More on Z-scores

Number of Standard Deviations from the Mean

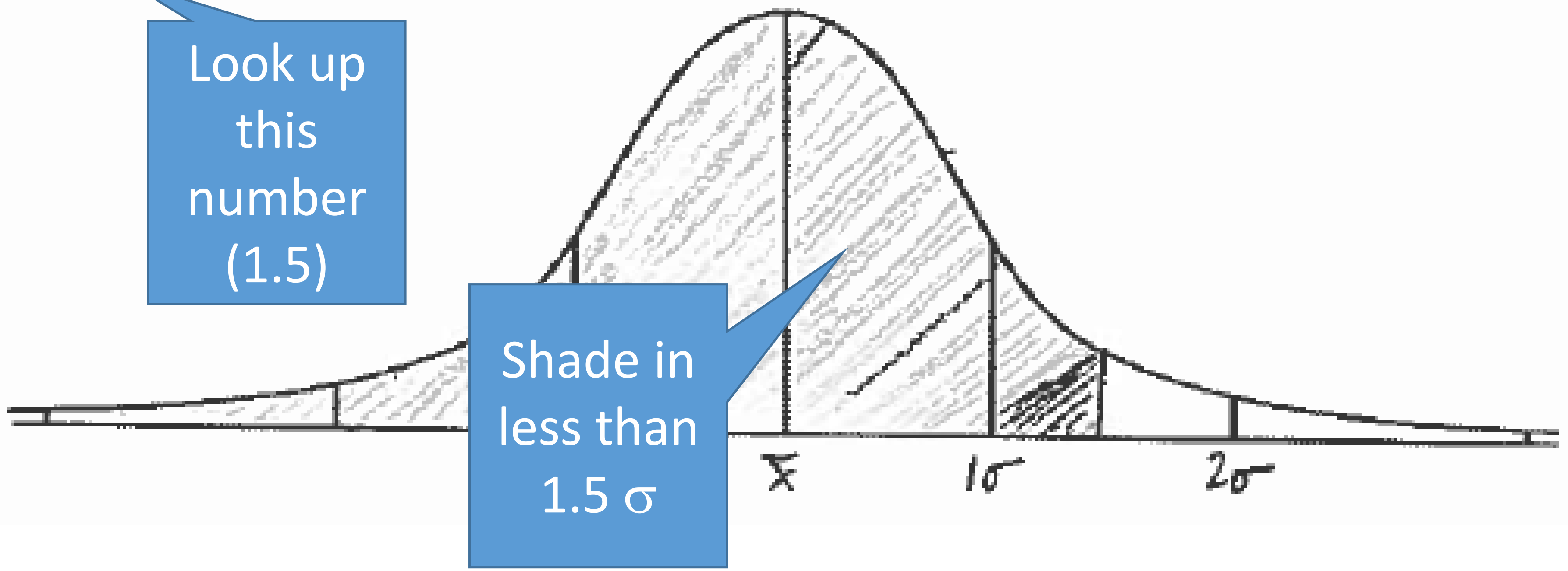
Filling in your worksheet

The first answer for each question, annotated

Graph Regions #1 – Under a Z-score

Shade the region indicated. Look up the z-score in the table.

