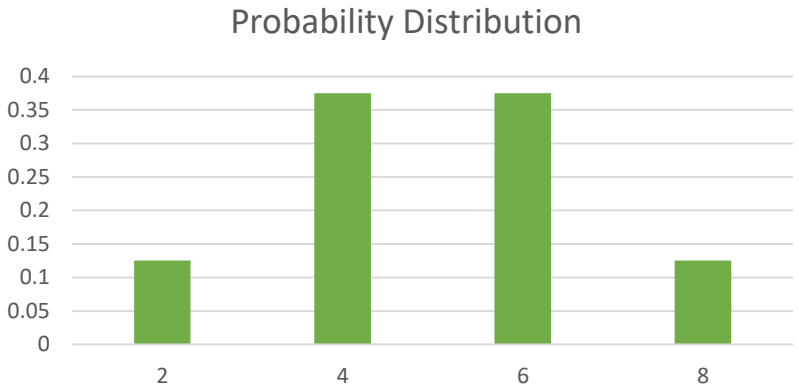
$$E(X) = \sum x \times P(x)$$

$$= 3.5 + 0.8 + 1.35 + 2.5$$

$$= 8.15$$

Calculate the Expected Value for this probability distribution.

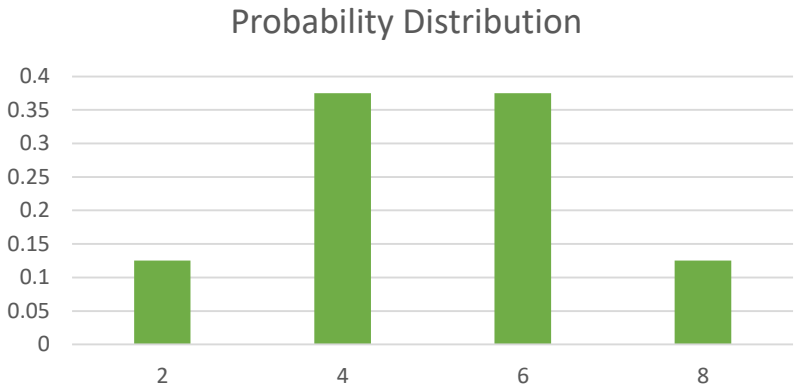
x	2	4	6	8
Freq	1	3	3	1
P(x)	1/8	3/8	3/8	1/8



$$E(X) = \sum x \times P(x)$$

Calculate the Expected Value for this probability distribution.

x	2	4	6	8
Freq	1	3	3	1
P(x)	1/8	3/8	3/8	1/8



$$E(X) = \sum x \times P(x)$$

$$= 2 \times \frac{1}{8} + 4 \times \frac{3}{8} + 6 \times \frac{3}{8} + 8 \times \frac{1}{8}$$

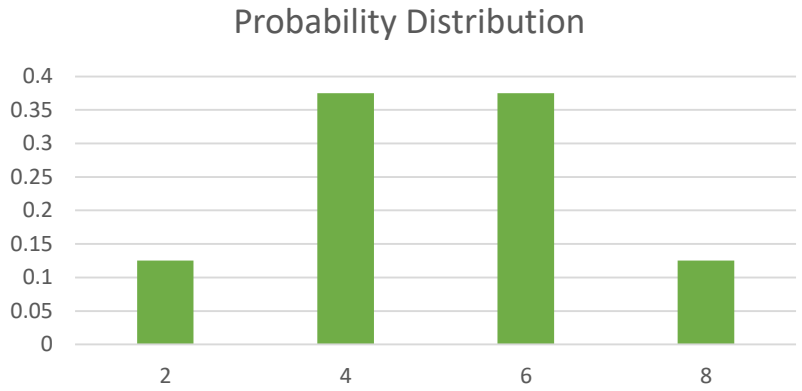
$$= \frac{2}{8} + \frac{12}{8} + \frac{18}{8} + \frac{8}{8}$$

$$= \frac{2 + 12 + 18 + 8}{8}$$

Now, let's look at the mean of this distribution.

