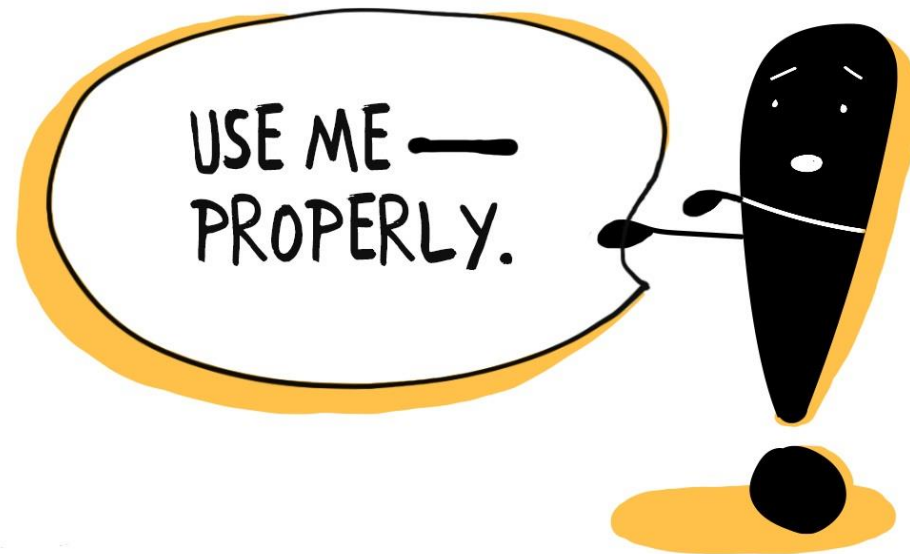
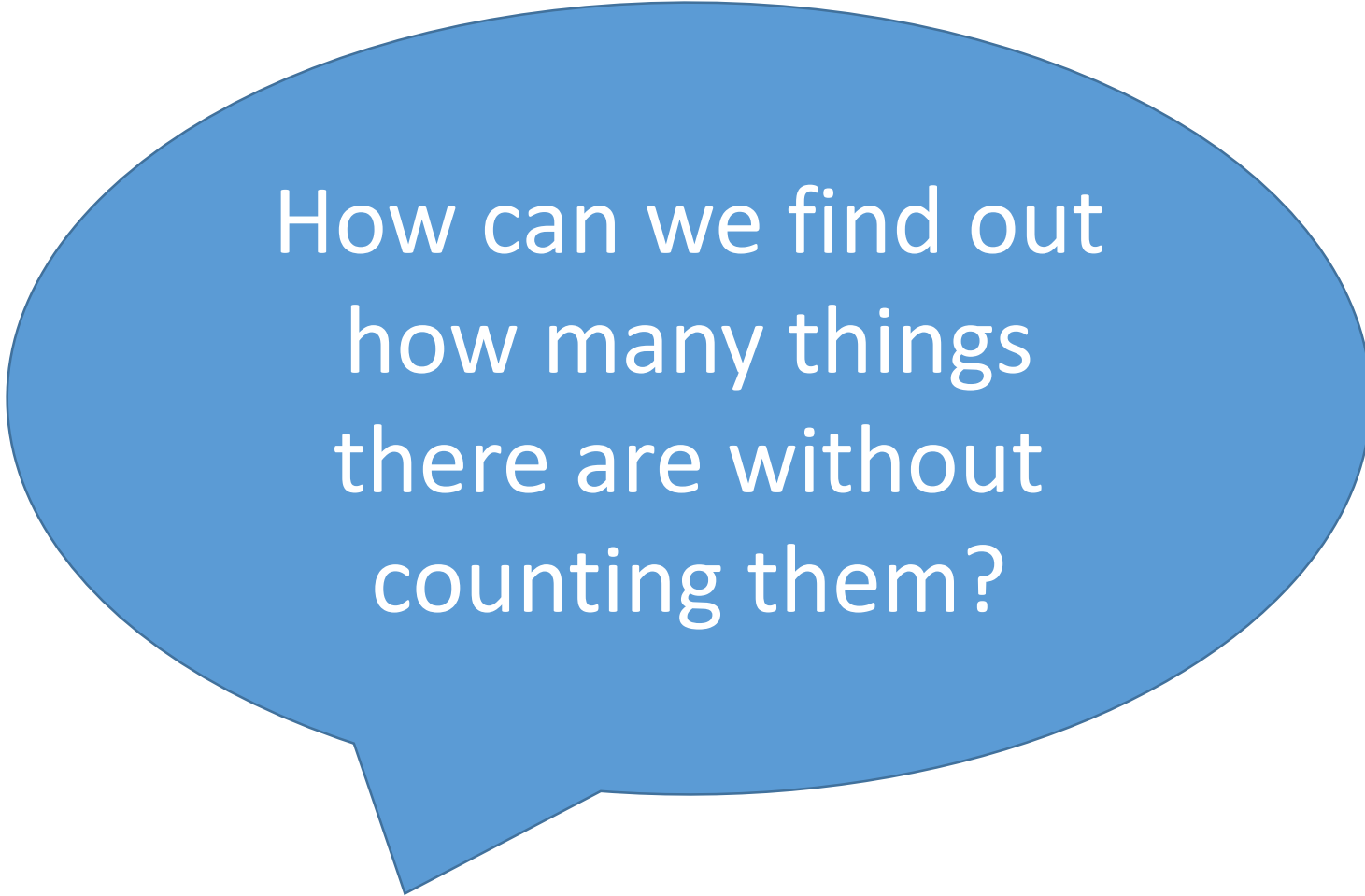


