

MDM4U – Sample Test 5 – Distributions – December 1, 2023

Name: Solutions

Knowledge	Application	Communication	Thinking	Total	Percent
25	25	22	20	92	%

Knowledge

1. A spinner has 5 sections is spun. Find the distribution and the expected value. /5

$X \sim \text{Uniform}(n=5)$

x	x=1 (red)	x=2 (blue)	x=3 (yellow)	x=4 (green)	x=5 (purple)
P(x)	$\frac{1}{5}$ <i>or 0.2</i>	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

(a) $E(x) = \frac{1+5}{2}$
 $= \frac{6}{2}$
 $= 3$

2. You have a bag of 12 marbles, 5 are blue. You pull out 3 marbles without replacement. /8

$X \sim \text{Hypergeometric}(n=12, a=5, r=3)$. Thus, $E(x) = ra/n = 1.25$ ← on average = expected value

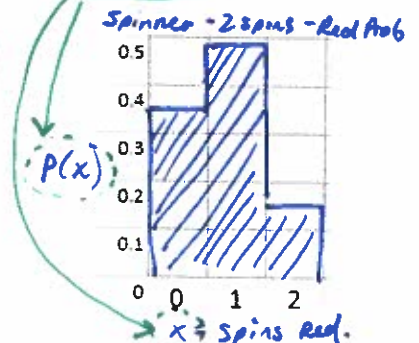
x	0 blue	1 blue	2 blue	3 blue
$C(a,x)$ Blue marbles	$C(5,0) = 1$	$C(5,1) = 5$	$C(5,2) = 10$	$C(5,3) = 10$
$C(n-a,r-x)$ Not-blue	$C(7,3) = 35$	$C(7,2) = 21$	$C(7,1) = 7$	$C(7,0) = 1$
Numerator $C(a,x) * C(n-a,r-x)$	35	105	70	10
$C(n,r)$ Total	$C(12,3) = 220$	$C(12,3) = 220$	$C(12,3) = 220$	$C(12,3) = 220$
P(x) = num / C(n,r)	0.159	0.48	0.32	0.05

- (a) When you draw out 3 marbles how many blue ones will you get on average? 1.25
- (b) What is the probability of getting 0 blue marbles? 0.159

3. A spinner has 2 red spaces out of 5 total. You spin it twice. Graph the distribution. Label the axis. /7

$X \sim \text{Binomial}(n=2, p=0.4)$. Thus $q = 0.6$

x	0 spins red	1 spin red	2 spins red
$C(n,x)$	$C(2,0) = 1$	$C(2,1) = 2$	$C(2,2) = 1$
p^x	$0.4^0 = 1$	$0.4^1 = 0.4$	$0.4^2 = 0.16$
q^{n-x}	$0.6^2 = 0.36$	$0.6^1 = 0.6$	$0.4^0 = 1$
P(x)	0.36	0.48	0.16



4. You are rolling a dice until you get a 6. What is the probability that it will take less than 3 rolls? /5

$X \sim \text{Geometric}(p=0.167)$, Thus $q = 0.833$

x	x=0 (1 st roll)	x=1 (2 nd roll)
q^x	$0.833^0 = 1$	$0.833^1 = 0.833$
p	0.167	0.167
P(x) = $q^x p$	0.167	0.139

- (a) What is the probability of getting a 6 in less than 3 rolls? 0.306

← add $P(0) + P(1)$



Application

5. For each box where it is possible: (a) sub in the values in the formula and (b) evaluate fully.

/21

	$P(x=2)$	$E(x)$	σ
$X \sim \text{Uniform } (n = 7)$ $\text{min} = 1$ $\text{max} = 7$	$P(x) = \frac{1}{n}$ $= \frac{1}{7}$	$E(x) = \frac{1+7}{2}$ $= \frac{8}{2}$ $= 4$	
$X \sim \text{Geometric } (p = 0.4)$ $q = 1 - 0.4$ $= 0.6$	$P(x) = (0.6)^2 (0.4)$ $= 0.144$	$E(x) = \frac{q}{p}$ $= \frac{0.6}{0.4}$ $= 1.5$	
$X \sim \text{Binomial } (n = 6, p = 0.3)$ $q = 1 - 0.3$ $= 0.7$	$P(x) = C(6,2) * 0.3^2 * 0.7^4$ $= 15 * 0.09 * 0.24$ $= 0.32$	$E(x) = n * p$ $= 6 * 0.3$ $= 1.8$	$\sigma = \sqrt{npq}$ $= \sqrt{6 * 0.3 * 0.7}$ $= 1.12$
$X \sim \text{HyperGeo } (n = 6, r = 3, a = 5)$	$P(x) = \frac{C(5,2) * C(1,1)}{C(6,3)}$ $= \frac{10 * 1}{20}$ $= 0.5$	$E(x) = \frac{r+a}{n}$ $= \frac{3+5}{6}$ $= 2.5$	