1. P(z < 1.5) = ..93.22.%



Graph Regions #2 – Over a Z-score

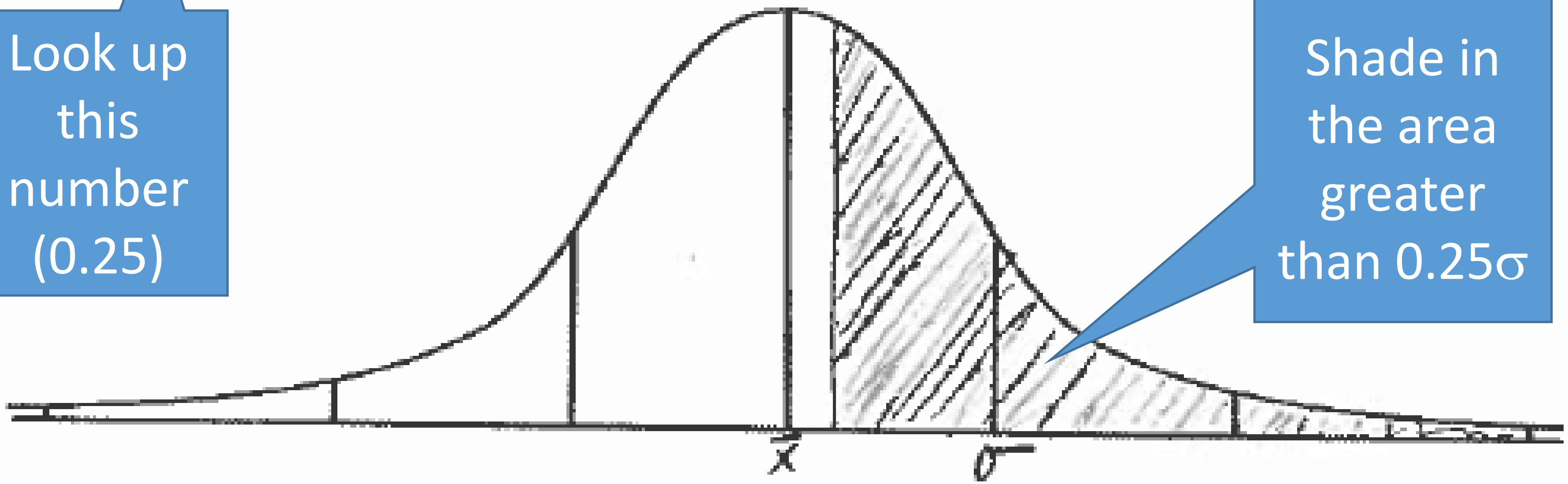
Shade the region indicated (Greater than!) z-score in the table.

