Conditional Probability

Given that…..

A.1.6 determine whether two events are independent or dependent and whether one event is conditional on another event, and solve related probability problems [e.g., calculate P(A and B), P(A or B), P(A given B)] using a variety of strategies (e.g., tree diagrams, lists, formulas)

A randomized study of 100 people was done for a new sleeping pill.

	Slept Well	Did not sleep well
Placebo	35	20
New Medication	30	15

A randomized study of 100 people was done for a new sleeping pill.

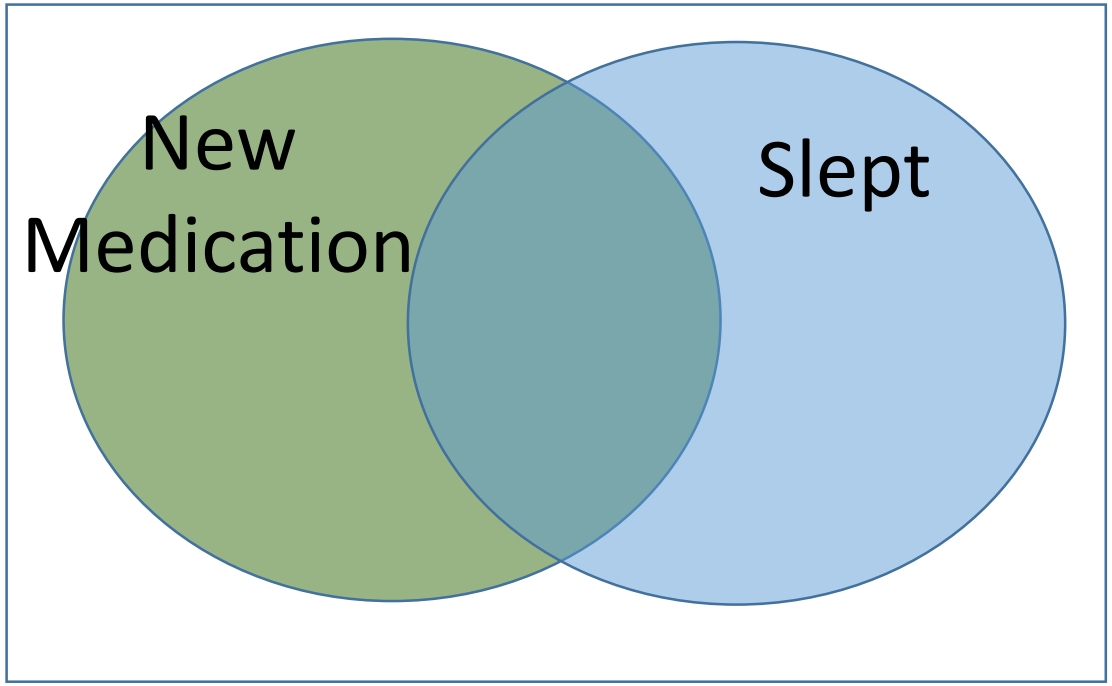
	Slept Well	**Did not sleep well**
Placebo	35	20
New Medication	30	15

Draw the Venn Diagram.

A randomized study of 100 people was done for a new sleeping pill.

	Slept Well	Did not sleep well
Placebo	35	20
New Medication	30	15

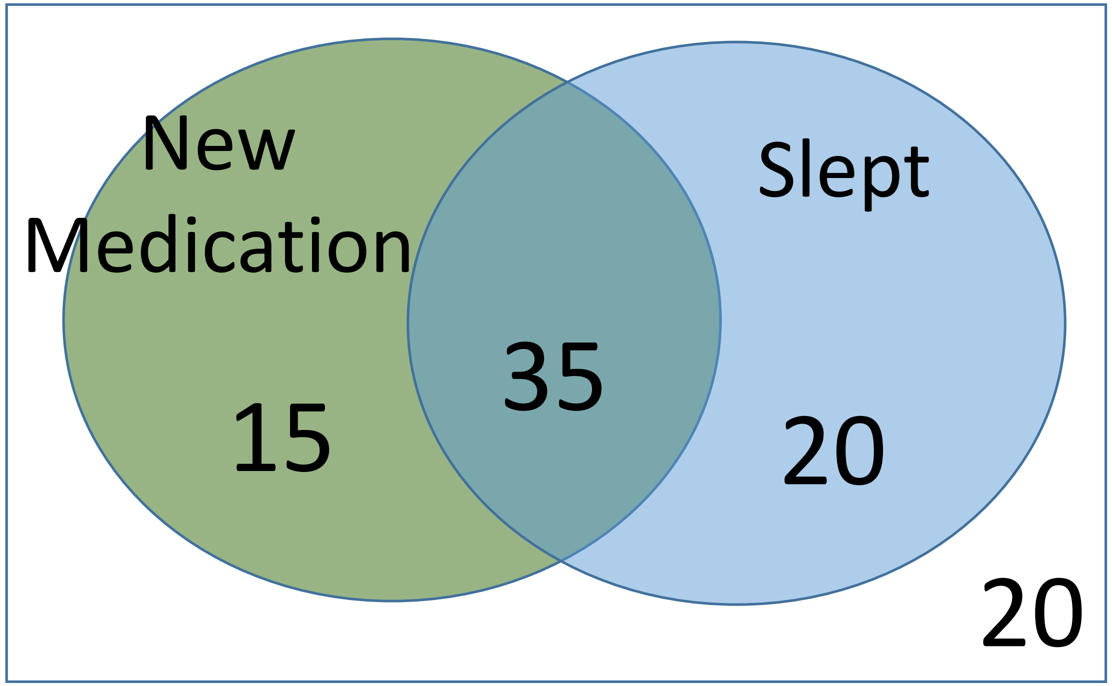
Draw the Venn Diagram.