Calculate the Expected Value for this probability distribution.

x	2	4	6	8
Freq	1	3	3	1
P(x)	1/8	3/8	3/8	1/8



$$E(X) = \sum x \times P(x)$$

$$= 2 \times \frac{1}{8} + 4 \times \frac{3}{8} + 6 \times \frac{3}{8} + 8 \times \frac{1}{8}$$

$$= \frac{2}{8} + \frac{12}{8} + \frac{18}{8} + \frac{8}{8}$$

$$= \frac{2 + 12 + 18 + 8}{8}$$

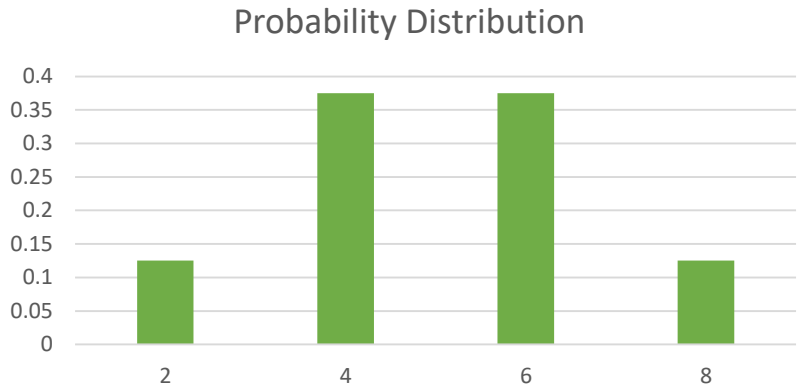
$$\bar{x} = \frac{\sum f \times x}{\sum f}$$

$$= \frac{2 \times 1 + 4 \times 3 + 6 \times 3 + 8 \times 1}{8}$$

$$= \frac{2 + 12 + 18 + 8}{8}$$

Calculate the Expected Value for this probability distribution.

x	2	4	6	8
Freq	1	3	3	1
P(x)	1/8	3/8	3/8	1/8



$$E(X) = \sum x \times P(x)$$

$$= 2 \times \frac{1}{8} + 4 \times \frac{3}{8} + 6 \times \frac{3}{8} + 8 \times \frac{1}{8}$$

$$= \frac{2}{8} + \frac{12}{8} + \frac{18}{8} + \frac{8}{8}$$

$$= \frac{2 + 12 + 18 + 8}{8}$$

$$= \frac{40}{8} \quad = 5$$

$$\bar{x} = \frac{\sum f \times x}{\sum f}$$

$$= \frac{2 \times 1 + 4 \times 3 + 6 \times 3 + 8 \times 1}{8}$$

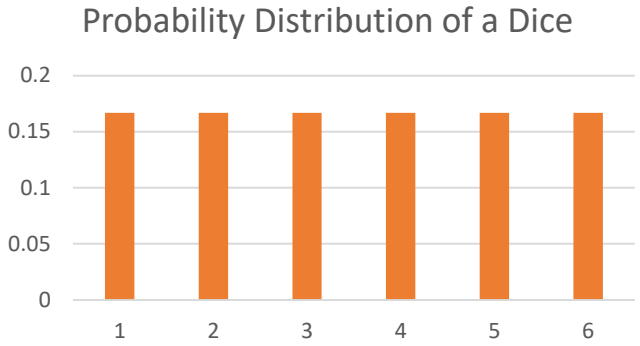
$$= \frac{2 + 12 + 18 + 8}{8}$$

$$= \frac{40}{8}$$

$$= 5$$

Calculate the expected value for this distribution:

