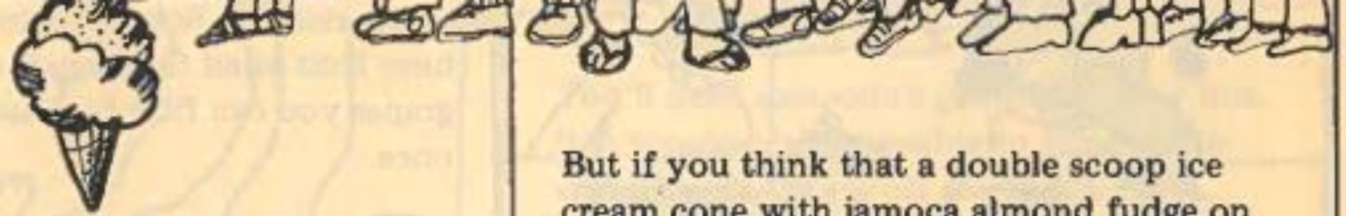
Combinations

The end of counting is in sight!

Word Problem Types

1. Counting AND
2. Counting OR
3. Factorial (n = r, using everything)
4. Permutations (r is less than n, using some)
5. Perms Restrictions – first place is known
6. Perms Restrictions – Subset is known, can move
7. Perms Restrictions – Subset is known, can move, can switch
8. Circular Permutations – fix head. (n-1)!
9. Factorial with Repeats – divide out repeats
10. Perms with Repeats – use cases
11. Combinations – order doesn't matter

VANILLA
BUTTERBRICKLE
CHOCOLATE CHIP
JAMOCA ALMOND FUDGE
BLUEBERRY MINT
MINT CHIP
FRESH PEACH
HONEY BANANA
CHOCOLATE
BITTERSWEET
FRENCH VANILLA
ROOTBEER MARBLE
STRAWBERRY
COFFEE
RUM RAISIN
BUTTER PECAN
ROCKY ROAD
COCONUT
31 flavors
CHOCOLATE MINT
WILD BLACKBERRY
BUTTERSCOTCH RIPPLE
TUTTI-FRUITI
BLACK WALNUT
PISTACHIO
LEMON
RUM NUTS
BANANA NUT
RASPBERRY
MAPLE WALNUT
PINEAPPLE
CREME DE MENTHE



Mathematicians eat ice cream just like everyone else. But sometimes they don't wait in line like everyone else. See the mathematician in this picture? She's the one holding up the line. That's because she is thinking. About ice cream. Actually, about double scoop ice cream cones. She's trying to figure out how many possible double scoop cones there are if there are 31 famous flavors to choose from.

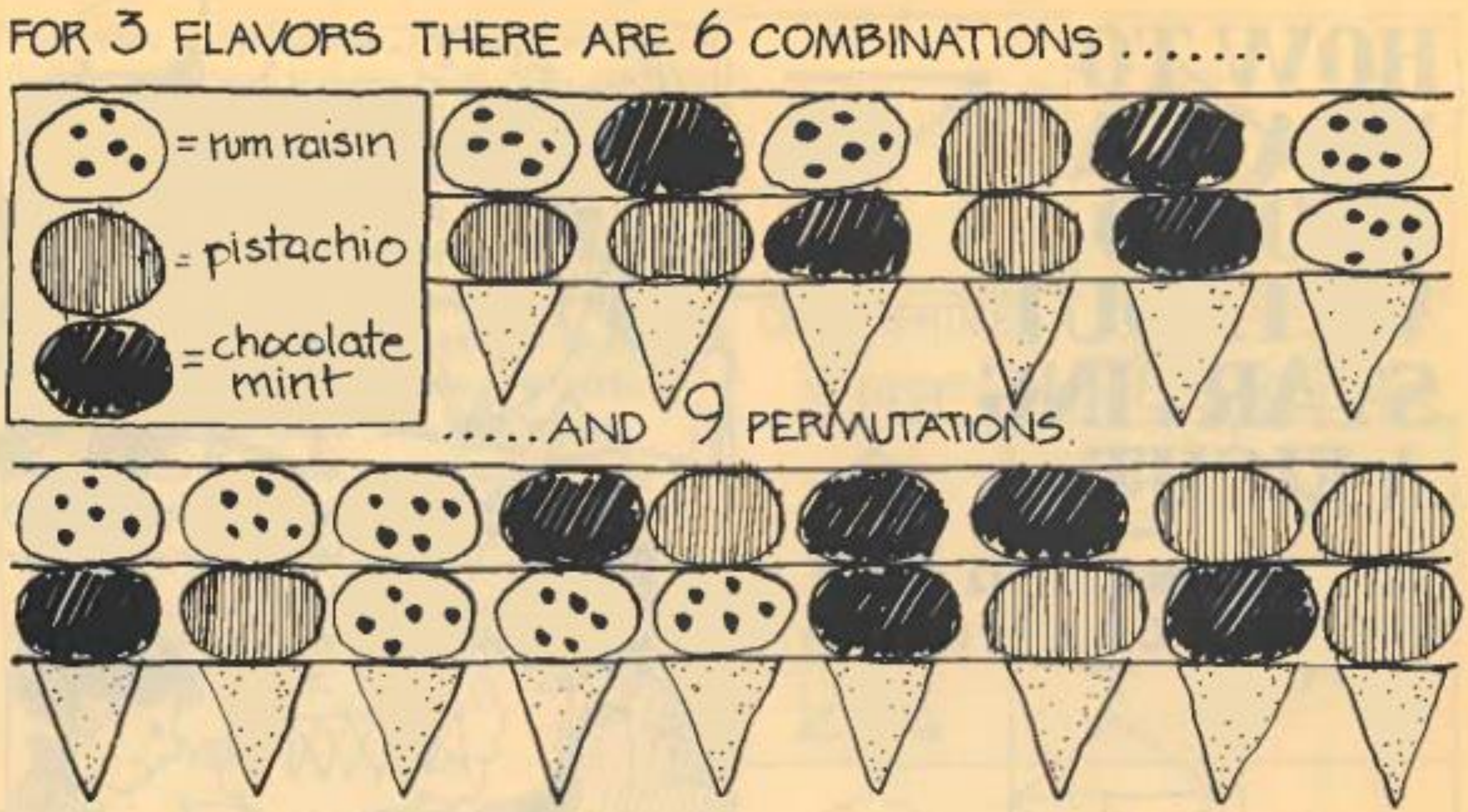
Now that's a real problem!