Factorial Notation





How can we find out
how many things
there are without
counting them?



When the licence plates have 5 numerical digits how many were there?



What about
when it went up
to 6 digits?



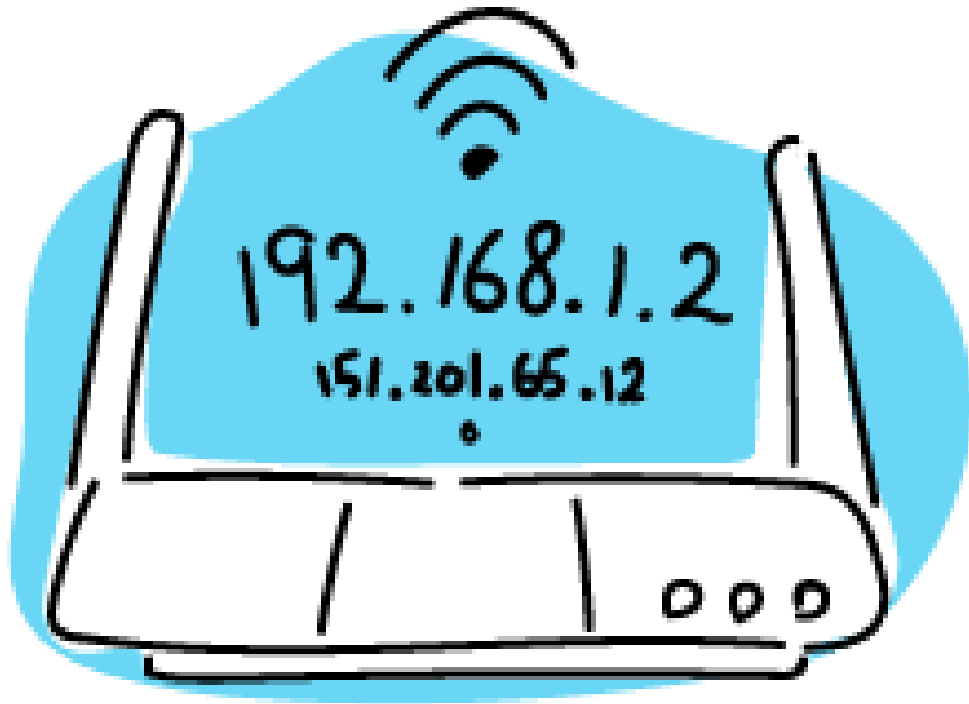
What about
when we added
letters?