x	1	2	3	4	5	6
P(x)	1/6	1/6	1/6	1/6	1/6	1/6

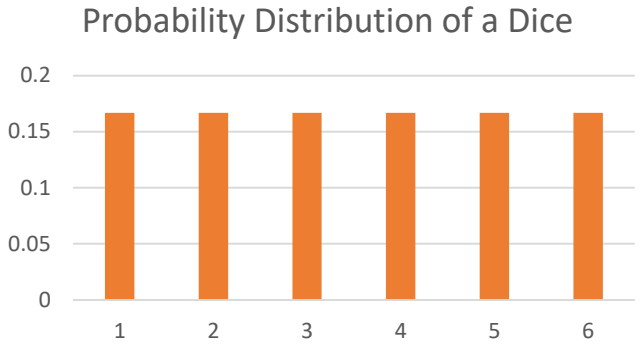


$$E(X) = \sum x \times P(x)$$

$$= 1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6}$$

Calculate the expected value for this distribution:

x	1	2	3	4	5	6
P(x)	1/6	1/6	1/6	1/6	1/6	1/6



$$E(X) = \sum x \times P(x)$$

$$= 1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6}$$

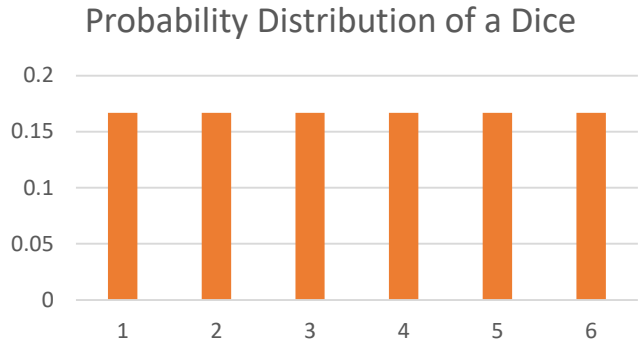
$$= \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6}$$

$$= \frac{21}{6}$$

$$= 3.5$$

Calculate the expected value for this distribution:

x	1	2	3	4	5	6
P(x)	1/6	1/6	1/6	1/6	1/6	1/6



$$E(X) = \sum x \times P(x)$$

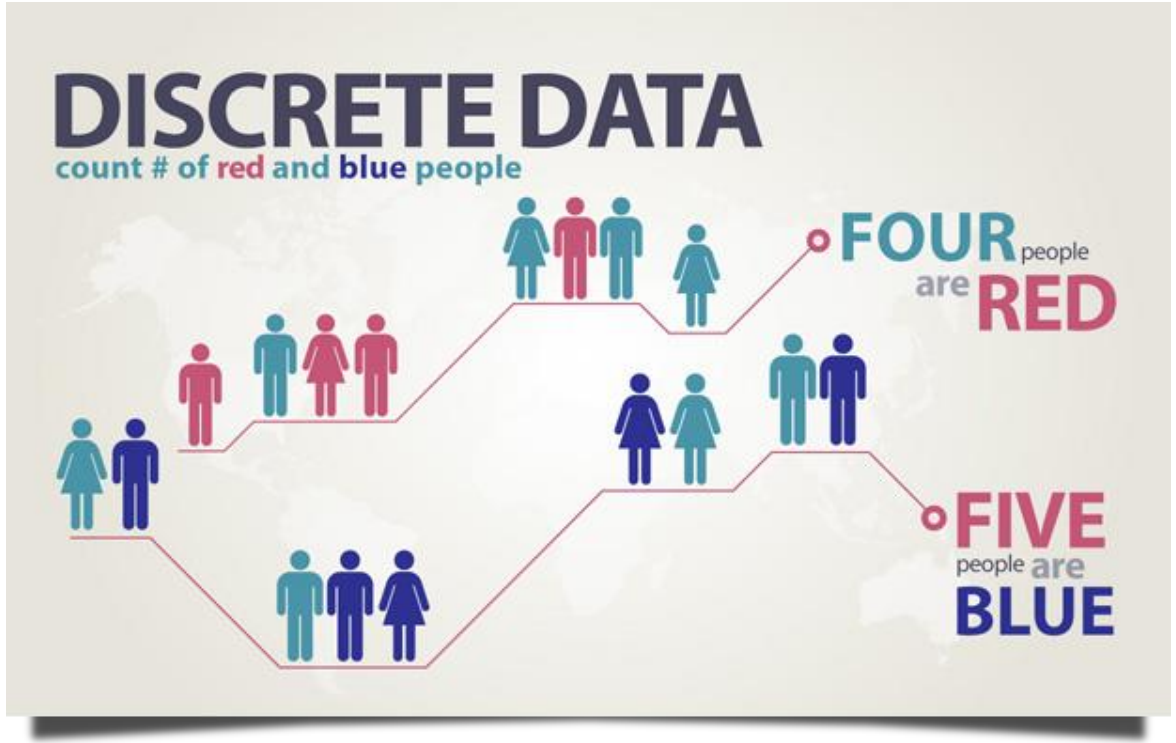
$$= 1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6}$$

