

## MDM4U – 2.3 – Combinations Questions

- Evaluate each expression
  - $C(6,3)$
  - $C(10,2)$
  - $C(6,4) / C(6,3)$
  - $C(7,6) - C(7,5)$
  - $C(4,0) + C(4,1) + C(4,2) + C(4,3) + C(4,4)$
- Evaluate each expression
  - $C(92,13) / C(90, 11)$
  - $C(n,2)$  if  $n \geq 2$
  - $C(n,3) / C(n,2)$  if  $n \geq 3$
  - $C(n, k) / C(n, k-1)$  if  $n \geq k \geq 1$
- Without evaluating any of the quantities involved, verify each equation:
  - $C(47,5) * C(42,11) = 47! / 5! 11! 31!$
  - $C(18,7) - C(15,7) = C(17,6) + C(16,6) + C(15,6)$
  - $C(72,15) * C(15,4) = C(72,4) * C(68, 11)$
  - $C(20, 4) * C(16, 4) * C(12, 4) * C(8,4) = 20! / 4!^5$
- A ballet choreographer wants 18 dancers for a scene. In how many ways can the choreographer choose the dancers if the company has 20 dancers? 24 dancers?
- Ursula LeGuin runs a small landscaping business. She has on hand 12 kinds of rose bushes, 16 kinds of small shrubs, 11 kinds of evergreen seedlings, and 18 kinds of flowering lilies. In how many ways can Ursula fill an order if a customer wants
  - 15 different varieties consisting of 4 roses, 3 shrubs, 2 evergreens, and 6 lilies?
  - either 4 different roses or 6 different lilies?
- A taxi is shuttling 11 students to a concert. The taxi can hold only 4 students, In how many ways can 4 students be chosen for
  - the taxi's first trip
  - the taxi's second trip
- In how many ways can a jury of 6 men and 6 women be chosen from a group of 10 men and 15 women?

### Combinations and Probability

- A club has 23 members, 20 of whom are married. A committee of 3 members is selected at random. Consider the events E, F, and G.  
E: Mr. and Mrs. Pascal are on the committee  
F: all the committee members are married  
G: Mr. Pascal is on the committee.  
Calculate the probability of each event
  - E
  - F
  - G
  - $E \cap F$
  - $E \cup F$
- A bridge hand consists of 13 cards dealt from a well-shuffled deck of 52 cards
  - Calculate the probability that a hand contains 7 hearts

- b. Calculate the probability that a hand contains a 7-card suit (7 cards belonging to the same suit).
10. A box of 50 tulip bulbs contains 5 that will not germinate. If you purchase a half-dozen bulbs to plant in your garden, find the probability of each event:
  - a. at least one bulb will not germinate
  - b. at most one bulb will germinate
11. A record club offers 5 free records from a list of 50 records as an incentive for joining the club. Suppose that records are selected at random
  - a. Calculate the probability that you select 2 favourite records and any 3 others
  - b. If you have already selected your 2 favourite records, what is the probability that your third favourite is also included in your selection of 5?
  - c. If you have already selected your 3 favourite records, what is the probability that the selection also includes the 2 records that you dislike most?
12. A dance committee for a class of 15 girls and 10 boys is formed by randomly selecting 4 students from the class.
  - a. Calculate the probability that the committee consists of John, Michael and 2 other boys
  - b. Calculate the probability that the committee consists of John, Michael and 2 girls
  - c. If John and Michael have already been chosen for the committee, what is the probability that the committee will consist of 3 boys and 1 girl.
  - d. If John and Michael have already been chosen for the committee, what is the probability that the committee will consist of 2 boys and 2 girls.
  - e. If John and Michael have already been chosen for the committee, what is the probability that the committee will consist of 3 girls and 1 boy.
13. In a lottery called “6-in-a-row”, six different numbers are chosen at random from 1 through 49, inclusive. If a ticket has been purchased, find the probabilities of the following events:
  - a. the first number on the ticket matches the first number in the draw.
  - b. the first two numbers match
  - c. the grand prize is won where all six numbers match.
14. In a track meet, five entrants of equal ability are competing. What is the probability that:
  - a. the finish will be in descending order of the entrants’ ages
  - b. Sandy will be first?
  - c. Shanaze is first and Dandy is second?
15. An infant typed three strokes on a keyboard. If all of the characters typed were letters of the alphabet, what is the probability that the characters that were struck were three consecutive letters in alphabetical order?
16. The starting line-up for a basketball team is to consist of two forwards and three guards. Two brothers, Matthew and Tony, play on the team and Matthew is a forward and Tony a guard. There are four forwards and six guards from which to choose the line-up. If the starting players are chosen at random, what is the probability that the two brothers will end up in the starting line-up?
17. Hans has 12 good friends, five of them male and seven of them female. He decides to have a dinner party but can only invite seven because his dining room

table can only seat eight people. He decides to invite his guests by lot. What is the probability that:

- a. there will be four males and four females at the party?
  - b. Rivka will be among those invited?
  - c. Hans will have only female guests?
18. A group of 12 people is going out on the town Saturday night. The group will take three cars with four people in each car. If they distribute themselves among the cars at random, what is the probability that Rafael and Chantal will be in the same car?
19. Ten boys and 12 girls decide to rent a 16-passenger van and a six-passenger car to drive to a rock concert in a nearby city. If the group is distributed randomly between the vehicles, what is the probability that:
- a. there are no boys in the car?
  - b. there are no girls in the car?
  - c. Alan and Margaret are both in the van?
  - d. there are more girls than boys in the car?

### Answers

1. a. 20, b. 45, c. 0.75, d. -14, e. 16. 2 a.  $161/3$ , b.  $n(n-1)/2$ , c.  $(n-2)/3$  d.  $(n-k+1)/k$

4. 190, 134596. 5. a. 283 billion, b. 19059. 6. a. 330, b. 35. 7. 1051050

8. (a) 0.011857 (b) 0.6437 (c) 0.13043 (d) 0.1129 (e) 0.644267 9. (a) 0.0088 (b) 0.035

10. (a) 0.4874 (b)  $2.84 \times 10^6$  11. (a) 0.00816 (b) 0.16917 (c) 0.002659

12. (a) 0.00553 (b) 0.0083 (c) 0.11067 (d) 0.415 (e) 0.875

13. (a)  $1/49$  (b)  $1/2352$  (c)  $9.932 \times 10^{-11}$  14. (a)  $1/120$  (b)  $1/5$  (c)  $1/20$  15.  $3/2197$  16.  $1/4$

17. (a) 0.44 (b)  $7/12$  (c)  $1/792$  18.  $1/11$  19. (a) 0.012 (b)  $10/3553$  (c) 0.52 (d) 0.42