It has to do with things called "permutations" and "combinations." If you think it doesn't matter which flavor is on top and which is on the bottom, then you're talking about *combinations.*

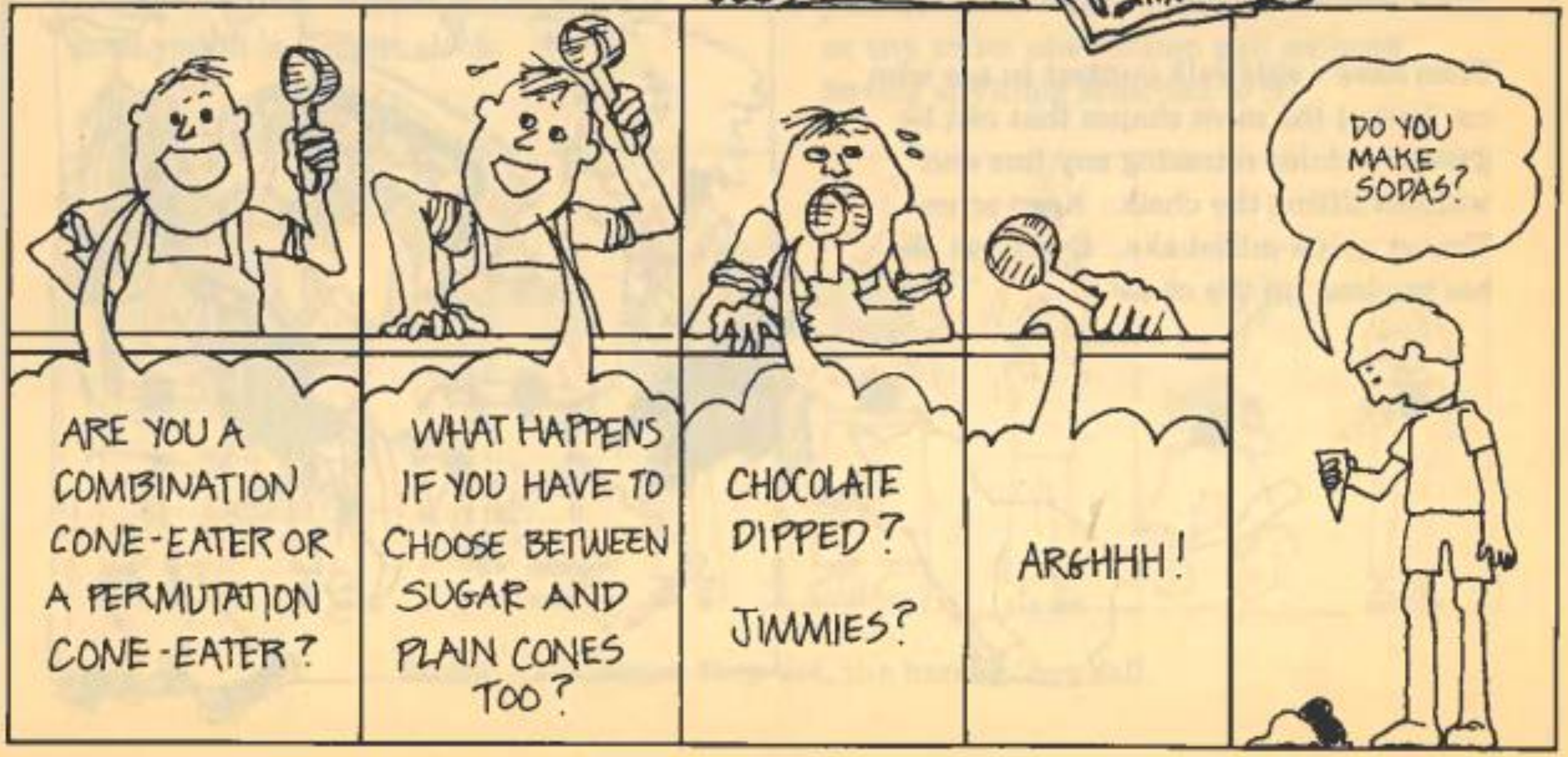
But if you think that a double scoop ice cream cone with jamoca almond fudge on top and strawberry on the bottom is *not* the same as strawberry on the top and jamoca almond fudge on the bottom, then you're talking about *permutations.*

There is a big difference.