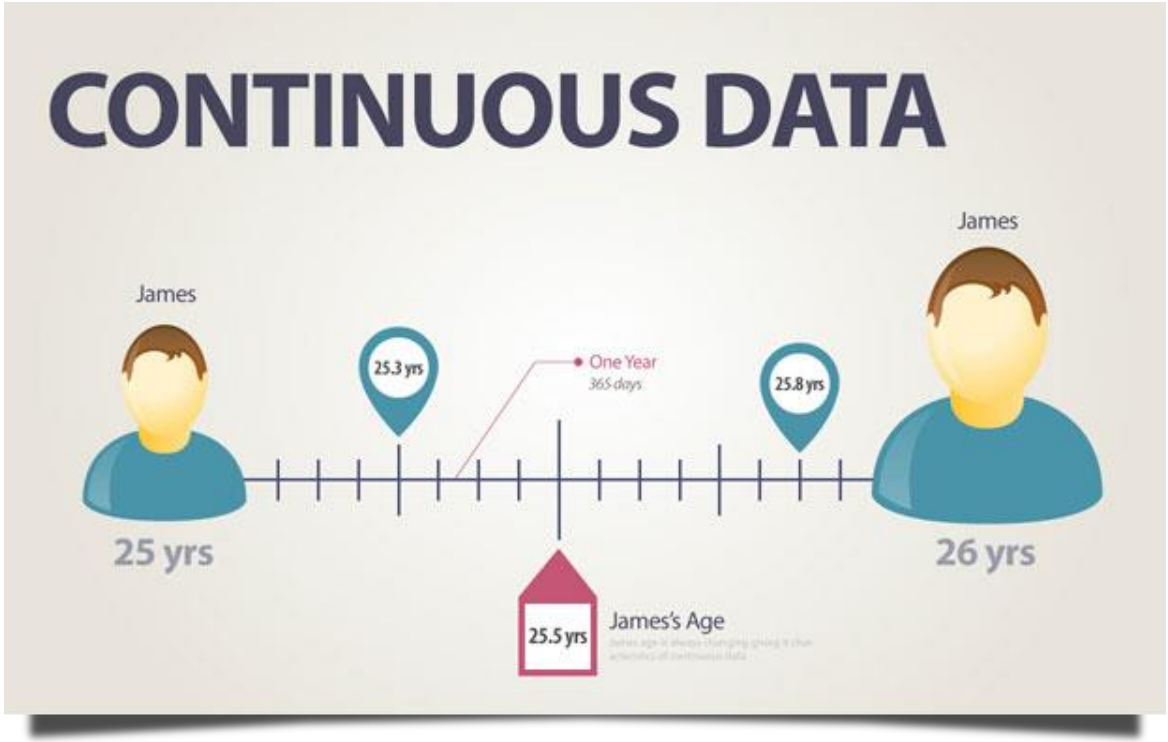
$$= \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6}$$

$$= \frac{21}{6}$$

$$= 3.5$$

There is a shortcut to find the expected value for a uniform probability distribution:
E(X) = (min of x + max of x)/2





Discrete Data results when the number of possible values is either a finite number or a countable number.

COUNTED

Continuous Data results from infinitely many possible values that correspond to some continuous scale that covers a range of values without gaps, interruptions or jumps.

