MDM4U – 2.1 – Factorial Notation

1. Express the following using factorial notation:

$(a)\ 4\times3\times2\times1 \quad (b)10\times9\times8 \quad (c)8\times7\times4\times3\times2 \quad (d)\dfrac{(35\times34\times33\times32)}{(27\times26\times25\times24\times23)}$

2. Match each expression on the top with the equivalent expression on the bottom.

$(a)\dfrac{14!}{13!} \quad (b)\dfrac{52!}{51!} \quad (c)\dfrac{101!}{99!} \quad (d)20\times10! \quad (e)90\times8! \quad (f)30\times4!$

$(i)10100 \quad (ii)6! \quad (iii)52 \quad (iv)10! \quad (v)14 \quad (vii)20!$

3. Find the value of each of the following.

$(a)\dfrac{8!}{5!} \quad (b)\dfrac{19!}{13!} \quad (c)\dfrac{21!}{17!4!} \quad (d)\dfrac{9!}{7!2!} \quad (e)\dfrac{155!}{152!} \quad (f)\dfrac{93!}{89!4!}$

4. Can the factorial operation be defined for negative numbers? Explain your answer.

5. Simplify, assuming that n is a whole number.

$(a)\ n(n-1)! \quad (b)\ n!(n+1) \quad (c)\ (n-1)!(n^2+n) \quad (d)\ n!(n^2+3n+2)$

$(e)\dfrac{n!}{(n-2)!} \quad (f)\dfrac{(n+2)!}{(n-1)!}$

6. Simplify the following as far as possible:

$(a)\dfrac{(n+5)!}{(n+3)!} \quad (b)\dfrac{n!}{(n-1)!} \quad (c)\dfrac{(n+1)!}{n!} \quad (d)\dfrac{(n-r+1)!}{(n-r)!}$

$(e)\dfrac{(n-r)!}{(n-r-1)!} \quad (f)\dfrac{(n-r+1)!}{(n-r-1)!}$

7. Solve for n, assume that n is a whole number.

$(a)\dfrac{(n+5)!}{(n+4)!}=7 \quad (b)\dfrac{(n+2)!}{n!}=20 \quad (c)\dfrac{n!}{2(n-2)!}=6 \quad (d)\dfrac{(n+1)!}{(n-1)!}=12$

$(e)\dfrac{(n+1)!}{n!}=9 \quad (f)\dfrac{n!}{(n-2)!}=20 \quad (g)\dfrac{3(n+1)!}{(n-1)!}=126 \quad (h)\dfrac{2n!}{(n-3)!}=84n$

8. The senior choir has rehearsed 5 songs for an assembly. In how many ways can the choir sing the songs?
9. In how many different orders can eight nominees for the student's council give their speeches at an assembly?
10. How many ways can you arrange the letters in the word FACTOR?
11. How many ways can Andrew Wiles arrange four different textbooks on a shelf in his office at Princeton?

Answers

1. a. 4! b. 10!/7! c.8!4!/6! d.35!22!/31!27!
2. a-v, b-iii, c-i, d-vii, e-iv, f-ii
3. a. 336 b. 19,535,040 c. 5985 d. 36 e. 3,652,110 f. 2,919,735
4. The book I got this from says “No”, but it is actually yes. Factorial needs are more complex definition before you can do it, anticipate 4^{th} year university math!
5. a. n! b.(n+1)! c. (n+1)! d. (n+2)! e. n(n-1) or n^2-n f. (n+2)(n+1)n
6. a. (n+4)(n+5) b. n c. n+1 d. n-r+1 e. n-r f. (n-r+1)(n-r)
7. a. 2 b. 3 c. 4 d. 3 e. 8 f. 5 g. 6 h. 8
8. 120
9. 40,320
10. 720
11. 24