

Probability Distributions

And Expected Value....

Consider this frequency information:

x	2	3	4	5	6	7	8
Freq	1	6	15	20	15	6	1

What we used last unit to make histograms and to do standard deviation.

$$\bar{x} = \frac{\sum f \times x}{\sum f}$$

$$\sigma = \sqrt{\frac{\sum f (x - \bar{x})^2}{\sum f}}$$

Consider this frequency information:

x	2	3	4	5	6	7	8
Freq	1	6	15	20	15	6	1
P(x)	1/64	6/64	15/64	20/64	15/64	6/64	1/64

We can calculate the probability of each x value.

$$P(x=5)$$

$$\begin{aligned}\sum f &= 1 + 6 + 15 + 20 + 15 + 6 + 1 \\ &= 64\end{aligned}$$

Consider this frequency information:

x	2	3	4	5	6	7	8
Freq	1	6	15	20	15	6	1
$P(x)$	$1/64$	$6/64$	$15/64$	$20/64$	$15/64$	$6/64$	$1/64$
$P(x)$	0.01563	0.09375	0.23438	0.3125	0.23438	0.09375	0.01563

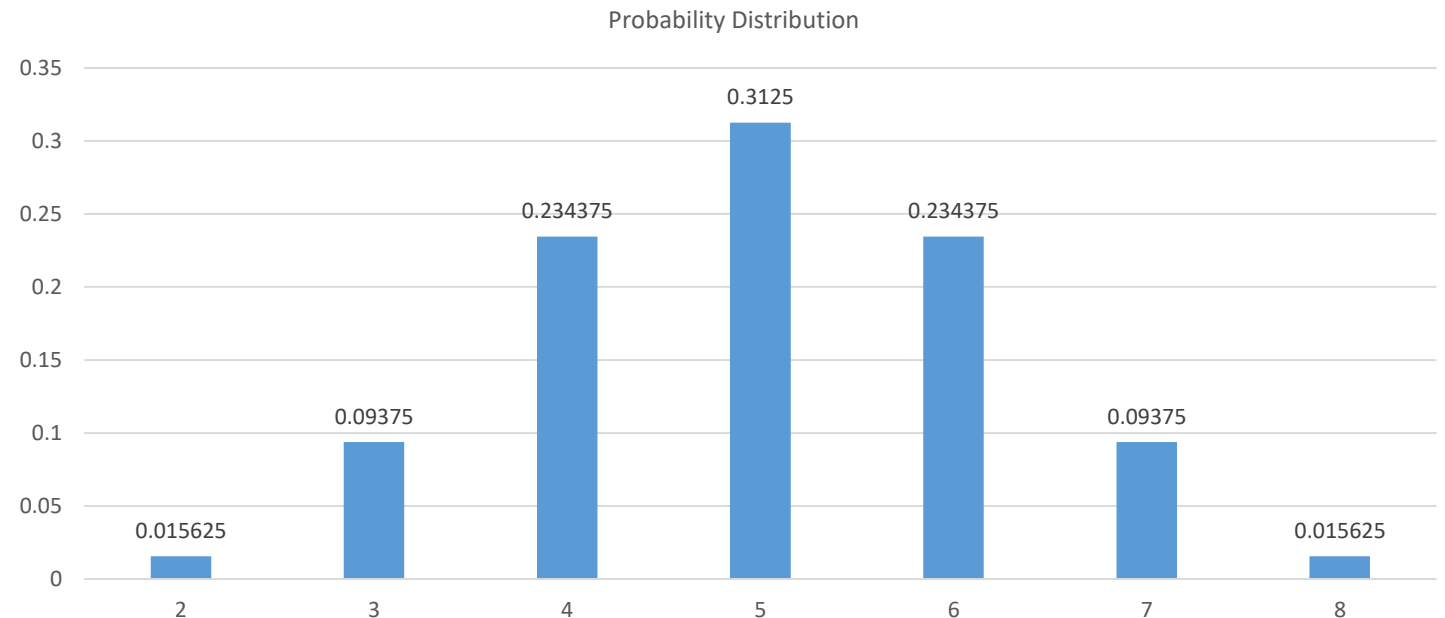
This can also
be expressed
as a decimal.

Consider this frequency information:

x	2	3	4	5	6	7	8
Freq	1	6	15	20	15	6	1
$P(x)$	$1/64$	$6/64$	$15/64$	$20/64$	$15/64$	$6/64$	$1/64$
$P(x)$	0.01563	0.09375	0.23438	0.3125	0.23438	0.09375	0.01563

Then, we can graph the probabilities.

Ta da! A probability distribution.



Create the probability distribution for rolling a dice.

x	1	2	3	4	5	6
P(x)						

In this case, we have no frequencies.

Just calculate the probabilities directly.

Create the probability distribution for rolling a dice.

x	1	2	3	4	5	6
P(x)	1/6	1/6	1/6	1/6	1/6	1/6

Create the probability distribution for rolling a dice.

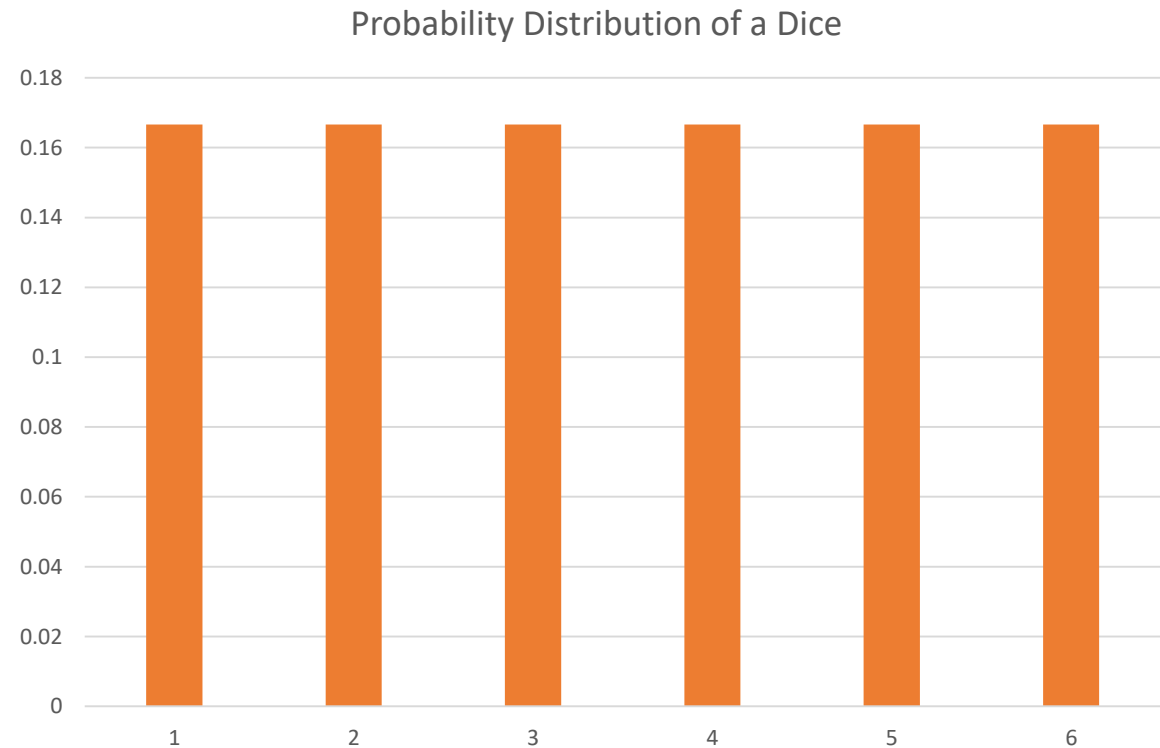
x	1	2	3	4	5	6
P(x)	1/6	1/6	1/6	1/6	1/6	1/6
P(x)	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667

Create the probability distribution for rolling a dice.

x	1	2	3	4	5	6
P(x)	1/6	1/6	1/6	1/6	1/6	1/6
P(x)	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667

Then, we can graph the probabilities.