MEASURED

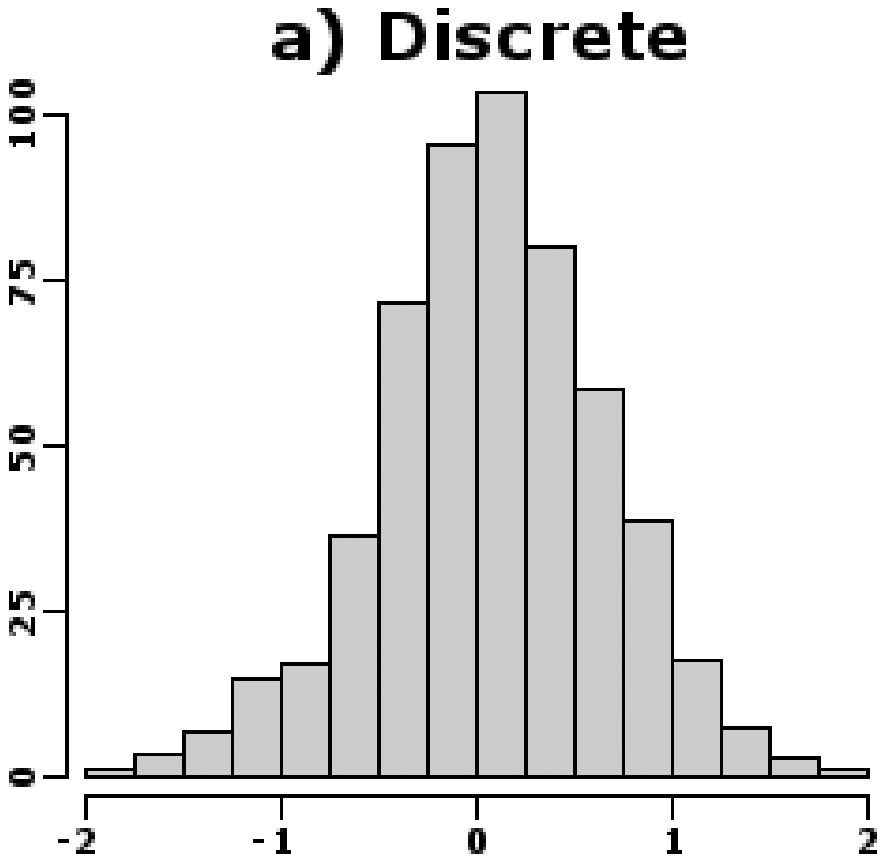
Examples

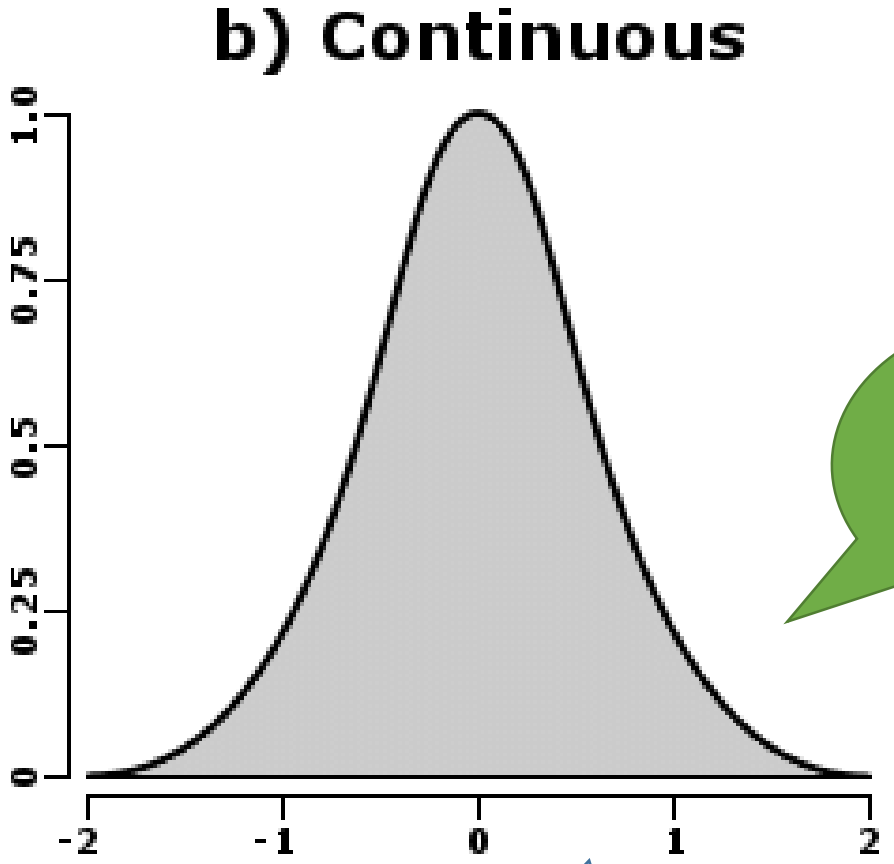
Discrete

- # of eggs in a basket
- # of kids in a class
- # of Facebook likes
- # of diaper changes in a day
- # of wins in a season
- # of votes in an election

Continuous

- Weight difference to 8 decimals before and after cookie binge.
- Wind speed
- Water temperature
- Volts of electricity