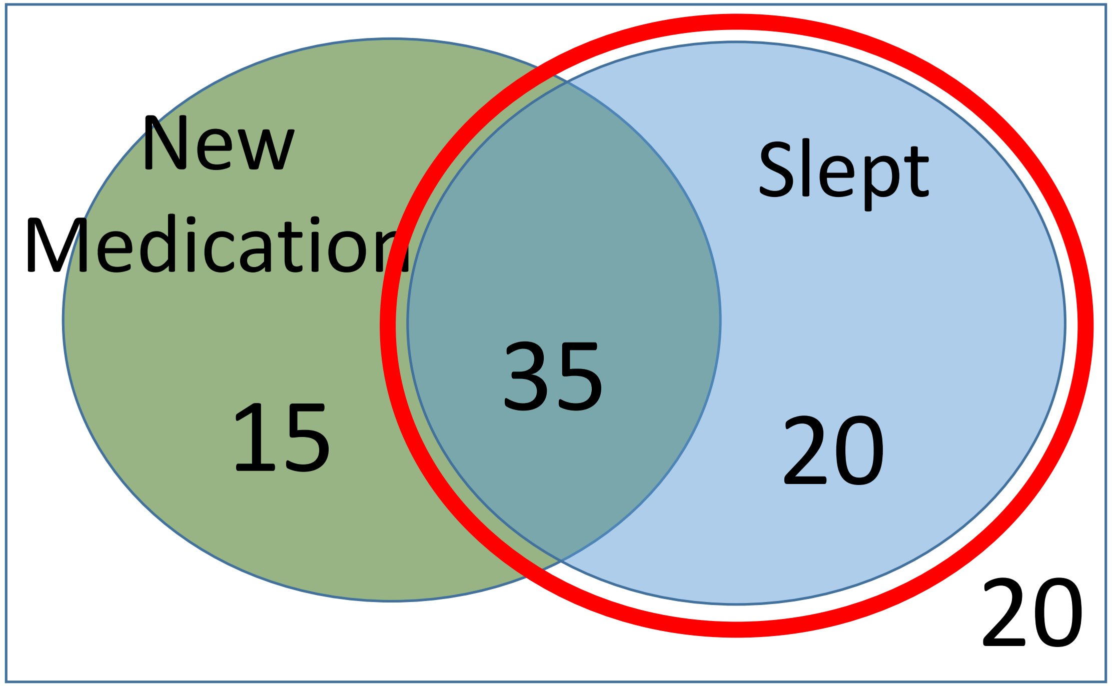


A randomized study of 100 people was done for a new sleeping pill.