Ta da! Another probability distribution.



Compare the distribution shapes

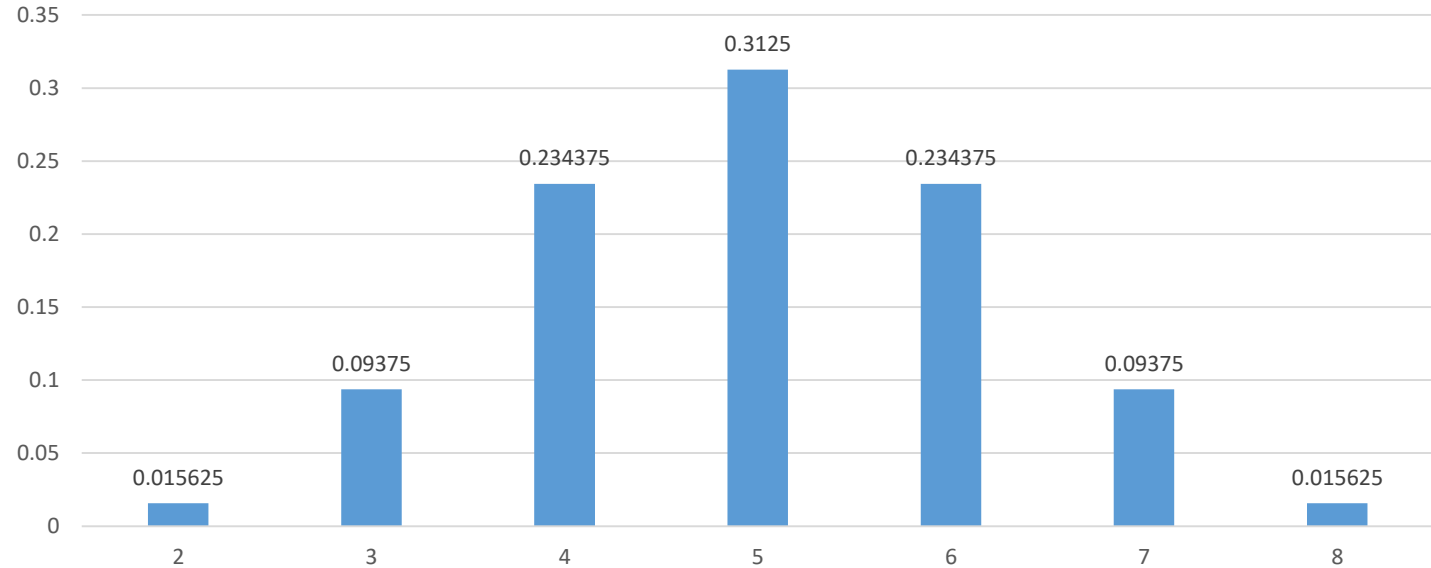
Symmetric?

Mode?

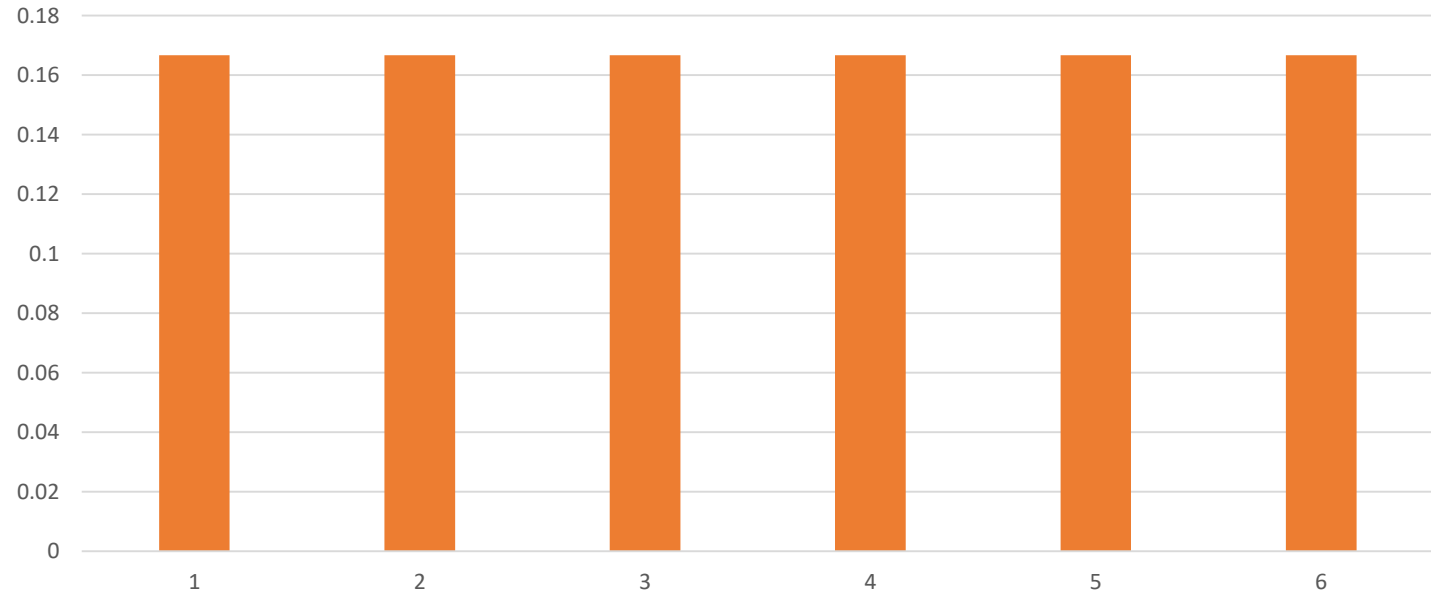
Median?

Standard Deviation Size?

Probability Distribution



Probability Distribution of a Dice



Create the probability distribution for this situation.

x	1	2	3	4
Freq	1	3	3	1

Create the probability distribution for this situation.