Last Unit

Probability Distribution

This Unit

Normal Distribution

HISTOGRAM

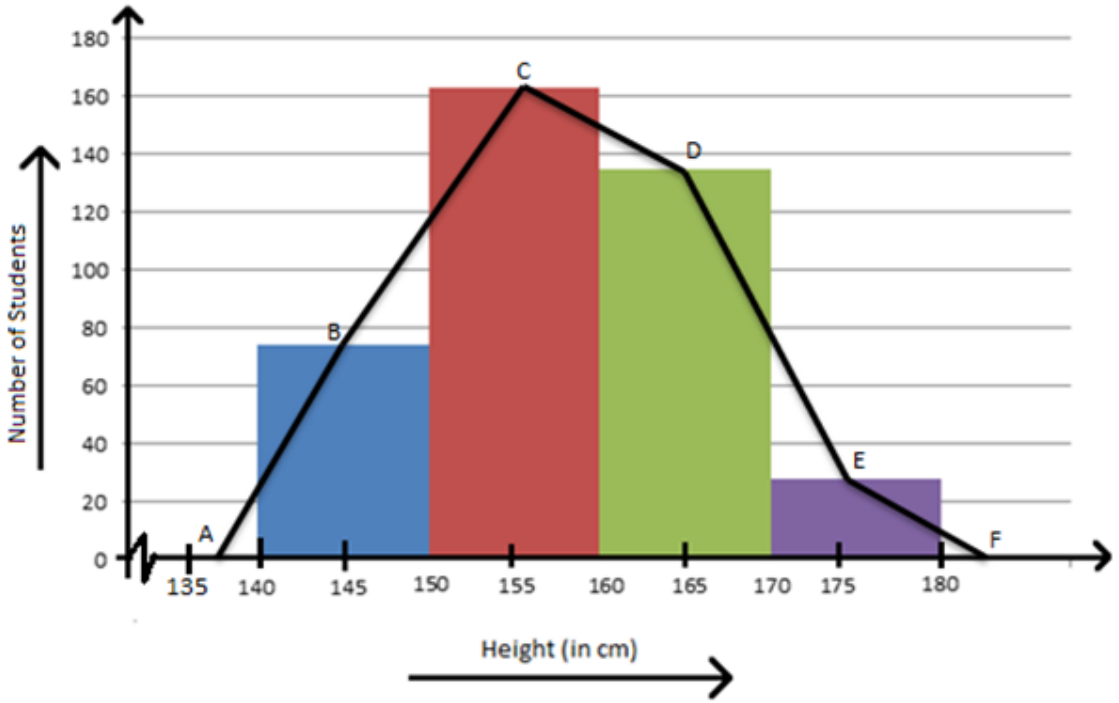
Tourists

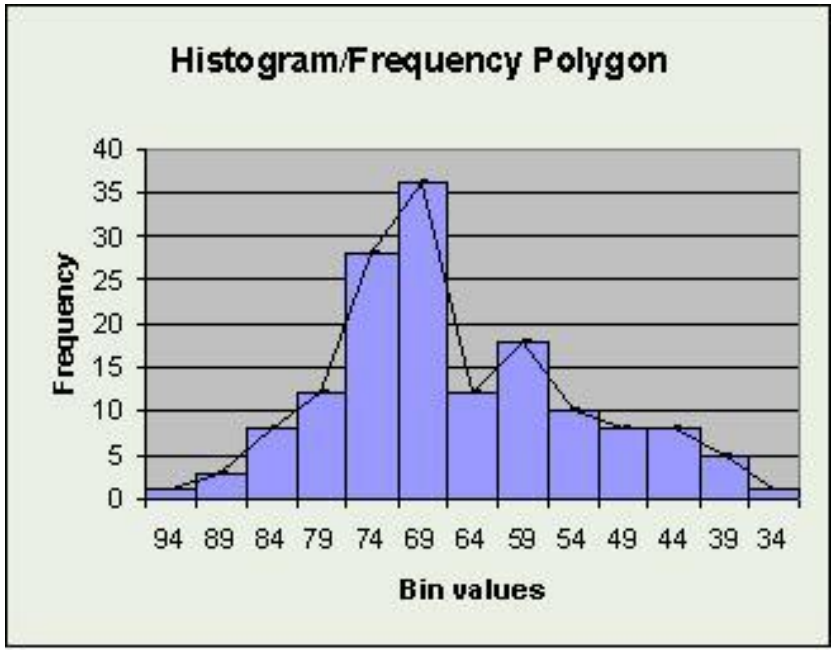
Over the past 4 years the number of tourists that visit our town has fluctuated.