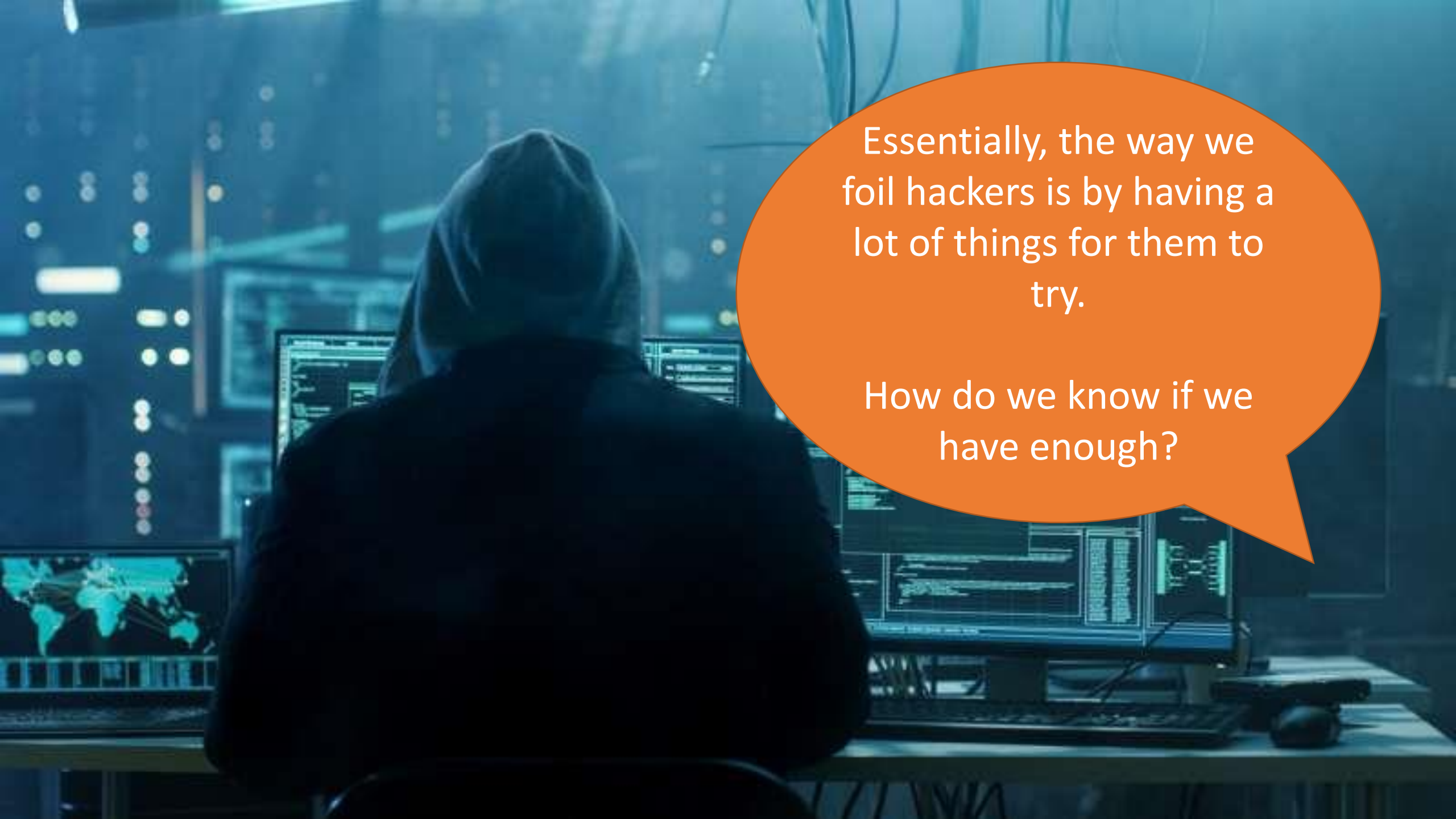
Now, how many?



How many
phone
numbers
exist?



How many
internet
addresses exist?

A person wearing a dark hoodie is seen from behind, sitting at a desk in a server room. They are looking at several computer monitors displaying code and data. The room is dimly lit with blue and green light from the screens and server racks in the background.

Essentially, the way we foil hackers is by having a lot of things for them to try.

How do we know if we have enough?

Suppose that we have 1 thing. How many ways can it be arranged?

Suppose that we have 1 thing. How many ways can it be arranged?

A

Suppose that we have 2 things. How many ways can they be arranged?

Suppose that we have 2 things. How many ways can they be arranged?

AB

BA

Suppose that we have 2 things. How many ways can they be arranged?

AB

BA

$$2 \times 1$$

Suppose that we have 3 things. How many ways can they be arranged?

Suppose that we have 3 things. How many ways can they be arranged?

ABC ACB

BAC BCA

CBA CAB

Suppose that we have 3 things. How many ways can they be arranged?

ABC ACB

BAC BCA

CBA CAB

$$3 \times 2 \times 1$$

Suppose that we have 4 things. How many ways can they be arranged?

Suppose that we have 4 things. How many ways can they be arranged?

ABCD ABDC ACBD ACDB ADCB ADBC

BACD BADC BCAD BCDA BDCA BDAC

CABD CBAD CADB CABD CDAB CDBA

DABC DBAC DBAC DBCA DCAB DCBA

Suppose that we have 4 things. How many ways can they be arranged?