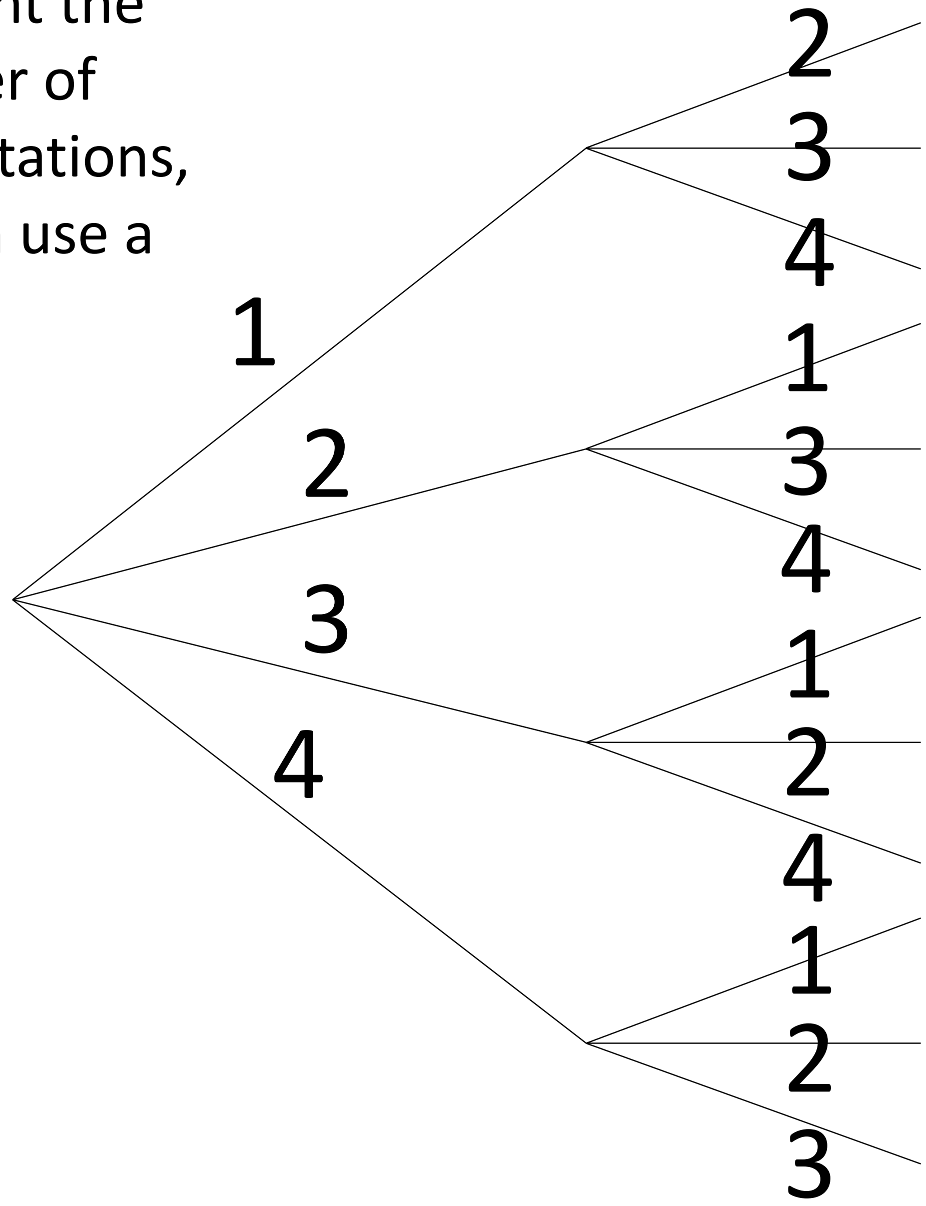
It's easier to start with a smaller number. What are your 3 most favorite flavors? Pistachio, chocolate mint, and rum raisin? Okay. How many different combinations are there? How about permutations? Hint: until you've done this a little, it may help to draw a picture.


FOR 3 FLAVORS THERE ARE 6 COMBINATIONS.......
= rum raisin
= pistachio
= chocolate mint
.....AND 9 PERMUTATIONS.

Now do a little research at the nearest ice cream parlor. Figure out how many different possibilities there are for the flavors they carry. (Stick with combinations.) After you've figured it out, see if you can win yourself a free cone by betting the ice cream man he can't guess how many combinations there are. If he tells you to get lost, try it with one of your friends. Or your big brother.



To count the number of permutations, we can use a tree.



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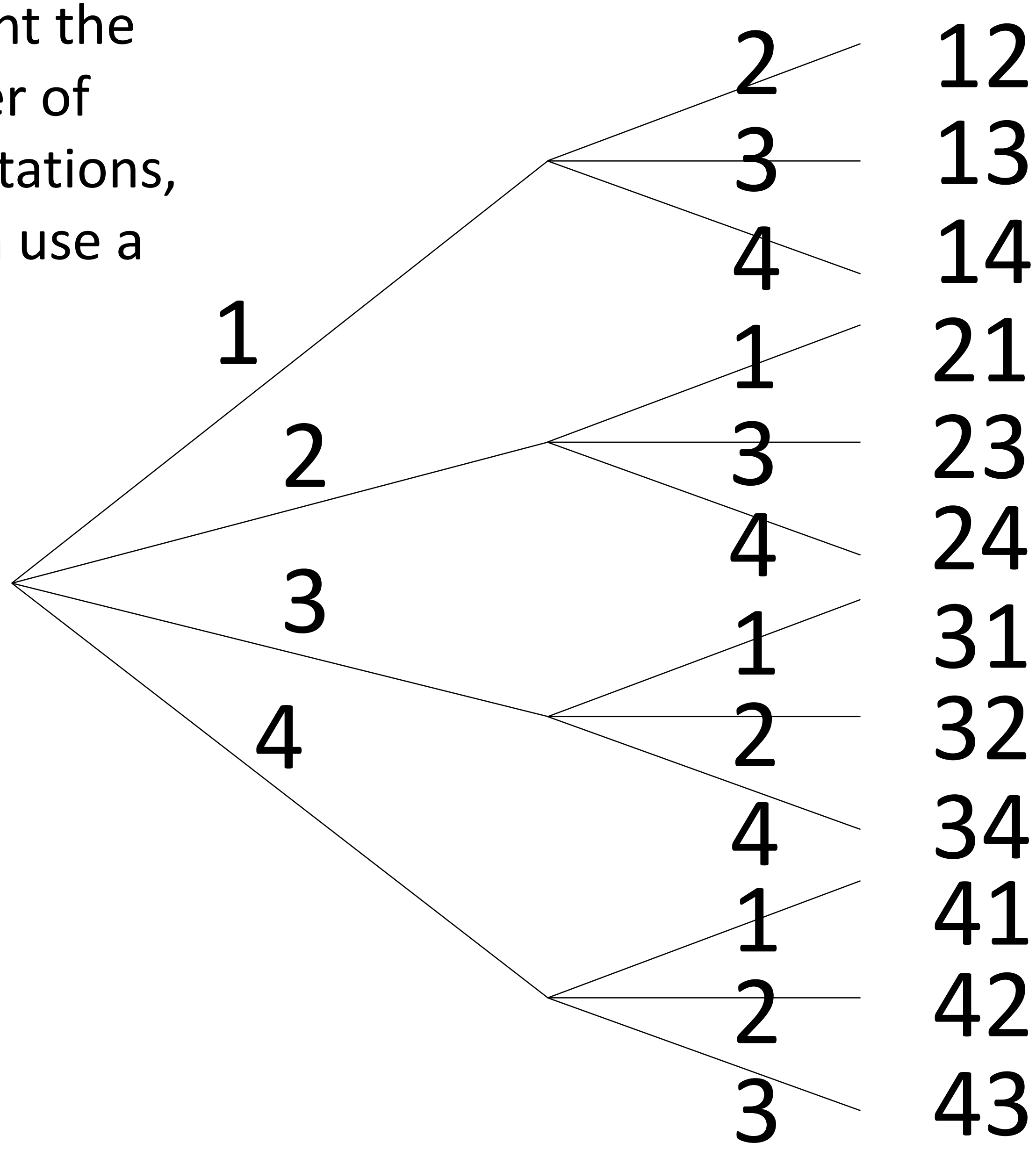