1. $P(z > 0.25)$ $= 1 - 0.5987$

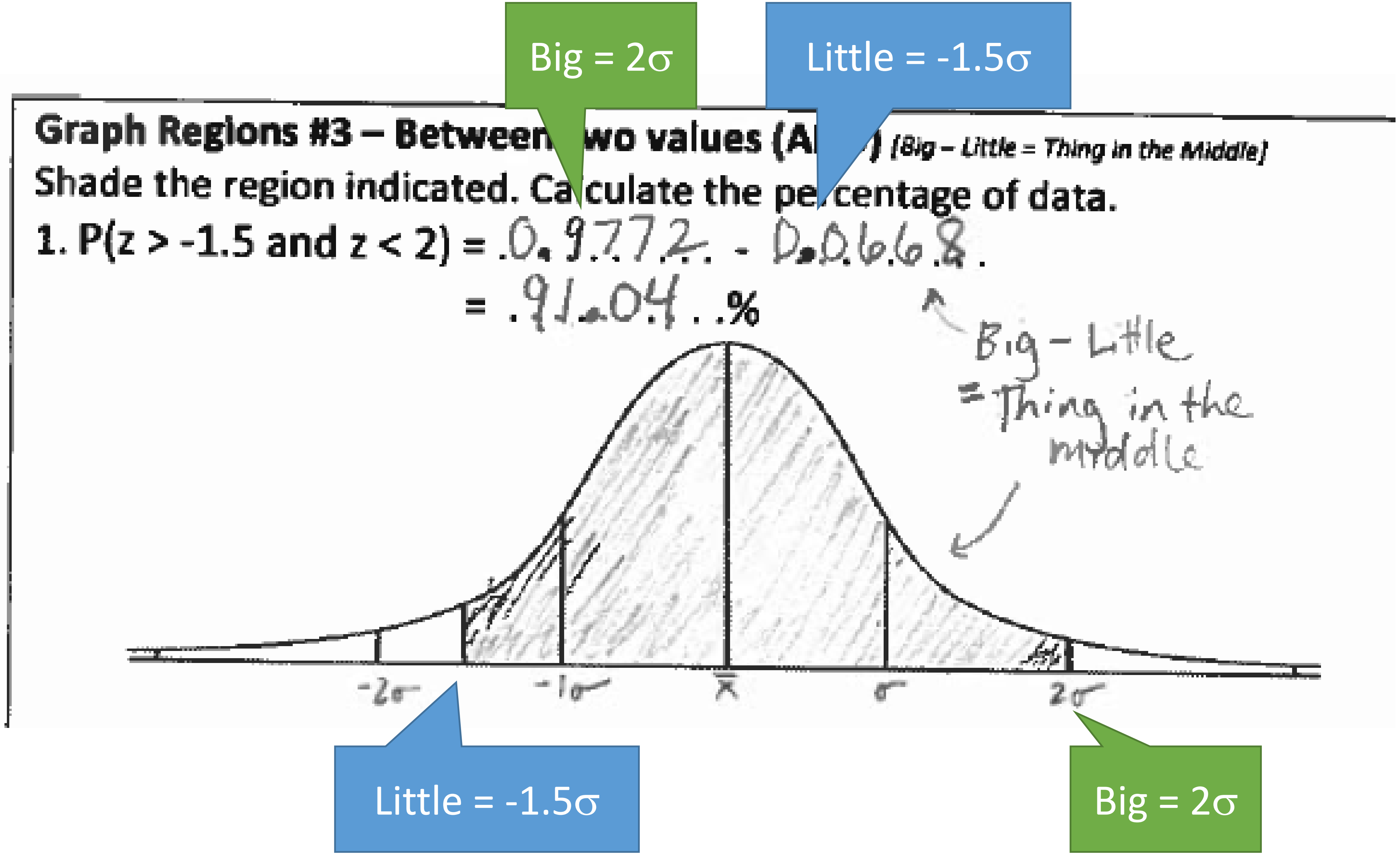
