

Z-Scores

Note: μ is \bar{x} (for our purposes)

1 The table shows Emma's midyear exam results. The exam results for each subject are normally distributed with the mean μ and standard deviation σ shown in the table.

Subject	Emma's score	μ	σ
English	48	40	4.4
Mandarin	81	60	9
Geography	84	55	18
Biology	68	50	20
Maths	84	50	15

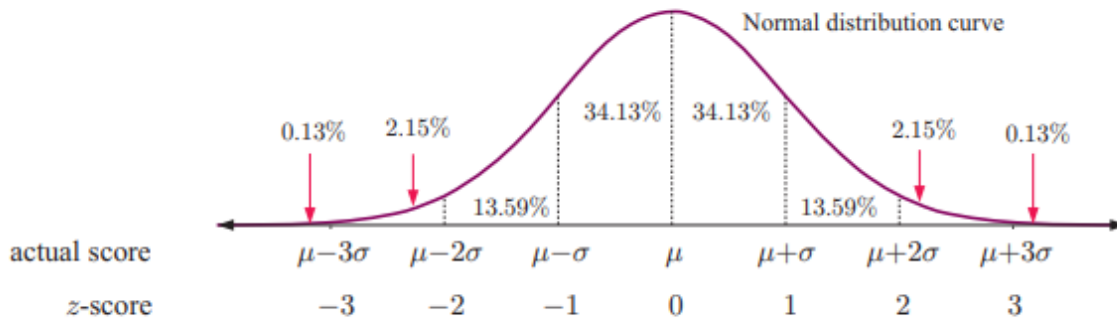
- Find the z -score for each of Emma's subjects.
- Arrange Emma's subjects from 'best' to 'worst' in terms of the z -scores.

2 The table alongside shows Sergio's results in his final examinations, along with the class means and standard deviations.

	Sergio	μ	σ
Physics	73%	78%	10.8%
Chemistry	77%	72%	11.6%
Mathematics	76%	74%	10.1%
German	91%	86%	9.6%
Biology	58%	62%	5.2%

- Find Sergio's Z -value for each subject.
- Arrange Sergio's performances in each subject in order from 'best' to 'worst'.

3 Consider the normal distribution probabilities:



Use the diagram to calculate the following probabilities. In each case sketch the Z -distribution and shade in the region of interest.

- $P(-1 < Z < 1)$
- $P(-1 \leq Z \leq 3)$
- $P(-1 < Z < 0)$
- $P(Z < 2)$
- $P(-1 < Z)$
- $P(Z \geq 1)$

5 The students of Class X sat a Physics test. The average score was 46 with a standard deviation of 25. The teacher decided to award an A to the top 7% of the students in the class. Assuming that the scores were normally distributed, find the lowest score that would achieve an A.

6 The length of fish from a particular species is normally distributed with mean 35 cm and standard deviation 8 cm. The fisheries department has decided that the smallest 10% of the fish are not to be harvested. What is the size of the smallest fish that can be harvested?



7 The length of a screw produced by a machine is normally distributed with mean 75 mm and standard deviation 0.1 mm. If a screw is too long it is automatically rejected. If 1% of screws are rejected, what is the length of the smallest screw to be rejected?

8 Pedro is studying Algebra and Geometry. He sits for the mid-year exams in each subject.

Pedro's Algebra mark is 56%, and the class mean and standard deviation are 50.2% and 15.8% respectively.

In Geometry he is told that the class mean and standard deviation are 58.7% and 18.7% respectively.

What percentage does Pedro need to have scored in Geometry, to have an equivalent result to his Algebra mark?



9 The volume of cool drink in a bottle filled by a machine is normally distributed with mean 503 mL and standard deviation 0.5 mL. 1% of the bottles are rejected because they are underfilled, and 2% are rejected because they are overfilled; otherwise they are kept for retail. What range of volumes is in the bottles that are kept?

Answers

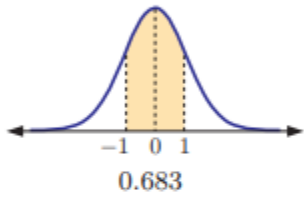
- 1 a z-scores: Geography ≈ 1.61
English ≈ 1.82 Biology = 0.9
Mandarin ≈ 2.33 Maths ≈ 2.27

b Mandarin, Maths, English, Geography, Biology

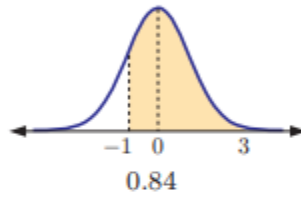
- 2 a Physics -0.463 , Chemistry 0.431 , Maths 0.198 ,
German 0.521 , Biology -0.769

b German, Chemistry, Maths, Physics, Biology

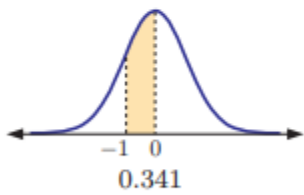
3 a



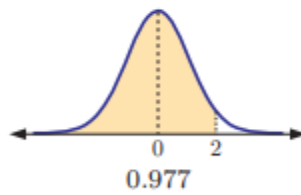
b



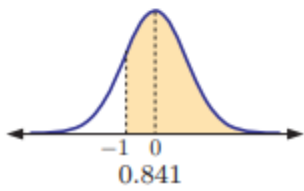
c



d



e



f



- 4 a $a \approx 21.4$ b $a \approx 21.8$ c $a \approx 2.82$
5 82.9 6 24.7 cm 7 75.2 mm 8 65.6%
9 between 502 mL and 504 mL