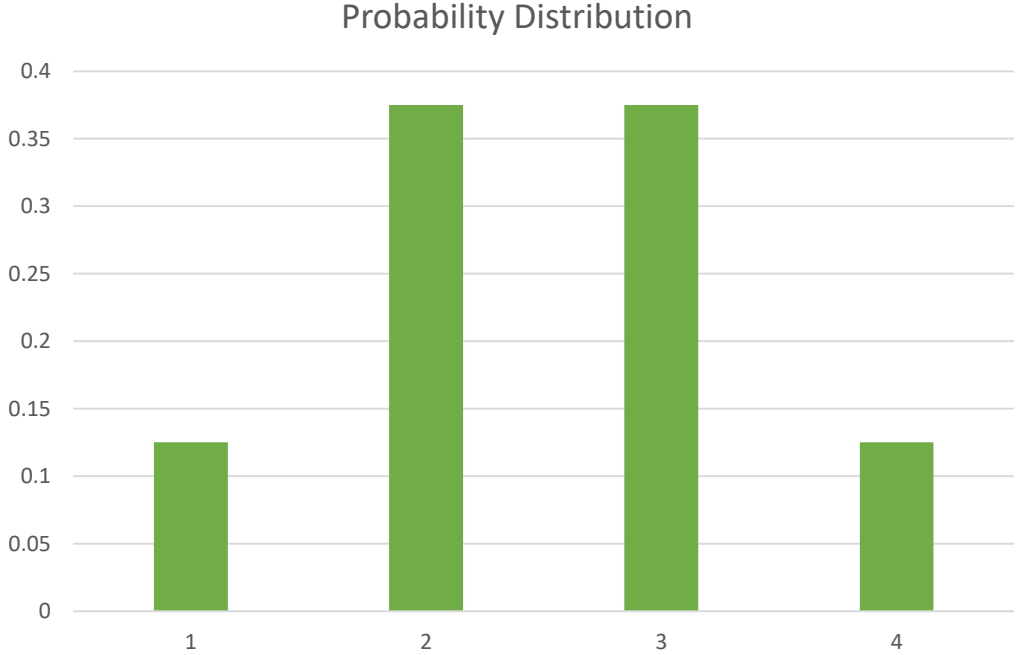
x	1	2	3	4
Freq	1	3	3	1
P(x)	1/8	3/8	3/8	1/8

Create the probability distribution for this situation.

x	1	2	3	4
Freq	1	3	3	1
P(x)	1/8	3/8	3/8	1/8
P(x)	0.125	0.375	0.375	0.125

Create the probability distribution for this situation.

x	1	2	3	4
Freq	1	3	3	1
P(x)	1/8	3/8	3/8	1/8
P(x)	0.125	0.375	0.375	0.125



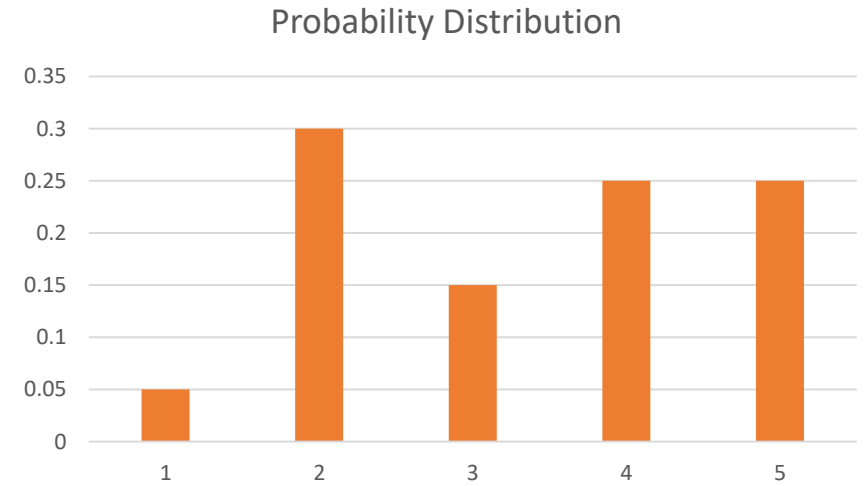


Expected
Value

$$E(X) = \sum x \times P(x)$$

Calculate the expected value for this distribution:

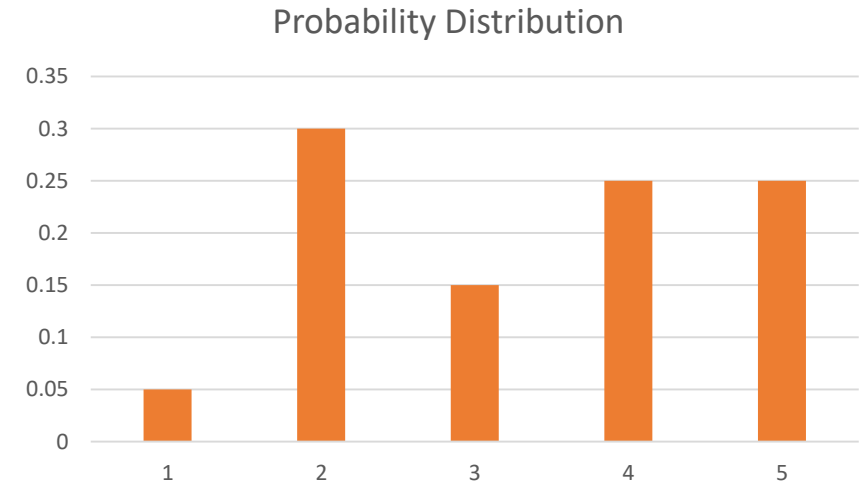
x	12	14	16	18	20
P(x)	0.05	0.3	0.15	0.25	0.25



$$E(X) = \sum x \times P(x)$$

Calculate the expected value for this distribution:

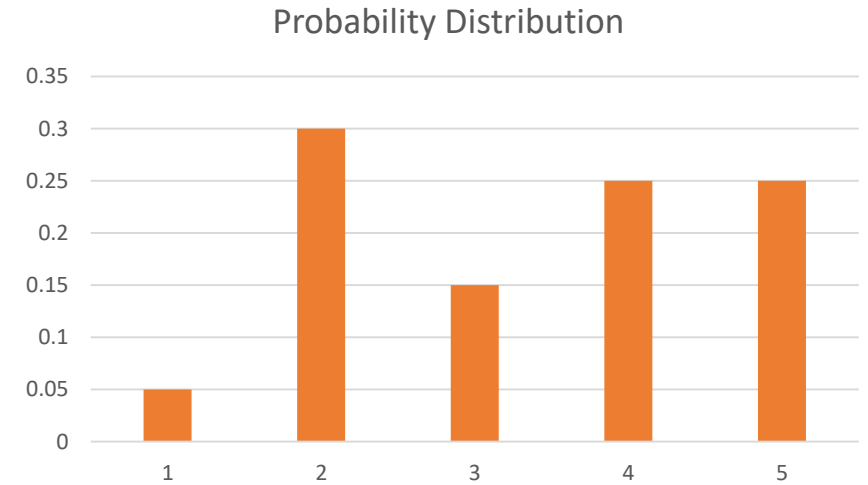
x	12	14	16	18	20
P(x)	0.05	0.3	0.15	0.25	0.25
X*P(x)					



$$E(X) = \sum x \times P(x)$$

Calculate the expected value for this distribution:

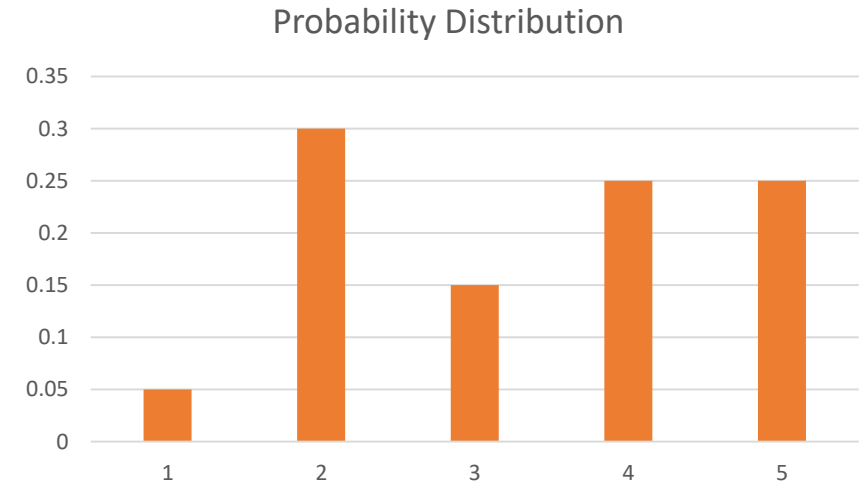
x	12	14	16	18	20
P(x)	0.05	0.3	0.15	0.25	0.25
X*P(x)	0.6	4.2	2.4	4.5	5



$$E(X) = \sum x \times P(x)$$
$$= 0.6 + 4.2 + 2.4 + 4.5 + 5$$

Calculate the expected value for this distribution:

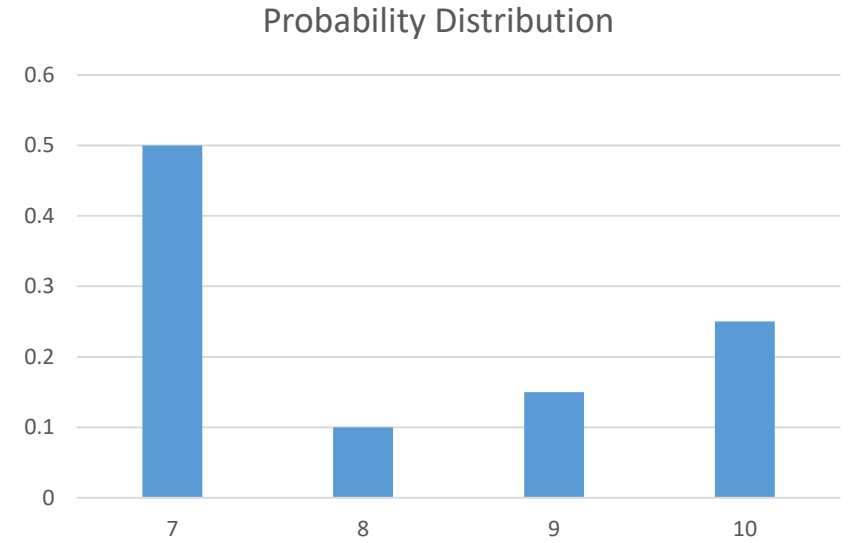
x	12	14	16	18	20
P(x)	0.05	0.3	0.15	0.25	0.25
X*P(x)	0.6	4.2	2.4	4.5	5



$$\begin{aligned} E(X) &= \sum x \times P(x) \\ &= 0.6 + 4.2 + 2.4 + 4.5 + 5 \\ &= 16.7 \end{aligned}$$

Calculate the expected value for this distribution:

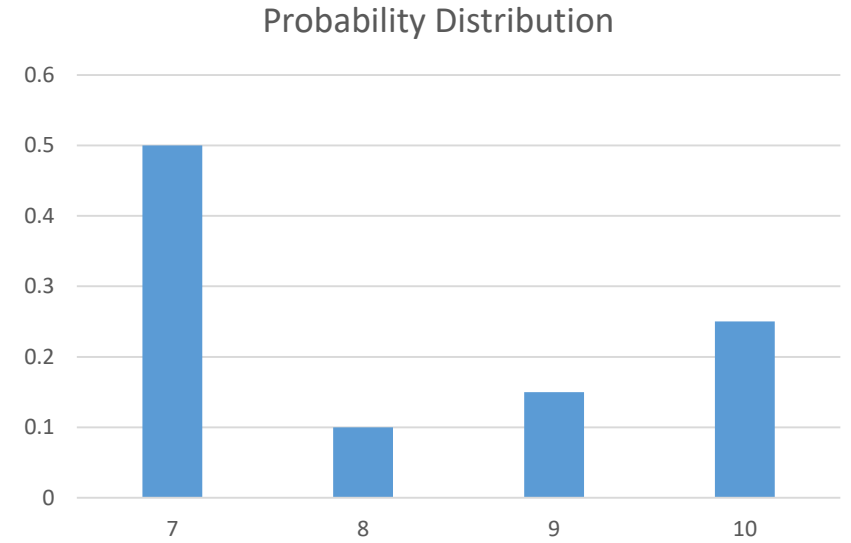
x	7	8	9	10
P(x)	0.5	0.1	0.15	0.25



$$E(X) = \sum x \times P(x)$$

Calculate the expected value for this distribution:

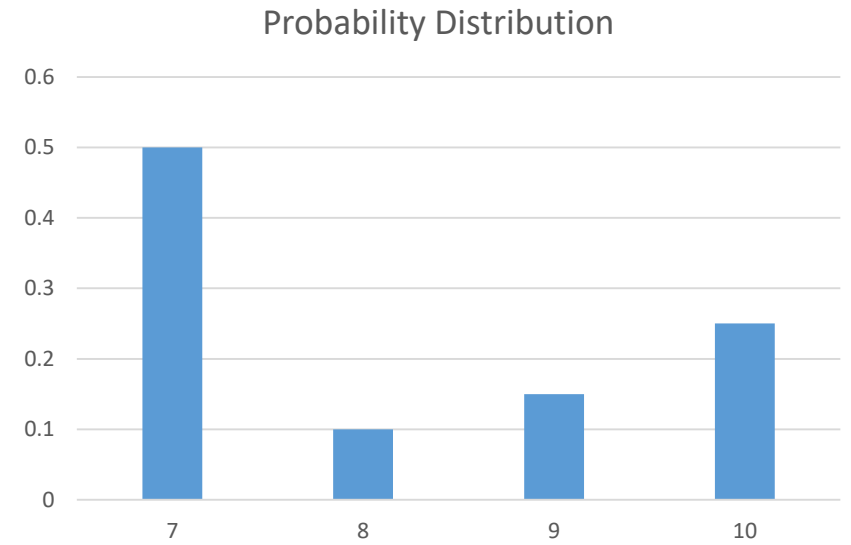
x	7	8	9	10
P(x)	0.5	0.1	0.15	0.25
X*P(x)				



$$E(X) = \sum x \times P(x)$$

Calculate the expected value for this distribution:

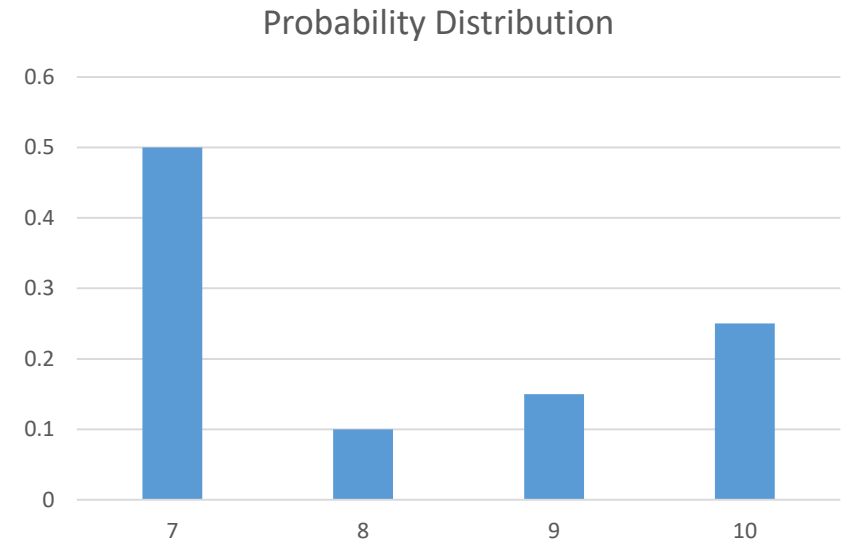
x	7	8	9	10
P(x)	0.5	0.1	0.15	0.25
X*P(x)	3.5	0.8	1.35	2.5



$$E(X) = \sum x \times P(x)$$
$$= 3.5 + 0.8 + 1.35 + 2.5$$

Calculate the expected value for this distribution:

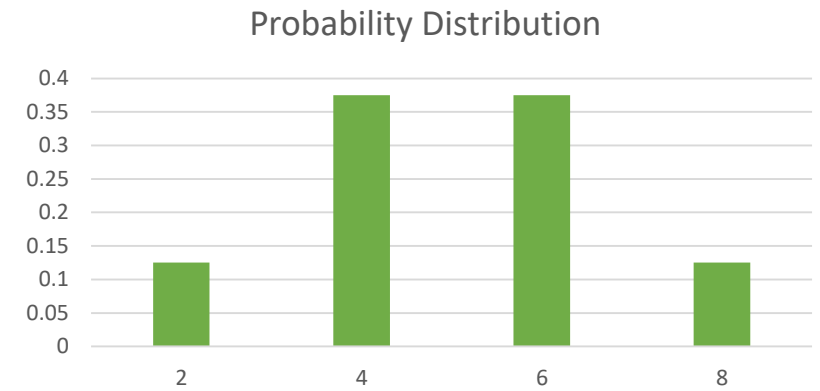
x	7	8	9	10
P(x)	0.5	0.1	0.15	0.25
X*P(x)	3.5	0.8	1.35	2.5



$$\begin{aligned} E(X) &= \sum x \times P(x) \\ &= 3.5 + 0.8 + 1.35 + 2.5 \\ &= 8.15 \end{aligned}$$

Calculate the Expected Value for this probability distribution.

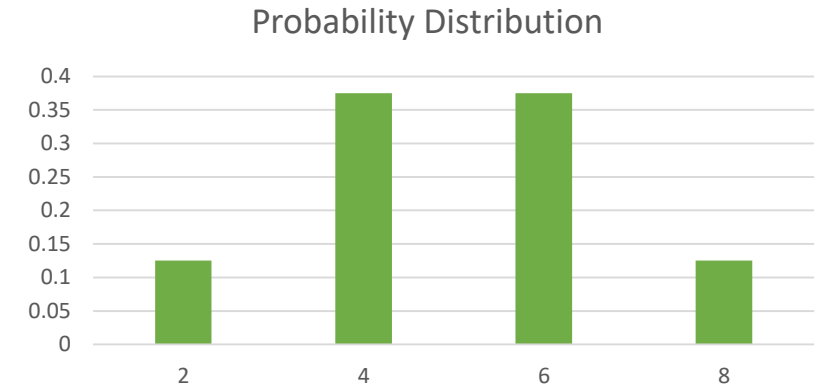
x	2	4	6	8
Freq	1	3	3	1
P(x)	1/8	3/8	3/8	1/8



$$E(X) = \sum x \times P(x)$$

Calculate the Expected Value for this probability distribution.

x	2	4	6	8
Freq	1	3	3	1
P(x)	1/8	3/8	3/8	1/8

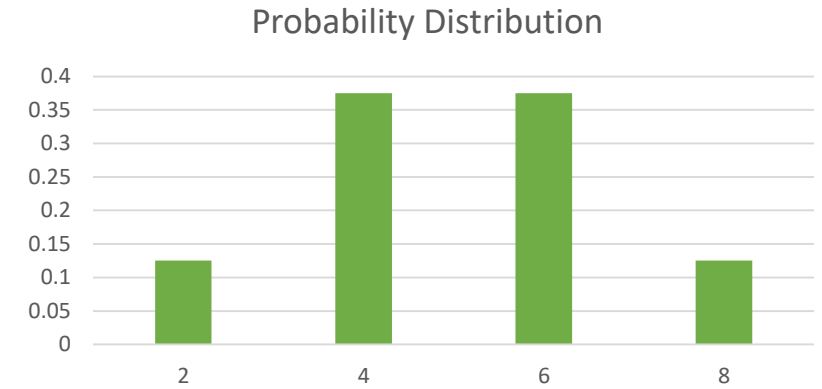


$$\begin{aligned} E(X) &= \sum x \times P(x) \\ &= 2 \times \frac{1}{8} + 4 \times \frac{3}{8} + 6 \times \frac{3}{8} + 8 \times \frac{1}{8} \\ &= \frac{2}{8} + \frac{12}{8} + \frac{18}{8} + \frac{8}{8} \\ &= \frac{2 + 12 + 18 + 8}{8} \end{aligned}$$

Now, let's look at the mean of this distribution.

Calculate the Expected Value for this probability distribution.

x	2	4	6	8
Freq	1	3	3	1
P(x)	1/8	3/8	3/8	1/8

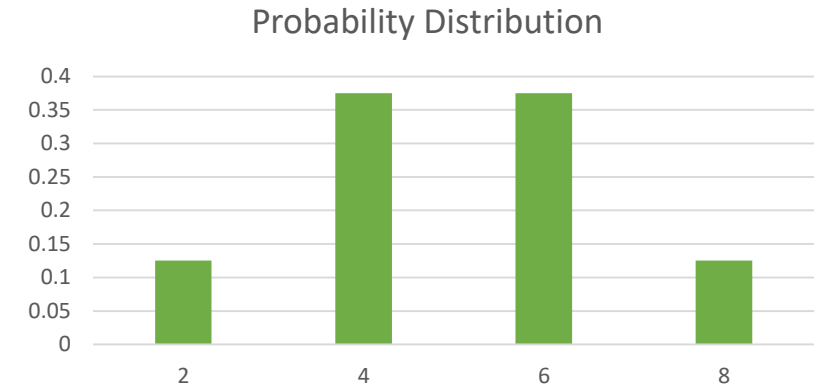


$$\begin{aligned}E(X) &= \sum x \times P(x) \\&= 2 \times \frac{1}{8} + 4 \times \frac{3}{8} + 6 \times \frac{3}{8} + 8 \times \frac{1}{8} \\&= \frac{2}{8} + \frac{12}{8} + \frac{18}{8} + \frac{8}{8} \\&= \frac{2 + 12 + 18 + 8}{8}\end{aligned}$$

$$\begin{aligned}\bar{x} &= \frac{\sum f \times x}{\sum f} \\&= \frac{2 \times 1 + 4 \times 3 + 6 \times 3 + 8 \times 1}{8} \\&= \frac{2 + 12 + 18 + 8}{8}\end{aligned}$$

Calculate the Expected Value for this probability distribution.