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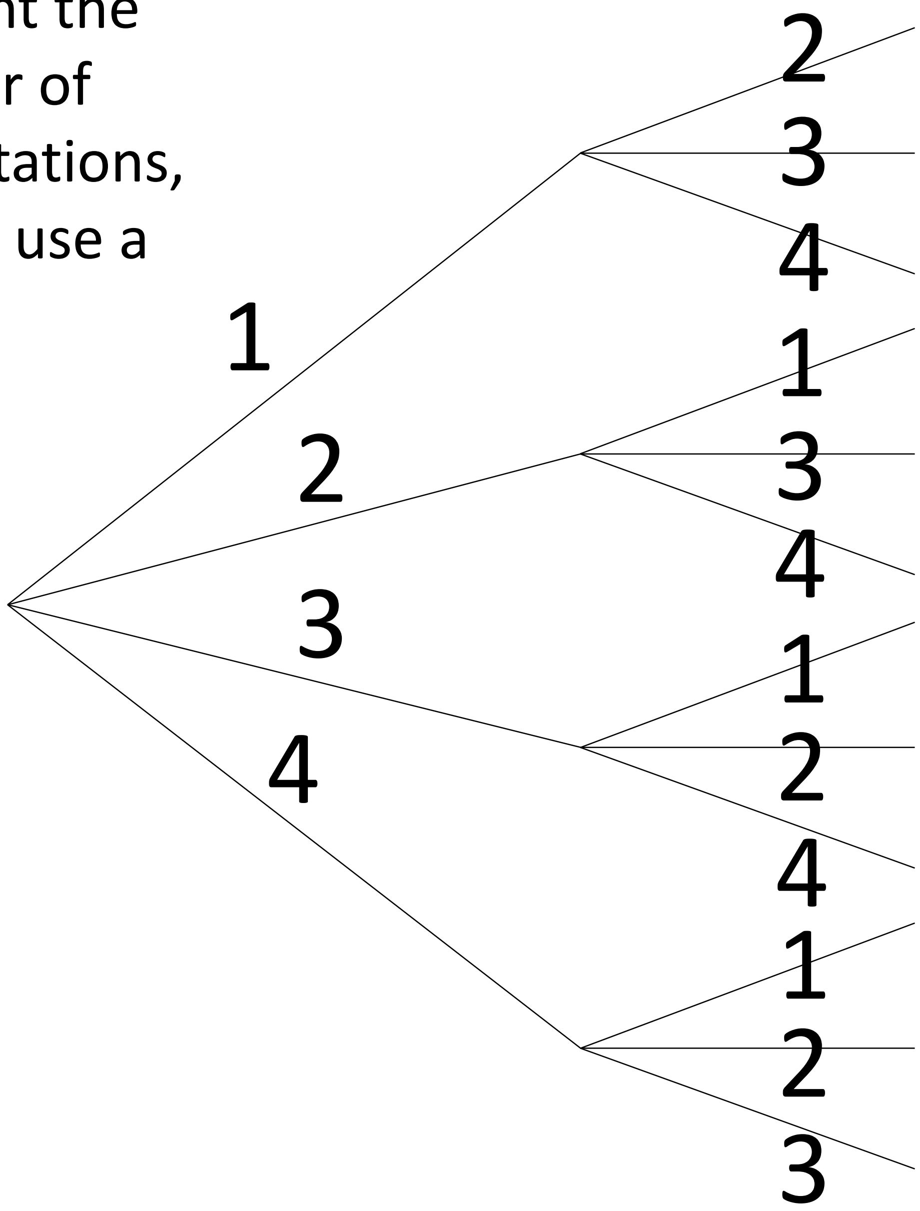

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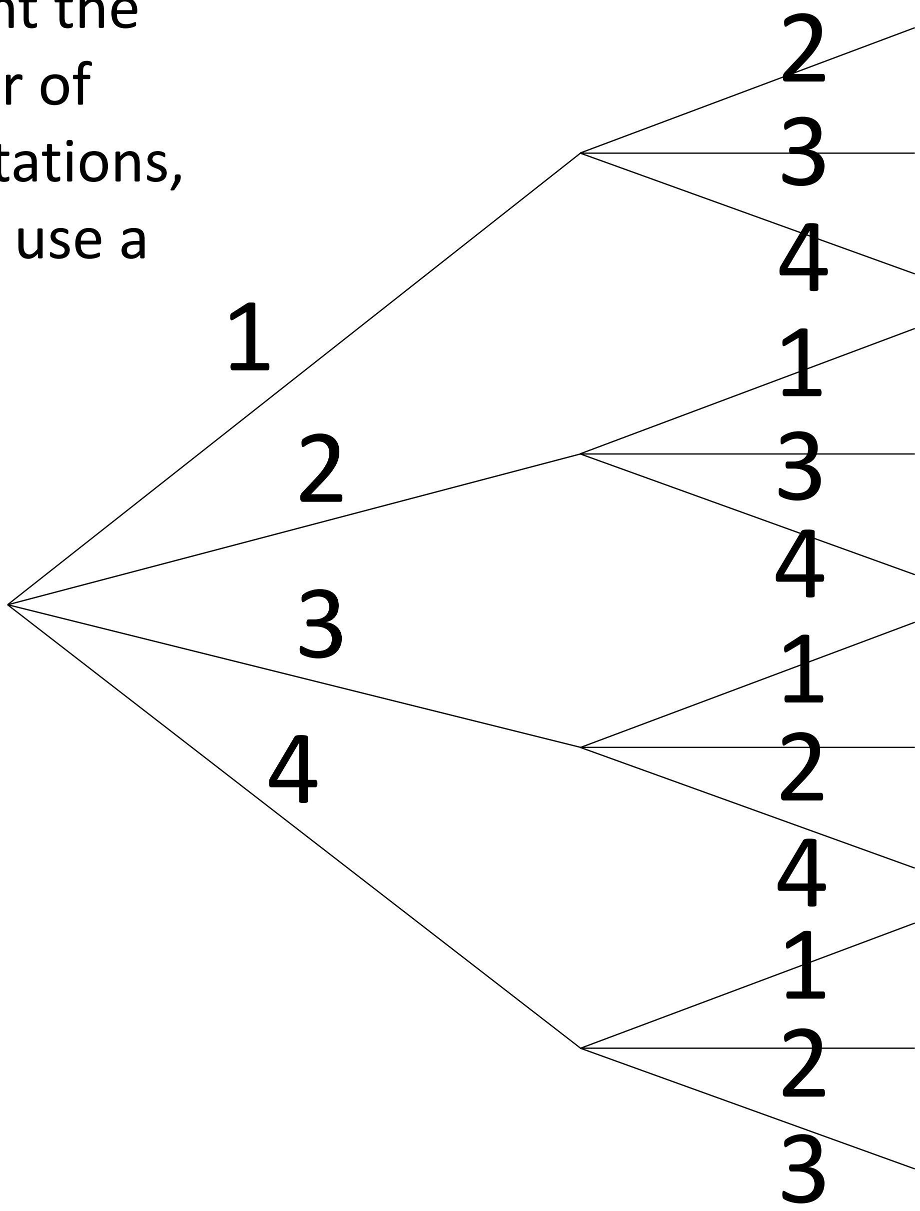