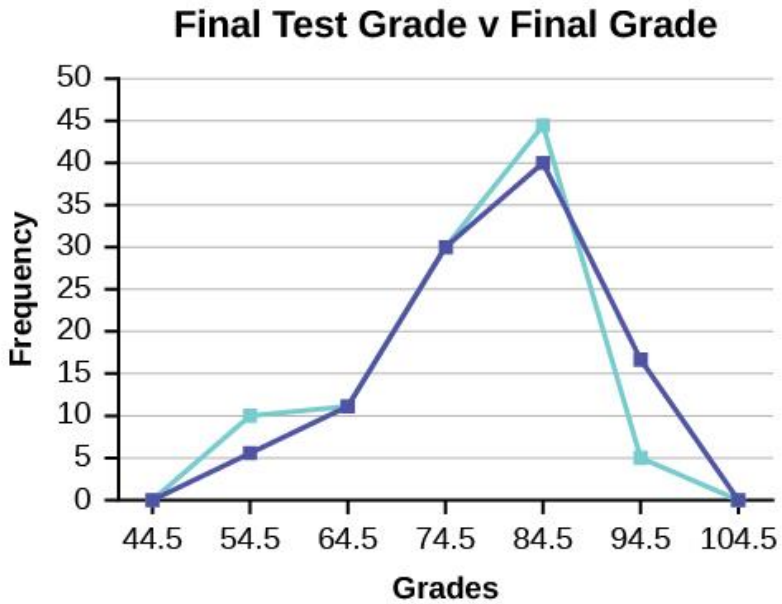
FREQUENCY POLYGON

WOMEN CLIMBERS

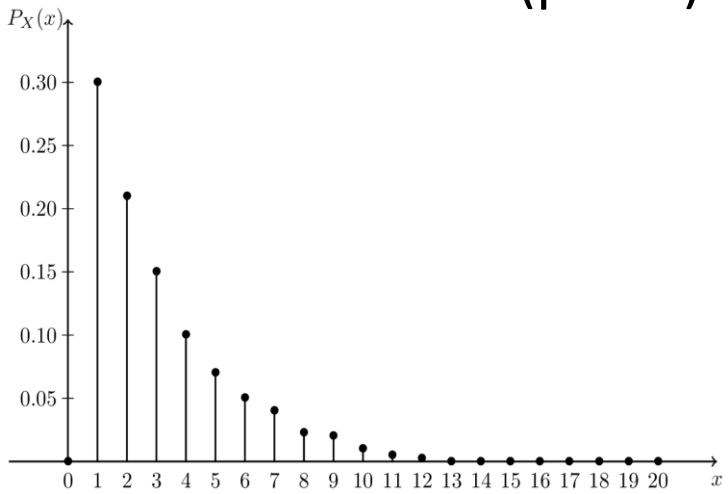
Over the past 4 years the number of women climbers in the northeast has been a declining.