$= 40.13\%$

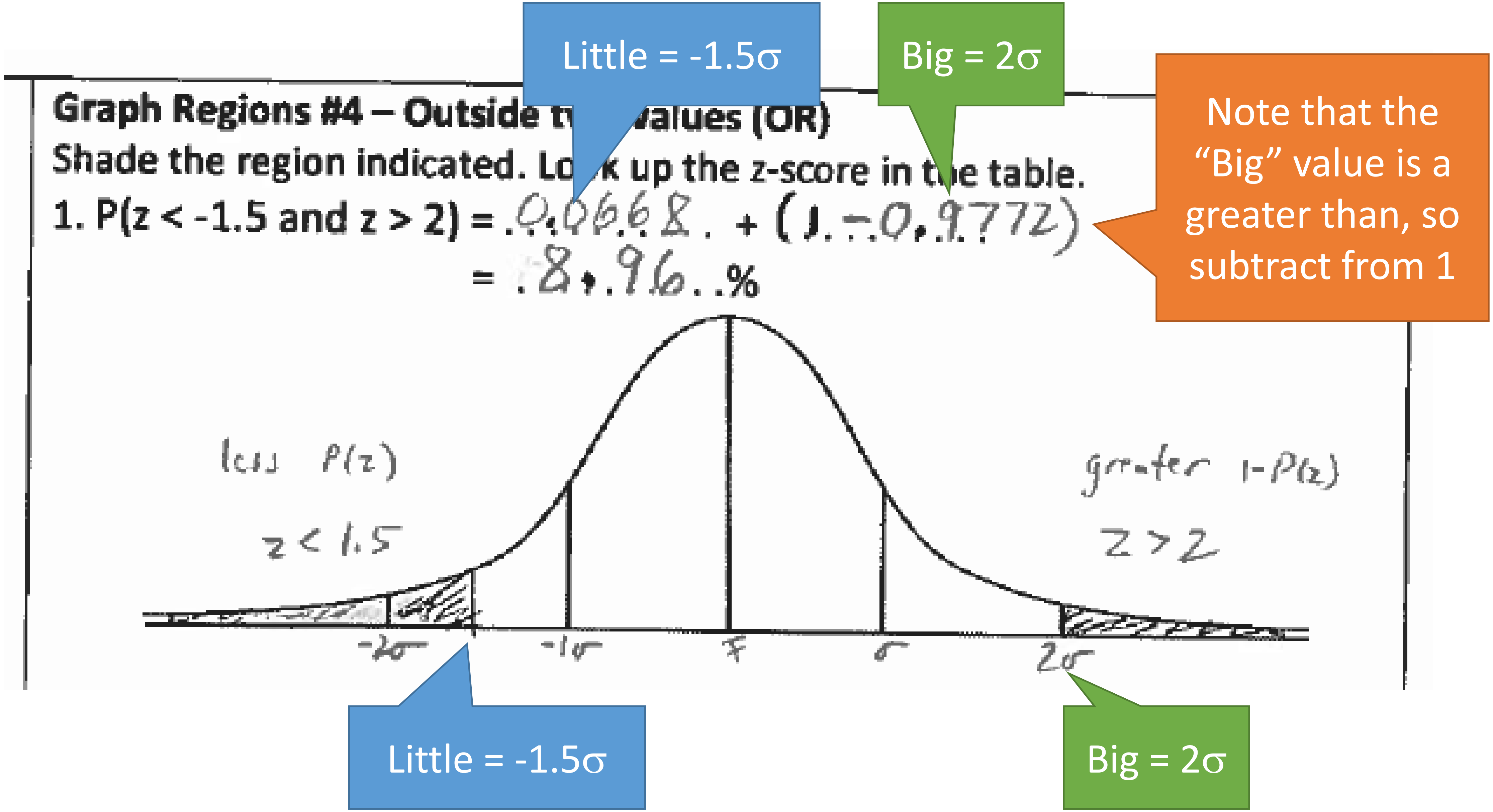
Look up this number (0.25)