x	2	4	6	8
Freq	1	3	3	1
P(x)	1/8	3/8	3/8	1/8

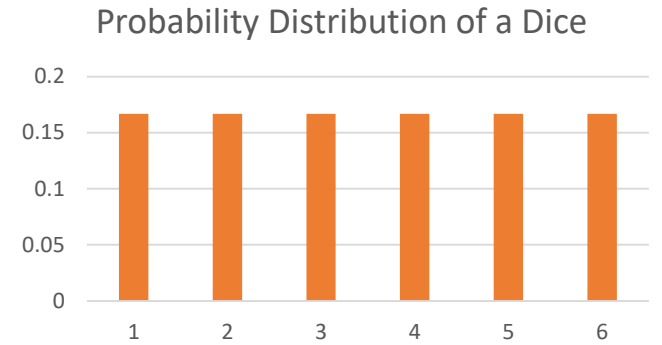


$$\begin{aligned}E(X) &= \sum x \times P(x) \\&= 2 \times \frac{1}{8} + 4 \times \frac{3}{8} + 6 \times \frac{3}{8} + 8 \times \frac{1}{8} \\&= \frac{2}{8} + \frac{12}{8} + \frac{18}{8} + \frac{8}{8} \\&= \frac{2 + 12 + 18 + 8}{8} \\&= \frac{40}{8} = 5\end{aligned}$$

$$\begin{aligned}\bar{x} &= \frac{\sum f \times x}{\sum f} \\&= \frac{2 \times 1 + 4 \times 3 + 6 \times 3 + 8 \times 1}{8} \\&= \frac{2 + 12 + 18 + 8}{8} \\&= \frac{40}{8} \\&= 5\end{aligned}$$

Calculate the expected value for this distribution:

x	1	2	3	4	5	6
P(x)	1/6	1/6	1/6	1/6	1/6	1/6

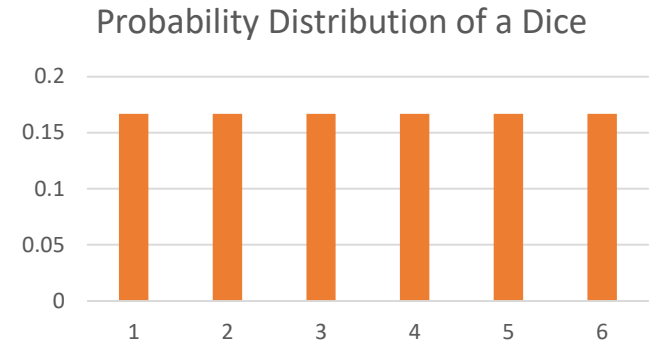


$$E(X) = \sum x \times P(x)$$

$$= 1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6}$$

Calculate the expected value for this distribution:

x	1	2	3	4	5	6
P(x)	1/6	1/6	1/6	1/6	1/6	1/6



$$E(X) = \sum x \times P(x)$$

$$= 1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6}$$

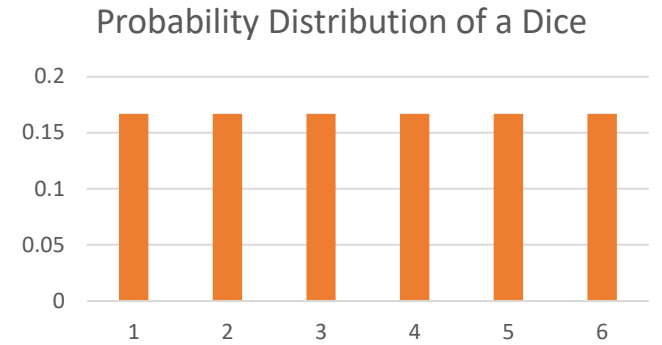
$$= \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6}$$

$$= \frac{21}{6}$$

$$= 3.5$$

Calculate the expected value for this distribution:

x	1	2	3	4	5	6
P(x)	1/6	1/6	1/6	1/6	1/6	1/6



$$E(X) = \sum x \times P(x)$$

$$= 1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6}$$

$$= \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6}$$

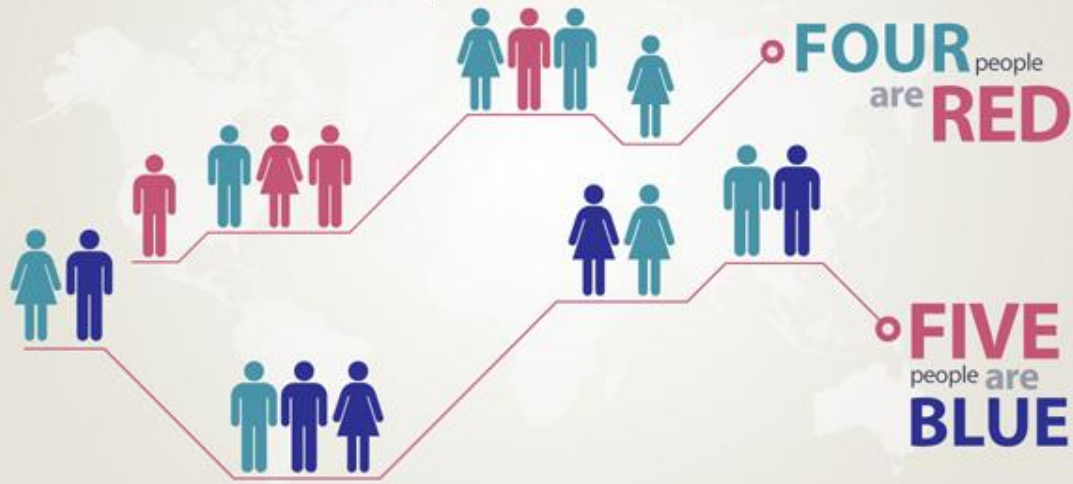
$$= \frac{21}{6}$$

$$= 3.5$$

There is a shortcut to find the expected value for a uniform probability distribution:
 $E(X) = (\text{min of } x + \text{max of } x)/2$

DISCRETE DATA

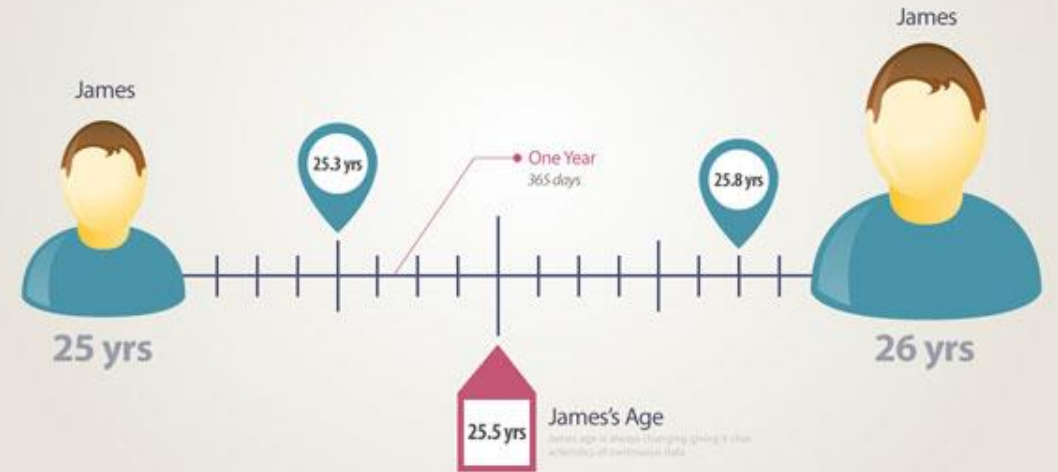
count # of red and blue people



Discrete Data results when the number of possible values is either a finite number or a countable number.

COUNTED

CONTINUOUS DATA



Continuous Data results from infinitely many possible values that correspond to some continuous scale that covers a range of values without gaps, interruptions or jumps.