6. What are the formulas found in the indicated cells of this spreadsheet?

/9

	A	B	C	D	E	F	G
1	Binomial Distribution						
2	(a) Fill in n, p and q						
3	n	5	p	0.3	q	0.7	
4							
5	(b) Create the probability distribution for 5 dice.						
6	x	0	1	2	3	4	5
7	C part	1	5	10	10	5	1
8	p part	1	0.3	0.09	0.027	0.008	0.002
9	q part	0.168	0.24	0.343	0.49	0.7	1
10	P(x)	0.168	0.36	0.309	0.132	0.028	0.002
11							
12	(c) Calculate the E(x) and standard deviation						
13	E(x)	1.5	σ	1.025			
14							
15	(d) Calculate these probabilities using sum						
16	P(x>=3)	0.163					

F3	= 1 - D3
B7	= combin (\$B\$3, B6)
B8	= \$D\$3 ^ (B6)
B9	= \$F\$3 ^ (\$B\$3 - B6)
B10	= B7 * B8 * B9 could be absolute
B13	= B3 * D3 could be absolute
D13	= sqrt(B3 * D3 * F3)
B16	= sum (E10 : G10)

Communication

7. Which probability distribution would be used to solve the problem? /5

- (a) The distribution of the number of hearts in a 5-card hand dealt from a standard card deck. *without replacement*
- (b) The distribution for rolling each side of a 12-sided die.
- (c) The distribution created when you roll a die 30 times, and graph the number of 4s rolled. *at the same time*
- (d) The distribution created when you dealt cards until you get a club.
- (e) The distribution created by measuring the heights of 1023 grade 2s. *continuous*

HyperGeometric
Uniform
Binomial
Geometric
Normal

8. Name each probability distribution. /5

HyperGeometric or Binomial	Uniform	Geometric	Normal	HyperGeometric or Binomial

9. Fill in the final column with the term or number indicated. /9

- (a) A continuous distribution.
- (b) A distribution with dependent trials. *without replacement = HGeo. independent = Binomial, well, and others*
- (c) What "x" stands for in a binomial distribution?
- (d) A trial at the basis of the Binomial distribution.
- (e) Discrete or continuous variable: Length of a pencil.
- (f) Discrete or continuous variable: Number of eggs.
- (g) The sum of the 10th row of Pascal's triangle. $2^{\text{row}-1} = 2^{10-1} = 2^9$
- (h) Another term for the weighted mean.
- (i) The distribution type for the random bug walk. *until Bug off bridge*

Normal
HyperGeometric
of successes
Bernoulli; (Bernoulli) Sorry
Continuous
Discrete
512
Expected Value
Geometric

10. Define and explain the importance of the term "hypergeometric distribution". /3

- a discrete probability distribution
- it considers the probability of success for a number of draws that are dependent or occur without replacement
- each draw is either a success or failure
- it can be used to calculate probabilities in card games and in combinations situations (order doesn't matter) like choosing team members.

Thinking

All questions should be organized using titles. If formulas are used, they should be shown.

11. How many ways can you spell each day of the week starting from the top and going to the bottom? /5

(a) Total: 20

T₁

U₁ U₁

E₁ E₂ E₁

S₁ S₃ S₃ S₁

D₄ D₆ D₄

A₁₀ A₁₀

Y₂₀

(b) Total: 32

M₁

O₁ O₁

N₁ N₂ N₁

D₁ D₃ D₃ D₁

A₁ A₄ A₆ A₄ A₁

Y₁ Y₅ Y₁₀ Y₁₀ Y₅ Y₁ → add up.

12. Why can't these be calculated? Circle the problematic value and explain why it is a problem. /6

<p>X ~ Uniform (n = <u>2</u>); min = 10, max = 12</p> <p>if min = 10, max = 12, there are 3 numbers: 10, 11, 12.</p> <p>however, it says n is 2; it should say 3.</p>	<p>X ~ Binomial (n = <u>0.39</u>, p = 0.39)</p> <p>n is the number of trials. that is a count; it is a discrete value. it must be a positive whole number. not a decimal.</p>	<p>X ~ HyperGeo (n = <u>12</u>, r = <u>14</u>, a = 3)</p> <p>n is the total number of things (say, people for a team). r is the places to put them (say positions on the team). choose questions have more of n than r. an error here.</p>
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13. In a raffle, there are 6 prize chickens worth \$100, 4 turkeys worth \$200, a pony worth \$500 and an okapi worth \$1000. 400 tickets are sold. /9

Item	Chicken	Turkey	Pony	Okapi	No Win	Σ
Value (x)	\$ 100	\$ 200	\$ 500	\$ 1000	\$ 0	
Frequency	6	4	1	1	388	400
P(x)	0.015	0.01	0.0025	0.0025	/	1
x * P(x)	1.5	2	1.25	2.5	0	7.25

What is the expected value of a ticket to the consumer? ... \$ 7.25 ...

If the raffle organizers wish to make a profit of 50% on the raffle, how much should they charge per ticket?

set up a ratio ← title
 50% to 100% ← another title

$$\frac{7.25}{0.5} = \frac{x}{1}$$

\$ 14.5 = x ← \$ appears

... They should charge \$14.50.
 ↑ conclusion

you don't need all the pink, but some is nice.