Subtract from 1 to find the other side



Shade in the area greater than 0.25σ


Big = 2σ
Little = -1.5σ
Graph Regions #3 – Between two values (Al…) (Big – Little = Thing in the Middle)
Shade the region indicated. Calculate the percentage of data.
1. P(z > -1.5 and z < 2) = 0.9772 - 0.0668
= 91.04 %
Big - Little = Thing in the middle
-2σ
-1σ
x̄
σ
2σ
Little = -1.5σ
Big = 2σ


Little = -1.5σ
Big = 2σ
Note that the "Big" value is a greater than, so subtract from 1
Graph Regions #4 – Outside t... Values (OR)
Shade the region indicated. Lo...k up the z-score in t...e table.
1. P(z < -1.5 and z > 2) = 0.0668 + (1 − 0.9772)
= 8.96 %
less P(z)
z < 1.5
greater 1-P(z)
z > 2
-2σ
-1σ
x̄
σ
2σ
Little = -1.5σ
Big = 2σ

Some Tricky Examples

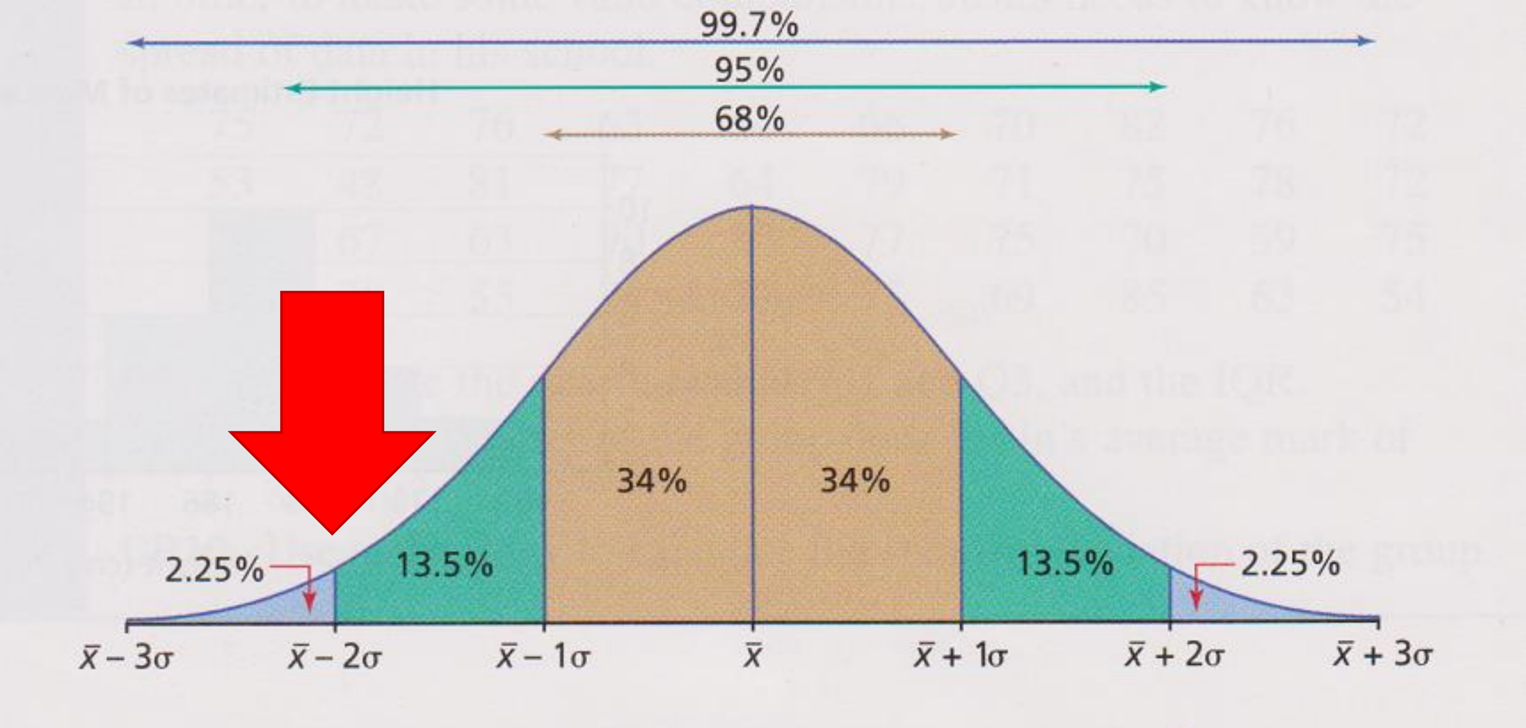
Thinking Questions

4 The results of a test are normally distributed. Harri gained a z-score equal to -2.

a Interpret this z-score with regard to the mean and standard deviation of the test scores.

b What proportion of students obtained a better score than Harri?

c The mean test score was 151 and Harri's actual score was 117. Find the standard deviation of the test scores.



The graph of the normal distribution $X \sim N(\bar{x}, \sigma^2)$

a. A z-score of -2 means that Harri's test mark is 2 standard deviations below the mean.

Recall that the z-score is the number of standard deviations something is away from the mean.

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c The mean test score was 151 and Harri's actual score was 117. Find the standard deviation of the test scores.

b. $P(z>-2) = 1 - P(z<-2)$

$= 1 - 0.0228$

$= 0.9772$

97.72 % of students obtained a better score than Harri.

The Z-score Probability Table tells us the % BELOW the value.

We subtract from 1 (or 100%) to find % OVER the value

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c The mean test score was 151 and Harri's actual score was 117. Find the standard deviation of the test scores.

c.

$$z = \frac{x - \bar{x}}{\sigma}$$

$$-2 = \frac{117 - 151}{\sigma}$$

$$\sigma = \frac{117 - 151}{-2}$$

$$\sigma = 17$$

Sub in the values we know into the z-scores formula (z, x and mean).

Then, solve for the σ

7 The life of a Xenon-brand battery is normally distributed with mean 33.2 weeks and standard deviation 2.8 weeks.

b For how many weeks can the manufacturer expect the batteries to last before 8% of them fail?

b)

This time, instead of starting with the z-score and getting the percentage, we find the percentage, and work back to the z-score.

8% has a z-score of -1.4

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.[illegible]05	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.[illegible]07	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.[illegible]10	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.[illegible]13	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.[illegible]19	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.[illegible]26	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.[illegible]35	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.[illegible]47	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.[illegible]62	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.[illegible]82	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.[illegible]07	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.[illegible]39	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.[illegible]79	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.[illegible]28	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.[illegible]87	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.[illegible]59	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.[illegible]46	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.[illegible]48	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.[illegible]68	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985

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8% has a z-score of -1.4

$$z = \frac{x - \bar{x}}{\sigma}$$

$$-1.4 = \frac{x - 33.2}{2.8}$$

$$-1.4(2.8) = x - 33.2$$

$$-3.92 = x - 33.2$$

$$29.28 = x$$

Sub in the values we know into the z-scores formula (z, σ and mean).

Thus, the manufacturer can expect the batteries to last 29.28 weeks before 8% of them fail.