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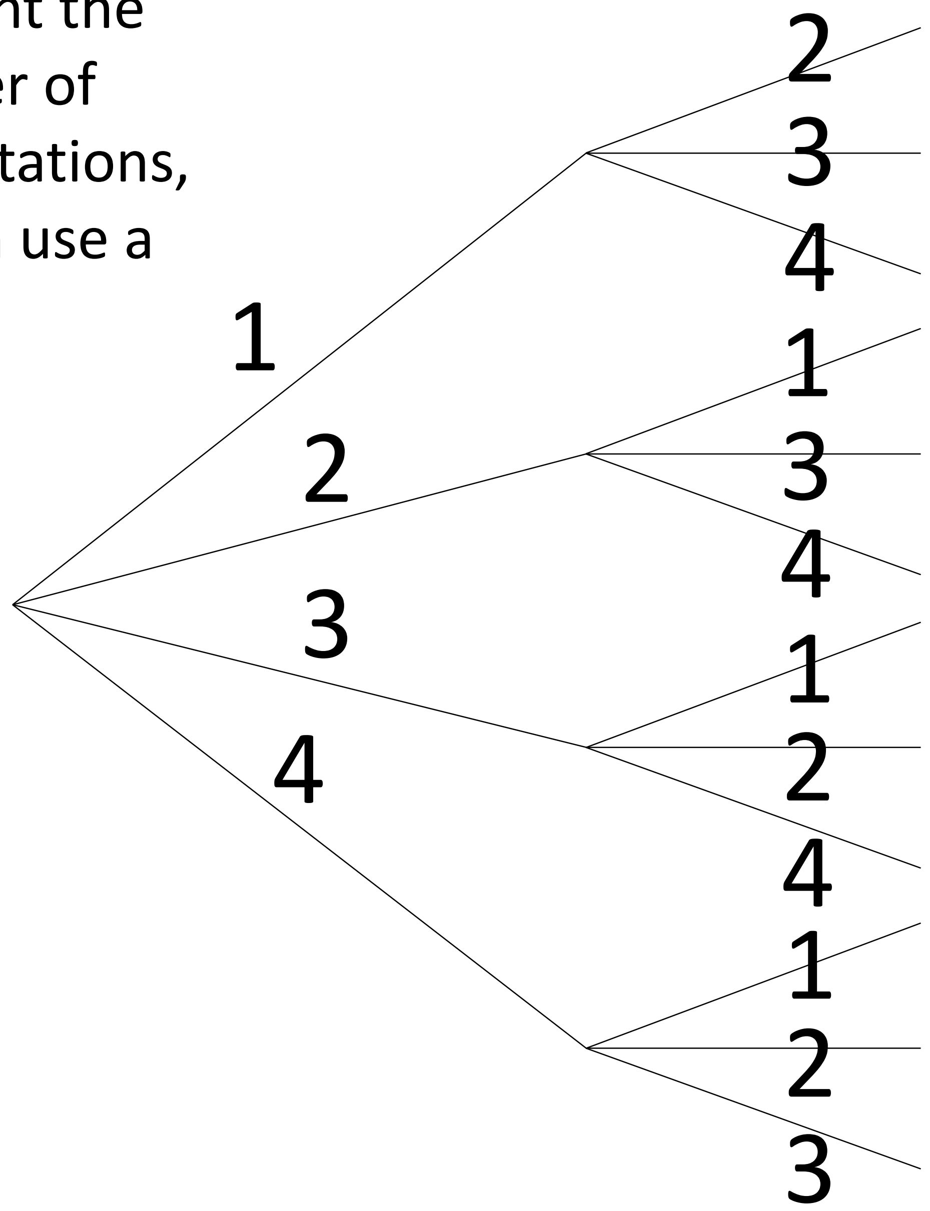