Number of Students
180
160
140
120
100
80
60
40
20
0
A
B
C
D
E
F
135
140
145
150
155
160
165
170
175
180
Height (in cm)

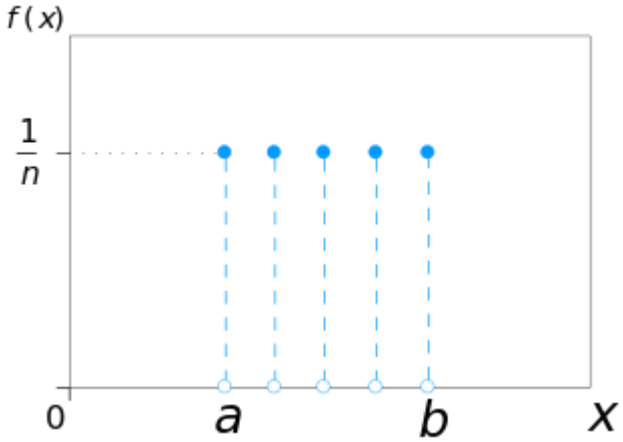

Histogram/Frequency Polygon
Frequency
40
35
30
25
20
15
10
5
0
94
89
84
79
74
69
64
59
54
49
44
39
34
Bin values


Final Test Grade v Final Grade
Frequency
50
45
40
35
30
25
20
15
10
5
0
44.5
54.5
64.5
74.5
84.5
94.5
104.5
Grades

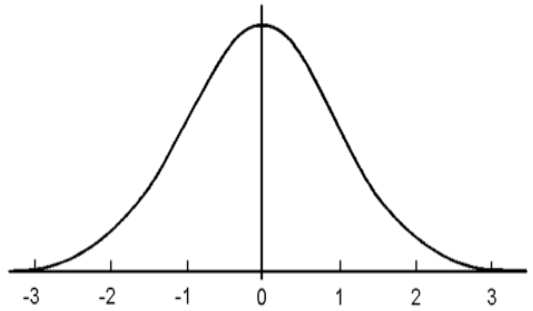
X ~ Geometric(p=0.3)



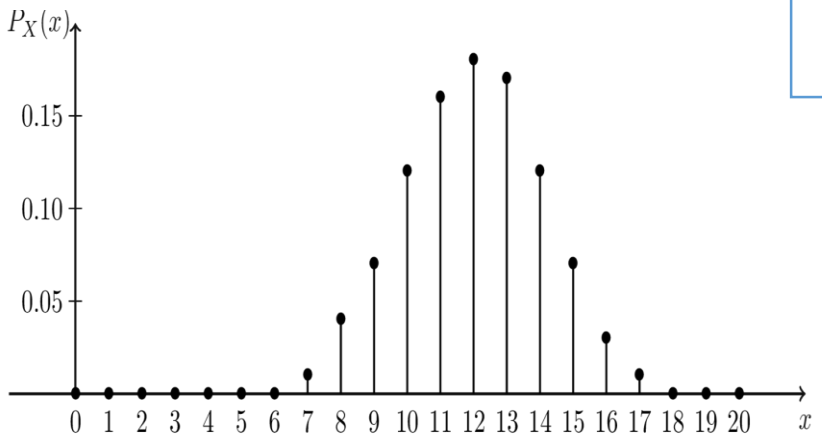
X ~ Uniform (n=5)



X ~ N(0, 1^2)



X ~ Binomial(n=20, p=0.6)



X ~ Hypergeometric (N=80, k=30, n=25)