Permutations

P(n,r)

What are each of the following?

1.0!

2.1!

3.2!

4.3!

5.4!

6.5!

7.6!

8.7!

How do you write:

$$92 \times 91 \times 90 \times 89?$$

How do you write:

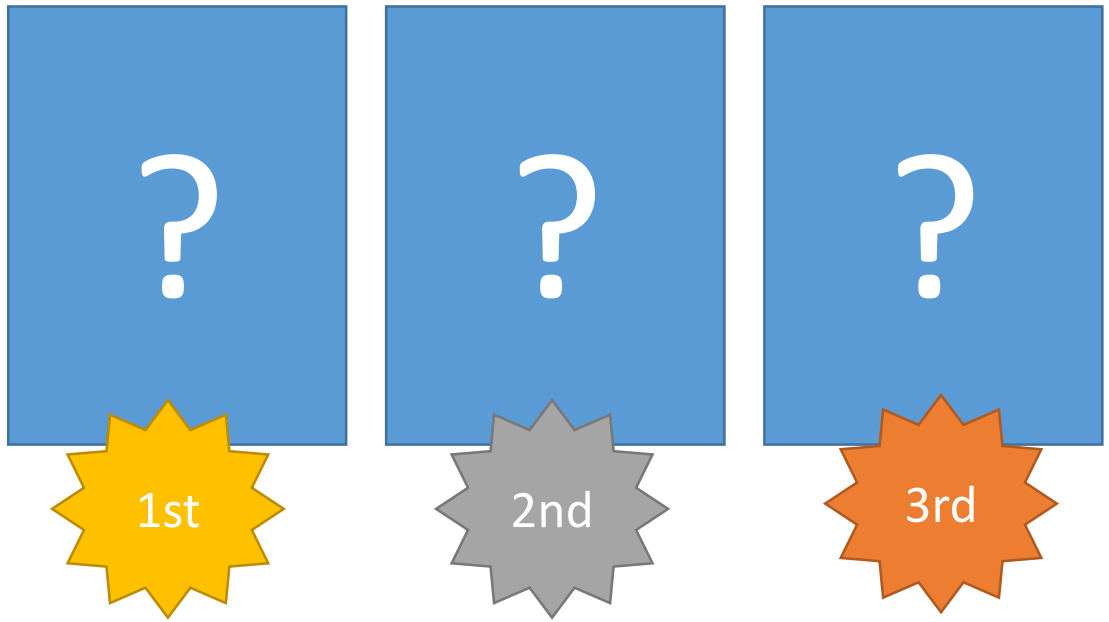
$$92 \times 91 \times 90 \times 89 = \frac{92!}{88!}$$

$$= \frac{92!}{(92-4)!}$$

$$= P(92,4)$$

$$= {}_{92}P_4$$

You have 10 horses in a race. How many ways can they place first, second or third?



You have 10 horses in a race. How many ways can they place first, second or third?

$$10 \times 9 \times 8$$

You have 10 horses in a race. How many ways can they place first, second or third?

$$10 \times 9 \times 8$$

$$= \frac{10!}{7!}$$

$$= \frac{10!}{(10-3)!}$$

You have 10 horses in a race. How many ways can they place first, second or third?

$$10 \times 9 \times 8$$

$$= \frac{10!}{7!}$$

$$= \frac{10!}{(10-3)!}$$

$$= P(10,3)$$

$$= {}_{10}P_3$$

$$P(n,r) \quad \text{or} \quad {}_nP_r$$

$$= \frac{n!}{(n-r)!}$$

- P stands for permutations
- n is the number of items to be arranged
- r is the number of places to put them

Permutations #1

Write as multiplication.

90 things, pick 3 = 90x89x88

1. **4 things, pick 2**

   4×3

2. **6 things, pick 3**

   $6 \times 5 \times 4$

3. **96 things, pick 3**

   $96 \times 95 \times 94$

4. **82 things, pick 2**

   82×81

5. **7 things, pick 4**

   $7 \times 6 \times 5 \times 4$

6. **5 things, pick 2**

   5×4

7. **10 things, pick 4**

   $10 \times 9 \times 8 \times 7$

8. **7 things, pick 4**

   $7 \times 6 \times 5 \times 4$

9. **n things, pick 2**

   $n \times (n-1)$

10. **n things, pick 1**

    n

11. **256 things, pick 3**

    $256 \times 255 \times 254$

12. **x things, pick 2**

    $x \times (x-1)$

13. **x things, pick 3**

    $x \times (x-1) \times (x-2)$

14. **23 things, pick 4**

    $23 \times 22 \times 21 \times 20$

Permutations #2

Write as a factorial fraction.