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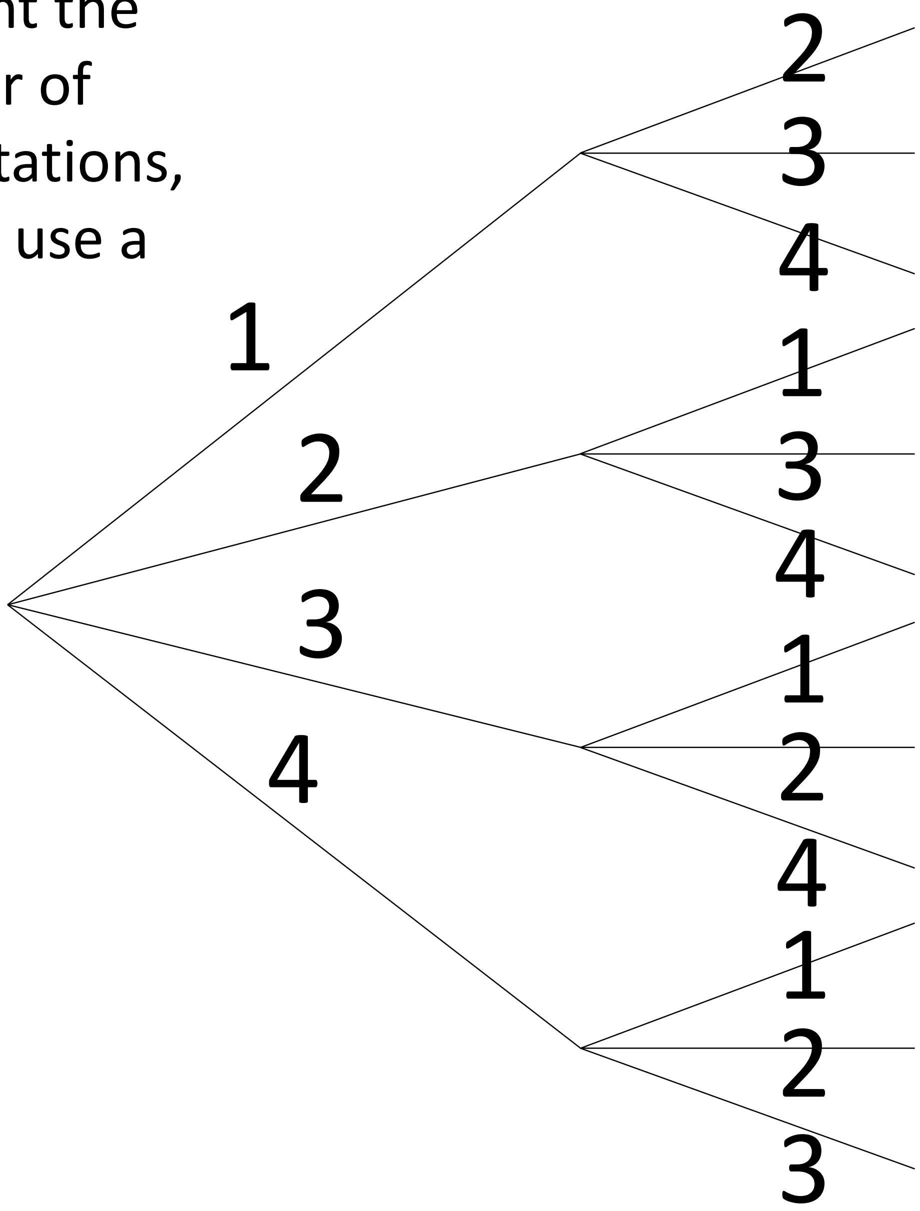

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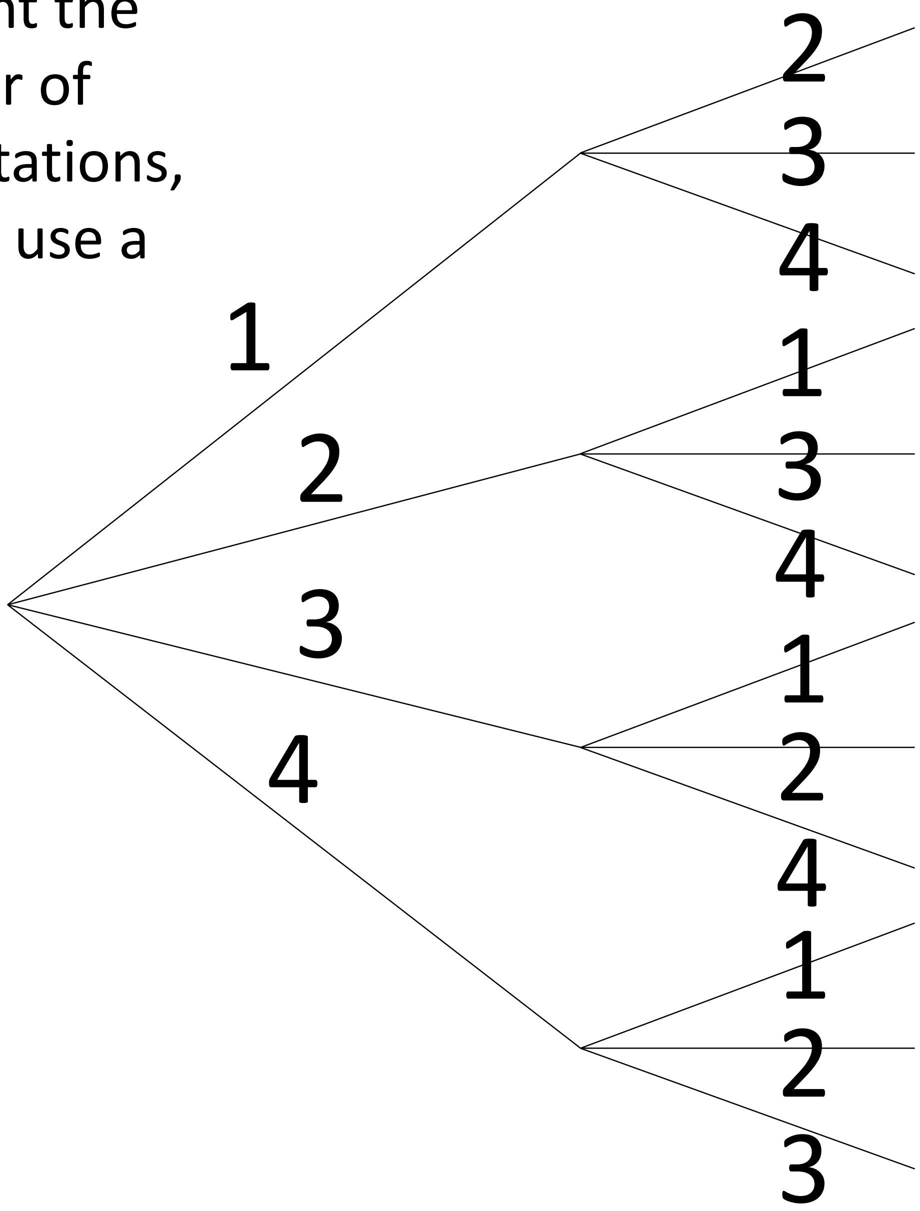