ABCD ABDC ACBD ACDB ADCB ADBC
BACD BADC BCAD BCDA BDCA BDAC
CABD CBAD CADB CABD CDAB CDBA
DABC DBAC DBAC DBCA DCAB DCBA

$$4 \times 3 \times 2 \times 1$$

A **factorial** is the product of an integer and all the integers below it.

$$1! = 1 = 1$$

$$2! = 2 = 1 \times 2$$

$$3! = 6 = 1 \times 2 \times 3$$

$$4! = 24 = 1 \times 2 \times 3 \times 4$$

$$5! = 120 = 1 \times 2 \times 3 \times 4 \times 5$$

	A	B	C	D
1	0	!	=	1
2	1	!	=	1
3	2	!	=	2
4	3	!	=	6
5	4	!	=	24
6	5	!	=	120
7	6	!	=	720
8	7	!	=	5040
9	8	!	=	40320
10	9	!	=	362880
11	10	!	=	3628800
12	11	!	=	39916800
13	12	!	=	479001600

- Factorials grow very quickly.
- This is why counting things by hand can be difficult.



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"THIS DECISION IS SO HARD!"

SNODGRASS
Dilbert



Get it?

	A	B	C	D
1	0	!	=	1
2	1	!	=	1
3	2	!	=	2
4	3	!	=	6
5	4	!	=	24
6	5	!	=	120
7	6	!	=	720
8	7	!	=	5040
9	8	!	=	40320
10	9	!	=	362880
11	10	!	=	3628800
12	11	!	=	39916800
13	12	!	=	479001600

How else can you express 5!

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$$5! = 1 \times 2 \times 3 \times 4 \times 5$$

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$$5! = 1 \times 2 \times 3 \times 4 \times 5$$

$$5! = 4! \times 5$$

$$5! = 1 \times 2 \times 3 \times 4 \times 5$$

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$$5! = 1 \times 2 \times 3 \times 4 \times 5$$

$$5! = 4! \times 5$$

$$5! = 1 \times 2 \times 3 \times 4 \times 5$$

$$5! = 3! \times 4 \times 5$$

Write using factorials

$$3 \times 4 \times 5 \times 6 =$$

Write using factorials

$$3 \times 4 \times 5 \times 6 = \frac{1 \times 2 \times 3 \times 4 \times 5 \times 6}{1 \times 2 \times 3}$$

Write using factorials

$$3 \times 4 \times 5 \times 6 = \frac{1 \times 2 \times 3 \times 4 \times 5 \times 6}{1 \times 2 \times 3}$$
$$= \frac{6!}{3!}$$

A
"Chopped"
off
factorial.

Simplify, then evaluate

$$\frac{7!}{4!} =$$

Simplify, then evaluate

$$\frac{7!}{4!} = \frac{7 \times 6 \times 5 \times 4!}{4!}$$

Simplify, then evaluate

$$\begin{aligned}\frac{7!}{4!} &= \frac{7 \times 6 \times 5 \times 4!}{4!} \\ &= 7 \times 6 \times 5\end{aligned}$$

Simplify, then evaluate

$$\begin{aligned}\frac{7!}{4!} &= \frac{7 \times 6 \times 5 \times 4!}{4!} \\ &= 7 \times 6 \times 5 \\ &= 210\end{aligned}$$

You have 5 different numbers, how many ways can they be arranged?



You have 5 different numbers, how many ways can they be arranged?

$$5 \times ? \times ? \times ? \times ?$$

You have 5 different numbers, how many ways can they be arranged?

$$5 \times 4 \times ? \times ? \times ?$$

You have 5 numbers, how many ways can they be arranged?

$$5 \times 4 \times 3 \times 2 \times 1$$

You have 5 numbers, how many ways can they be arranged?

$$5 \times 4 \times 3 \times 2 \times 1$$

Number of ways
= $5 \times 4 \times 3 \times 2 \times 1$
= $5!$

You have 5 numbers, how many ways can they be arranged?

$$5 \times 4 \times 3 \times 2 \times 1$$

$$\begin{aligned} \text{Number of ways} \\ &= 5 \times 4 \times 3 \times 2 \times 1 \\ &= 5! \\ &= 120 \end{aligned}$$

You have 10 different numbers, how many ways
4 unique digits be arranged on a licence plate?

The digits will
not repeat in this
example.

$$10 \times 9 \times 8 \times 7$$



You have 10 different numbers, how many ways
4 unique digits be arranged on a licence plate?

The digits will
not repeat in this
example.

$$10 \times 9 \times 8 \times 7$$

$$10 \times 9 \times 8 \times 7 = \frac{10!}{6!}$$



You have 10 different numbers, how many ways
4 unique digits be arranged on a licence plate?

The digits will
not repeat in this
example.

$$10 \times 9 \times 8 \times 7$$

$$10 \times 9 \times 8 \times 7 = \frac{10!}{6!} = 5,040$$