90 things, pick 5 $= \frac{90!}{85!}$

1. 4 things, pick 2 $= \frac{4!}{2!}$
2. 6 things, pick 3 $= \frac{6!}{3!}$
3. 9 things, pick 5 $= \frac{9!}{4!}$
4. 8 things, pick 3 $= \frac{8!}{5!}$
5. 7 things, pick 5 $= \frac{7!}{2!}$
6. 5 things, pick 2 $= \frac{5!}{3!}$
7. 10 things, pick 6 $= \frac{10!}{4!}$
8. 7 things, pick 4 $= \frac{7!}{3!}$
9. n things, pick 2 $= \frac{n!}{(n-2)!}$
10. n things, pick r $= \frac{n!}{(n-r)!}$
11. 9 things, pick 7 $= \frac{9!}{2!}$
12. x things, pick 3 $= \frac{x!}{(x-3)!}$
13. x things, pick y $= \frac{x!}{(x-y)!}$
14. 23 things, pick 9 $= \frac{23!}{14!}$

Permutations #3

Write as a factorial fraction.

$_{90}P_5 = \frac{90!}{85!}$

1. $_5P_4 = \frac{5!}{1!}$
2. $_6P_3 = \frac{6!}{3!}$
3. $_4P_2 = \frac{4!}{2!}$
4. $P(72,4) = \frac{72!}{68!}$
5. $_{35}P_5 = \frac{35!}{30!}$
6. $P(32,30) = \frac{32!}{2!}$
7. $_{76}P_3 = \frac{76!}{73!}$
8. $_{106}P_4 = \frac{106!}{102!}$
9. $P(90,4) = \frac{90!}{86!}$
10. $_{88}P_4 = \frac{88!}{84!}$
11. $P(60,30) = \frac{60!}{30!}$
12. $_{63}P_3 = \frac{63!}{60!}$
13. $_{46}P_2 = \frac{46!}{44!}$
14. $P(56,4) = \frac{56!}{52!}$

P(n,2)

P(8,2)
= 8 x 7

How many horses?

Put this in terms of the horse race.

How many places are we tracking?

8 horses into 2 places.
That happens in n x (n-1) ways.

Permutations #6

(Pick twos) Evaluate.

P(11,2) = 11x10 = 110

1. **P(5,2)** = 5x4 = 20
2. **P(6,2)** = 6x5 = 30
3. **P(7,2)** = 7x6 = 42
4. **P(3,2)** = 3x2 = 6
5. **P(9,2)** = 9x8 = 72
6. **P(10,2)** = 10x9 = 90
7. **P(8, 2)** = 8x7 = 56
8. **P(4,2)** = 4x3 = 12
9. **$_7P_2$** = 7x6 = 42
10. **$_5P_2$** = 5x4 = 20
11. **$_3P_2$** = 3x2 = 6
12. **$_4P_2$** = 4x3 = 12

Permutations #4

Evaluate. Notice: $_nP_{(n-1)} = n!$

$P(3,2) = 3! = 6$

1. $_7P_6$ $= 7! = 5040$

2. $P(7,6)$ $= 7! = 5040$

3. $_5P_4$ $= 5! = 120$

4. $_6P_5$ $= 6! = 720$

5. $P(6,5)$ $= 6! = 720$

6. $P(2,1)$ $= 2! = 2$

7. $_1P_0$ $= 1! = 1$

8. $_2P_1$ $= 2! = 2$

9. $P(3,2)$ $= 3! = 6$

10. $P(1,0)$ $= 1! = 1$

11. $_3P_2$ $= 3! = 6$

12. $_4P_3$ $= 4! = 24$

13. $P(4,3)$ $= 4! = 24$

14. $P(5,4)$ $= 5! = 120$

P(n,0)

P(9,0)

Put this in terms of the horse race.

How many horses?

How many places are we tracking?

P(n,0)

P(9,0)

9 horses into 0 places.
That happens in 1 way.

Put this in terms of the horse race.

How many horses?

How many places are we tracking?

P(n,1)

P(15,1)

Put this in terms of the horse race.

How many horses?

How many places are we tracking?