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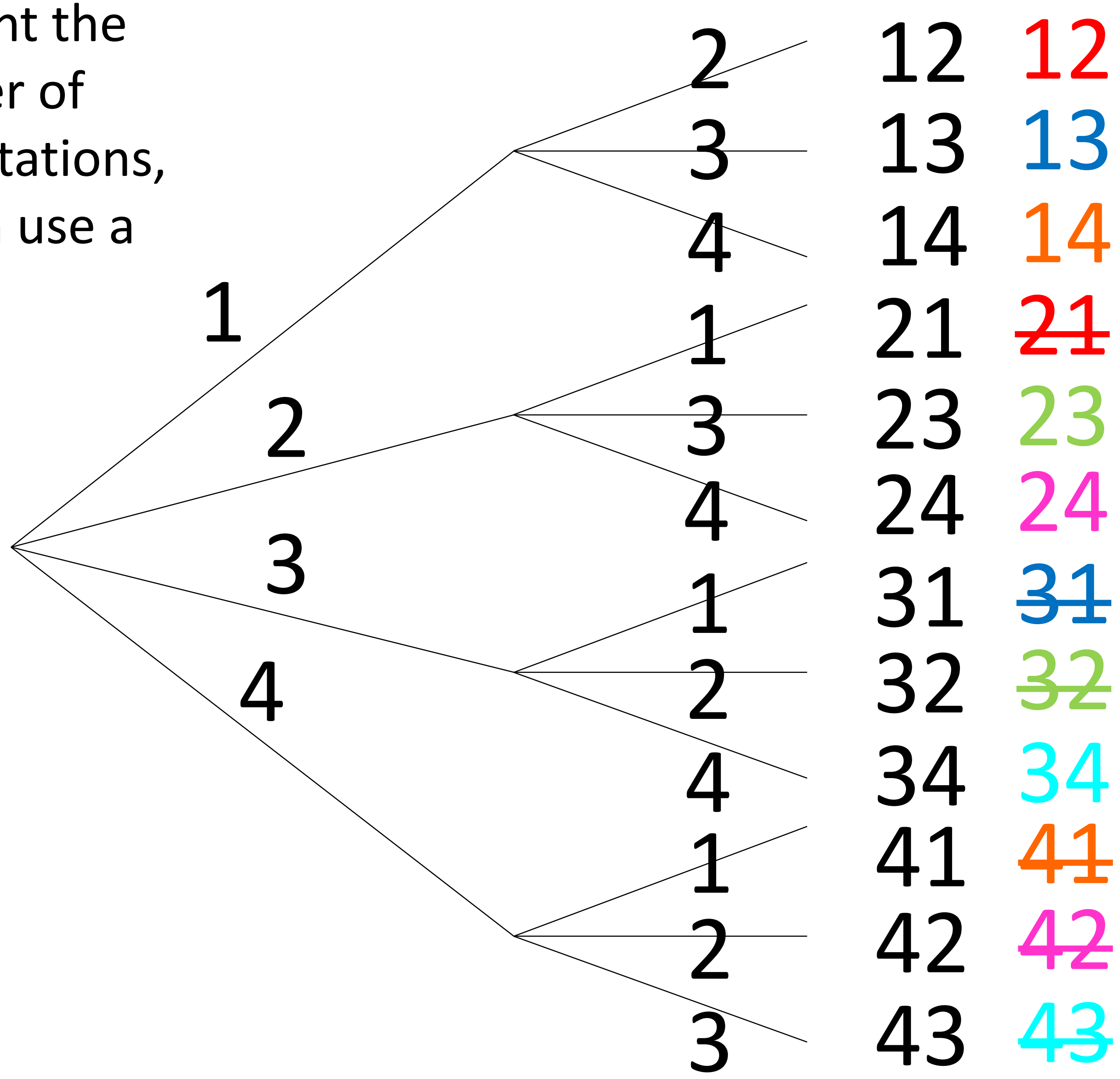

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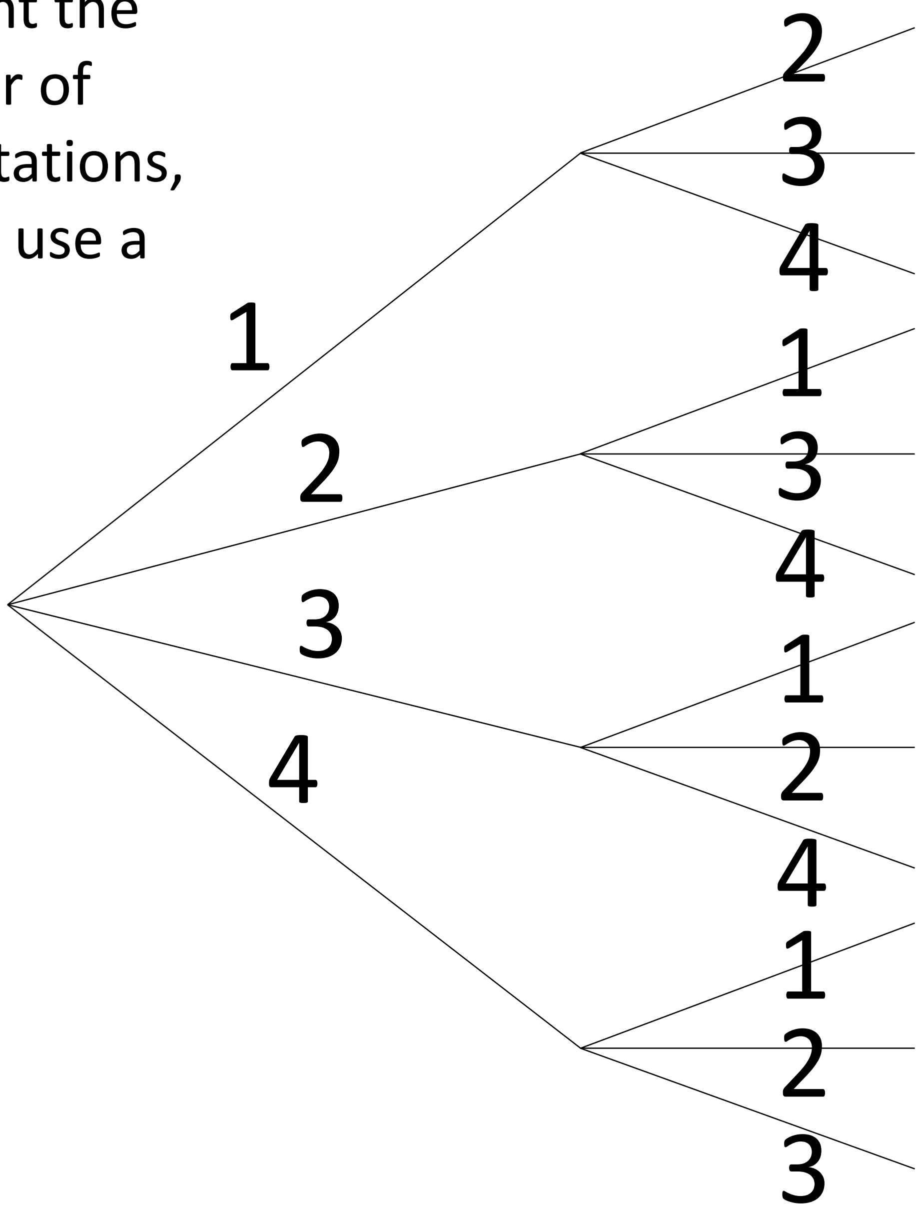