MEASURED

Examples

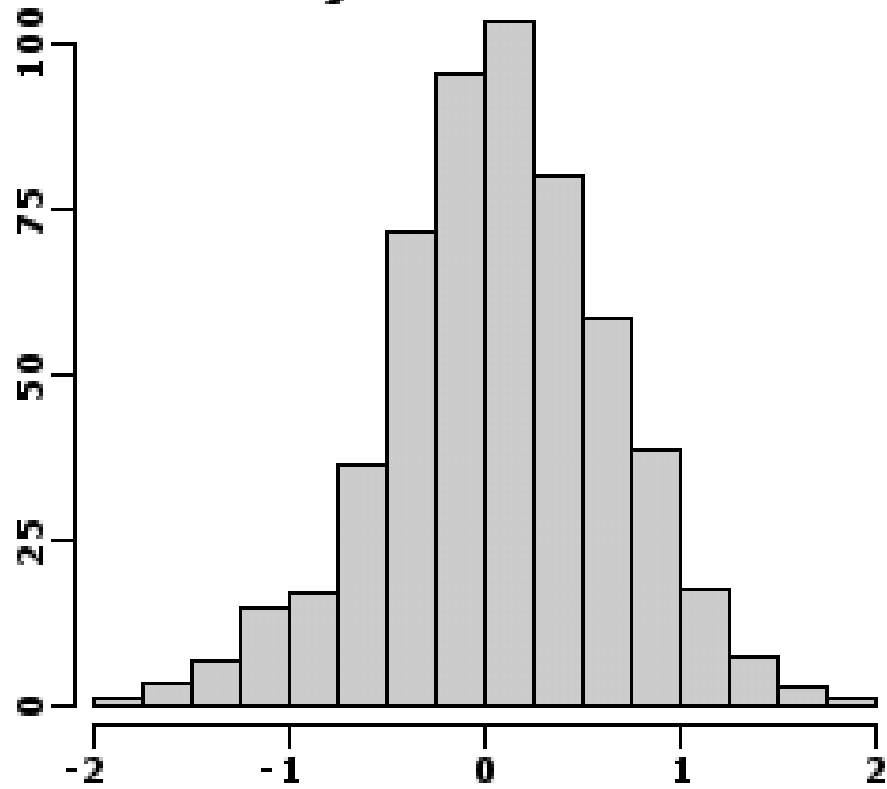
Discrete

- # of eggs in a basket
- # of kids in a class
- # of Facebook likes
- # of diaper changes in a day
- # of wins in a season
- # of votes in an election

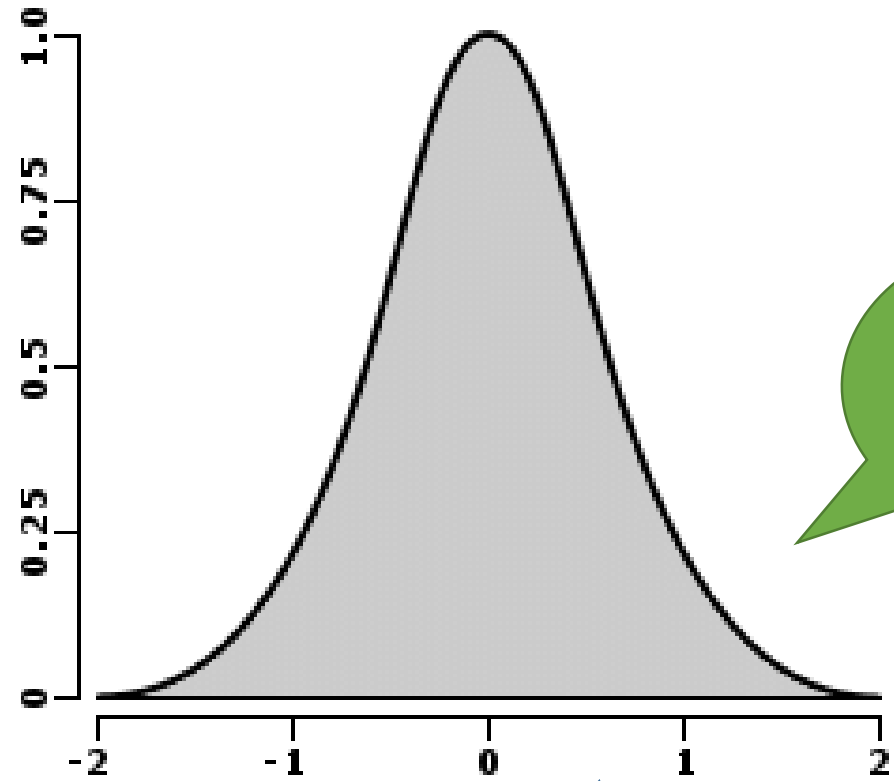
Continuous

- Weight difference to 8 decimals before and after cookie binge.
- Wind speed
- Water temperature
- Volts of electricity