P(n,1)

P(15,1)

15 horses into 1 place.
That happens in 15 ways.

How many horses?

Put this in terms of the horse race.

How many places are we tracking?

P(n,n)

P(15,15)

Put this in terms of the horse race.

How many horses?

How many places are we tracking?

P(n,n)

P(15,15)

15 horses into 15 places.
That happens in 15! ways.

Put this in terms of the horse race.

How many horses?

How many places are we tracking?

P(n,n+1)

P(23, 24)

Put this in terms of the horse race.

How many horses?

How many places are we tracking?

P(n,n+1)

P(23, 24)

23 horses into 24 places.
AHHH!! Bad news bears.

Put this in terms of the horse race.

How many horses?

How many places are we tracking?

Permutations #5

Evaluate these odd-ball permutations.

1. ${}_5P_0 = 1$
2. ${}_1P_1 = 1$
3. ${}_2P_0 = 1$
4. ${}_4P_1 = 4$
5. $P(1,1) = 1$
6. ${}_2P_2 = 2! = 2$
7. ${}_4P_4 = 4! = 24$
8. $P(1,0) = 1$
9. ${}_1P_2 =$ error
10. ${}_0P_1 =$ error
11. ${}_0P_0 = 1$
12. $P(0,0) = 1$
13. ${}_4P_0 = 1$
14. ${}_2P_1 = 2$
15. ${}_4P_6 =$ error

	Order matters No repeats	Can repeat
All Values	Factorial	Counting
Only some places	Permutation	Combination
	Order matters No repeats	Order not important

Permutations #7

[Factorial Questions]

Write a factorial to represent the number of ways you can:

1. Arrange 5 books on a shelf $5!$
2. Rearrange the letters in SONG $4!$
3. Send 6 cards to 6 friends. $6!$
4. Arrange 4 people for a picture $4!$
5. Have 4 horses finish a race $4!$
6. Seat 6 people in a theatre $6!$
7. Order 16 pool balls $16!$
8. Draw 48 balls in a lottery. $48!$
9. Arrange 12 eggs in a carton. $12!$
10. Arrange 3 cards in your hand. $3!$

Permutations #8

[Permutation questions]
Write a factorial fraction to represent the number of ways you can:

1. Arrange 7 out of 4 people for a picture $\frac{7!}{3!}$
2. Seat 6 of 9 people in a theatre $\frac{9!}{3!}$
3. Draw 6 balls from 48 in a lottery. $\frac{48!}{42!}$
4. Order 4 of 16 pool balls $\frac{16!}{12!}$
5. Arrange 3 of 5 cards in your hand. $\frac{5!}{2!}$
6. Arrange 3 out of 5 books on a shelf $\frac{5!}{2!}$
7. 3 letter words from SONG $\frac{4!}{1!}$
8. Send 4 of your 6 cards to friends. $\frac{6!}{2!}$
9. Top 3 placement of 12 horses in a race $\frac{12!}{9!}$
10. Arrange 12 of 14 eggs in a carton. $\frac{14!}{2!}$

Permutations #9

[Counting questions.]

Write an exponent to model how many ways can you make:

1. 3 digit electronic lock combination (each 0-60)

   60^3

2. 4 digit PIN 10^4

3. 5 letter password

   26^5

4. Outfits from 3 shirts, 4 pants, 2 hats

   $3 \times 4 \times 2$

5. Meals from 3 appetizers, 4 main courses, 5 desserts

   $3 \times 4 \times 5$

6. Postal Codes (L4F 5W3)

   $26^3 \times 10^3$

7. 3 digit internet colour (each 0-255)

   256^3

8. Phone numbers 905-345-234 9^9

9. License Plates (BPMW 834)

   $26^4 \times 10^3$

10. 4 piece IP address (each 0-255)

    256^4

Permutations #10

Is it a factorial, permutation or counting question?

1. Seat 6 of 9 people in a theatre perms
2. Arrange 4 people for a picture fact
3. Have 4 horses finish a race fact
4. Outfits from 3 shirts, 4 pants, 2 hats count
5. Draw 6 balls from 48 in a lottery. perm
6. Arrange 3 out of 5 books on a shelf perm
7. 5 letter password count
8. Rearrange the letters in SONG fact
9. Postal Codes (L4F 5W3) count
10. Top 3 placement of 12 horses in a race perm
11. 3 digit locker combination (each 0-60) count