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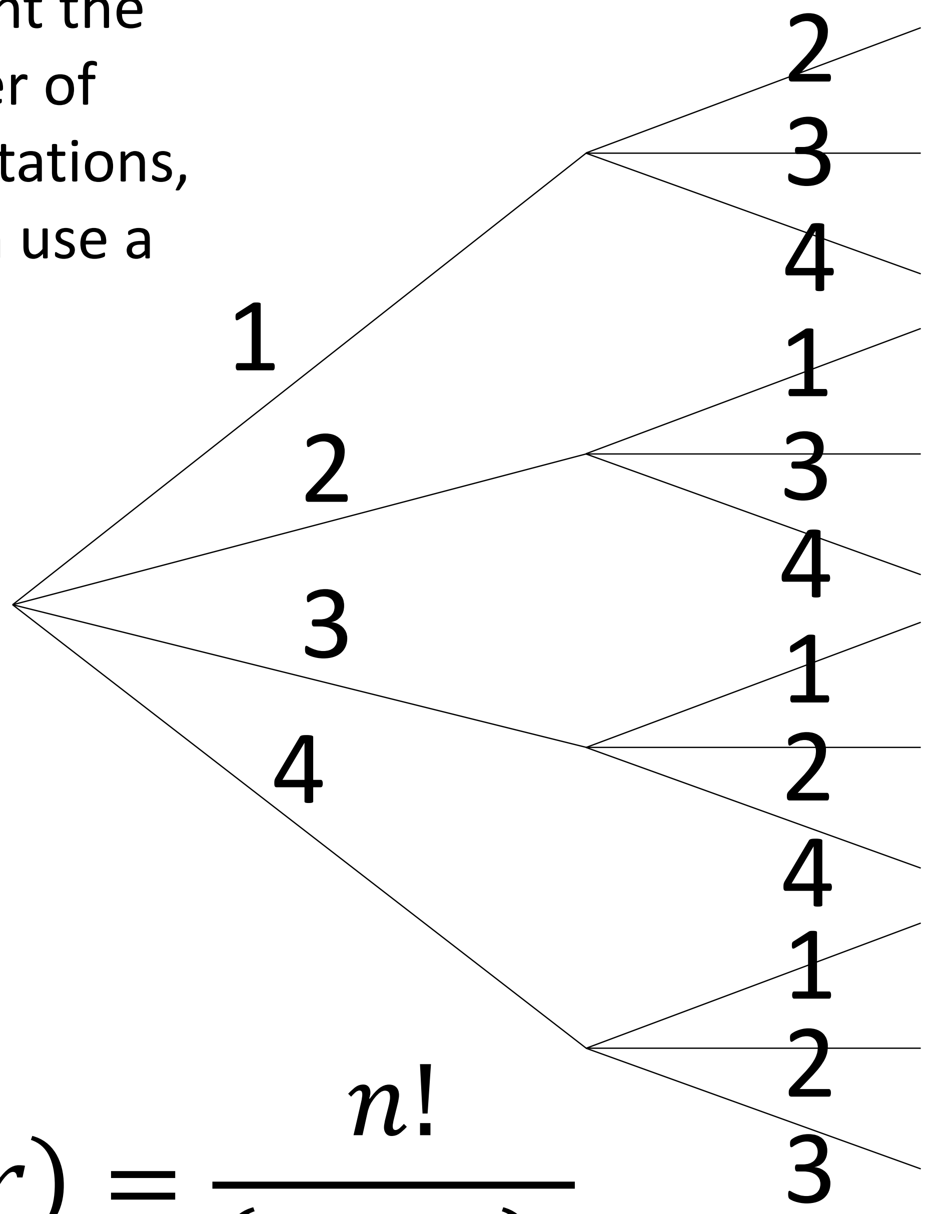

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$$P(n,r) = \frac{n!}{(n-r)!}$$

$$C(n,r) = \frac{n!}{r!(n-r)!}$$

What's the Difference?

In English we use the word "combination" loosely, without thinking if the **order** of things is important. In other words:

"My fruit salad is a combination of apples, grapes and bananas" We don't care what order the fruits are in, they could also be "bananas, grapes and apples" or "grapes, apples and bananas", its the same fruit salad.

"The combination to the safe is 472". Now we **do** care about the order. "724" won't work, nor will "247". It has to be exactly **4-7-2**.

So, in Mathematics we use more *precise* language:

- When the order doesn't matter, it is a **Combination**.
- When the order **does** matter it is a **Permutation**.

So, we should really call this a "Permutation Lock"!

	Order matters No repeats	Can repeat
All Values	Factorial	Counting
Only some places	Permutation	Combination
	Order matters No repeats	Order not important

Combinations Problems

Order doesn't matter.

For example, you have a class of 20 students and you want to choose 4 of them to empty the recycling. How many ways can this be done?

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Recycling Ways = ${}_{20}C_4$

= 4845

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$= 2376$