a) Discrete



b) Continuous



Probability
Distribution

This
Unit

Normal
Distribution

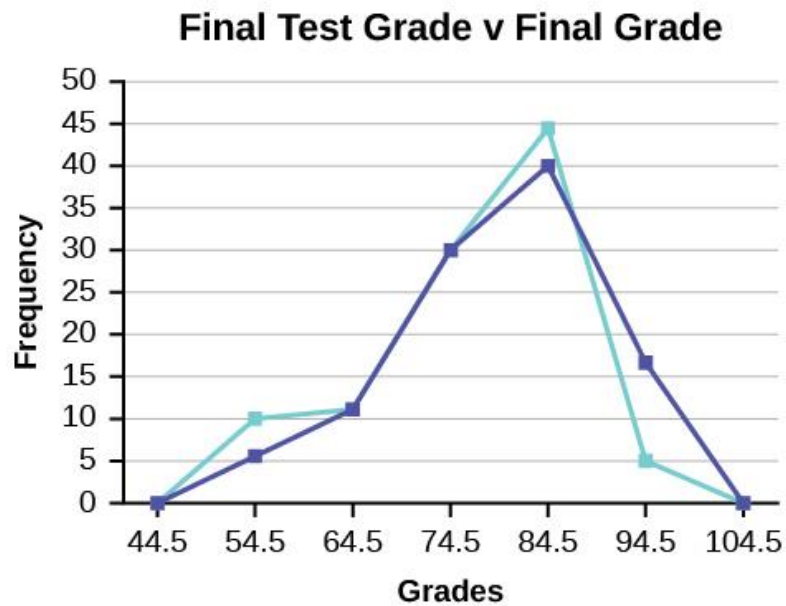
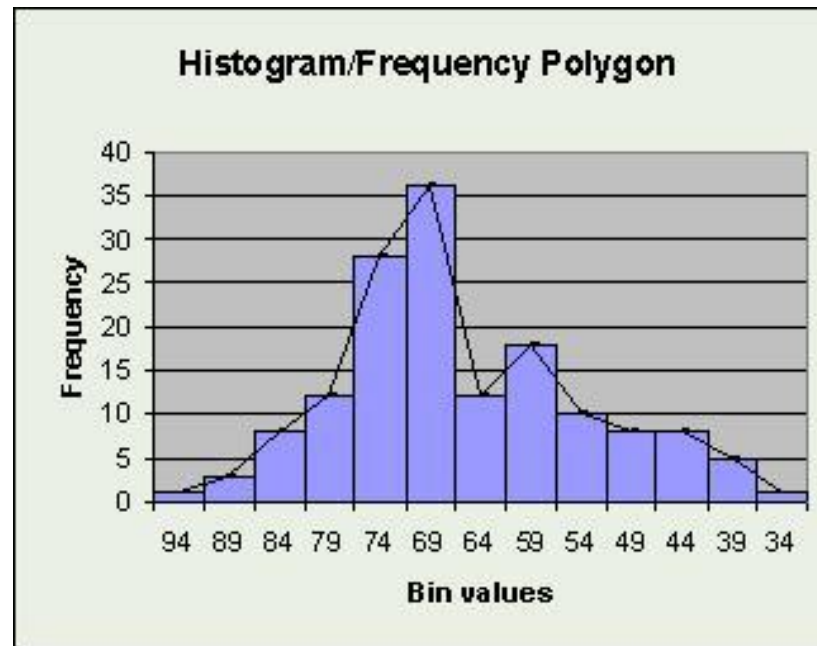
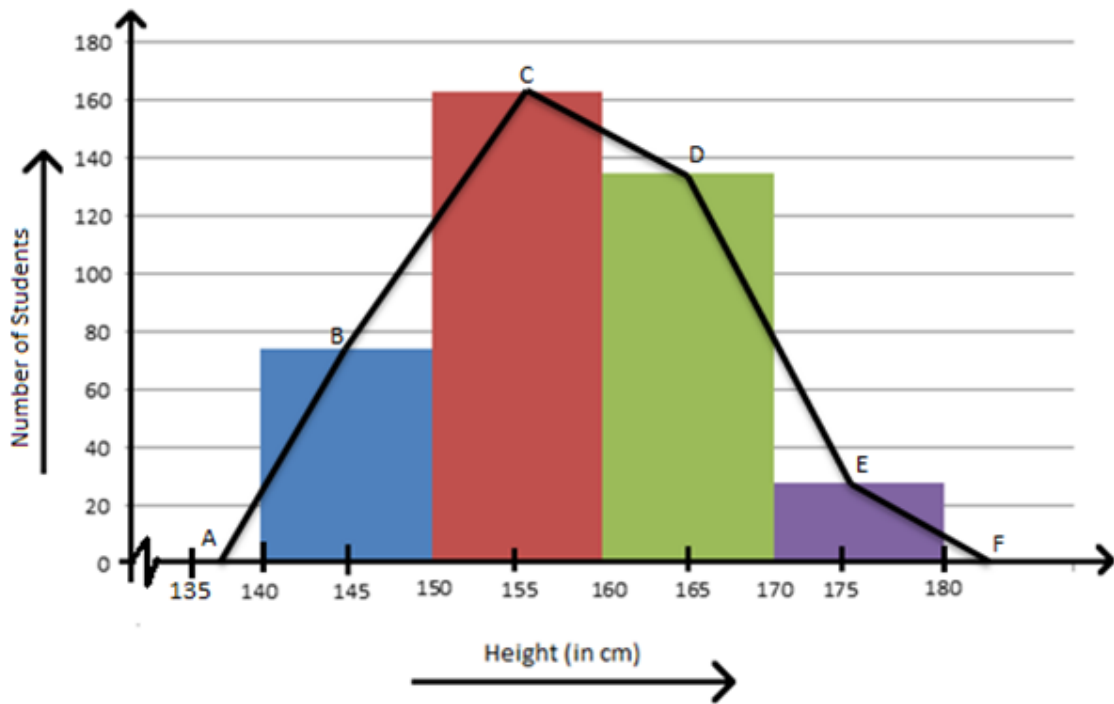
HISTOGRAM



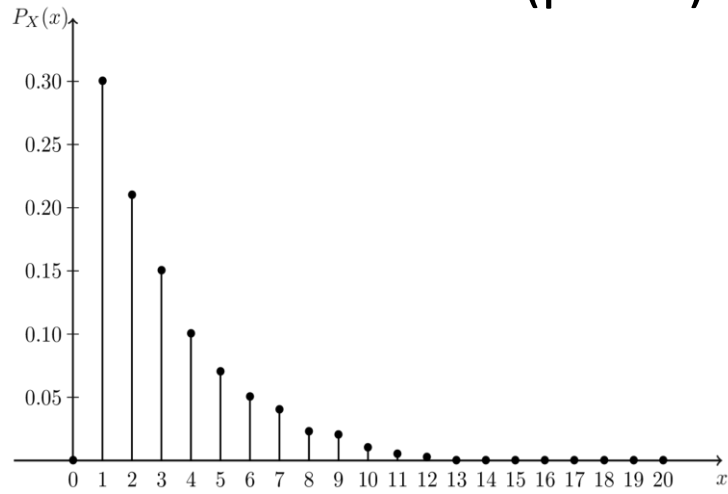
FREQUENCY POLYGON



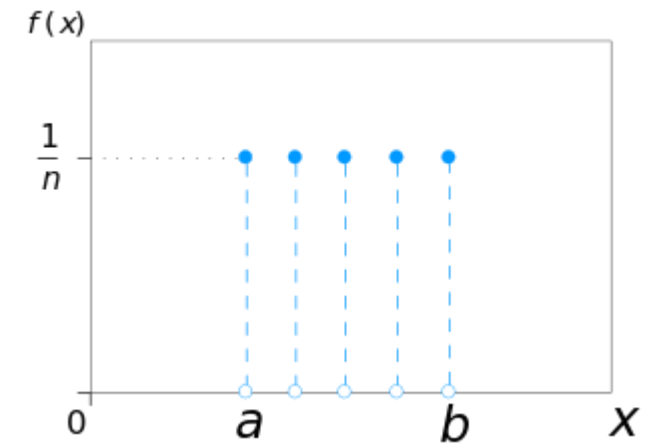
Over the past 4 years, the number of women climbers in the northeast has seen a decline.



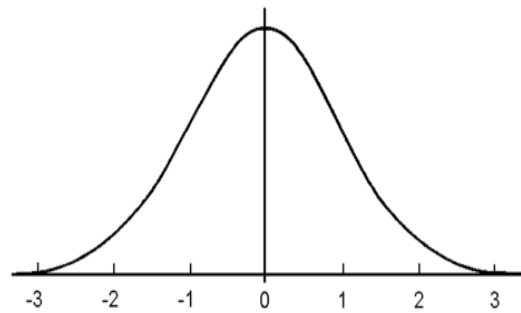
$X \sim \text{Geometric}(p=0.3)$



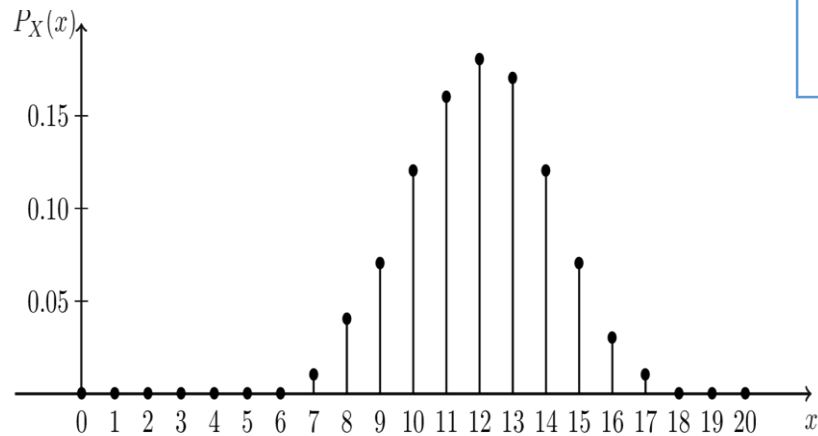
$X \sim \text{Uniform}(n=5)$



$X \sim N(0, 1^2)$



$X \sim \text{Binomial}(n=20, p=0.6)$



$X \sim \text{Hypergeometric}(N=80, k=30, n=25)$