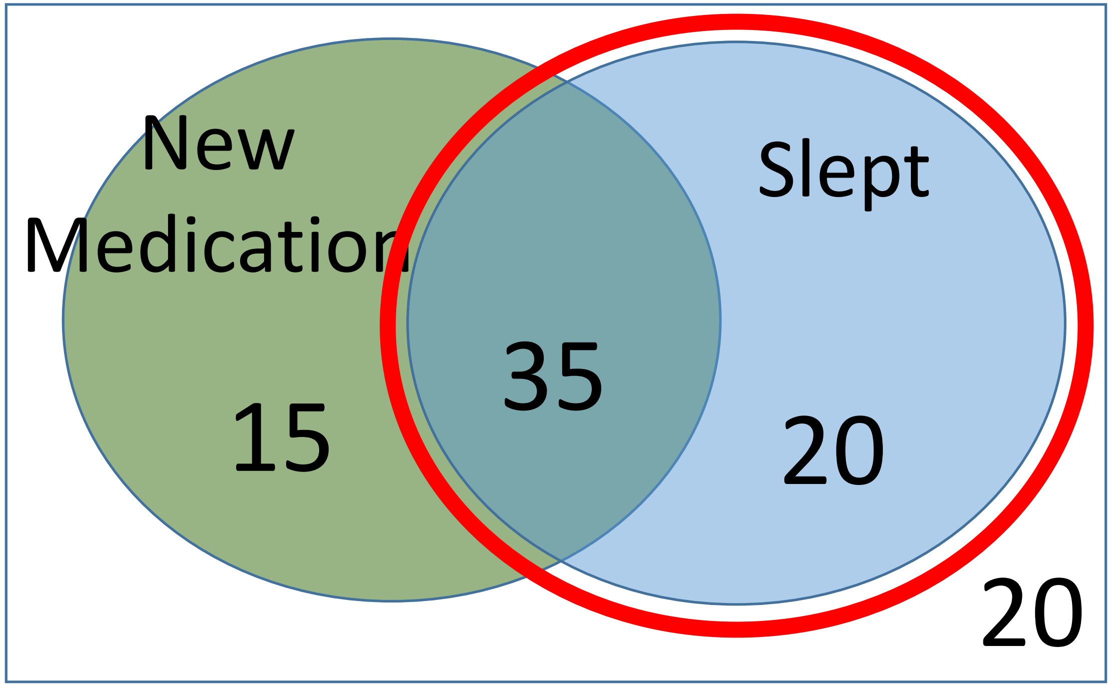
	Slept Well	Did not sleep well
Placebo	35	20
New Medication	30	15

Draw the Venn Diagram.



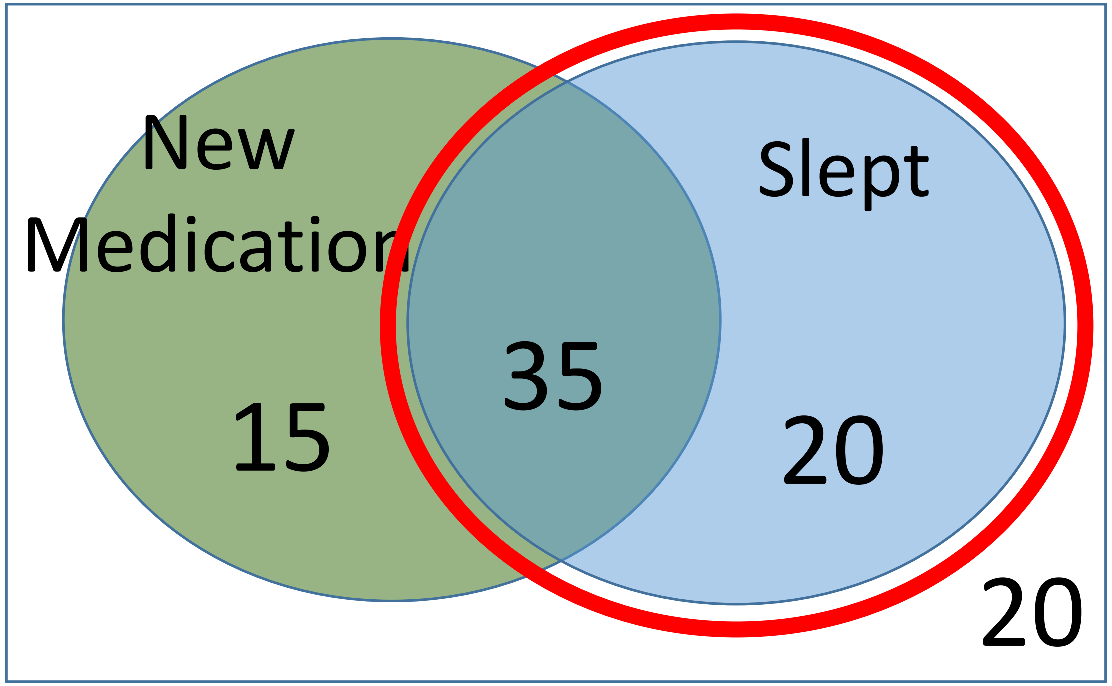


Given that they slept well, what was the probability they had medication?



Given that they slept well, what was the probability they had medication?

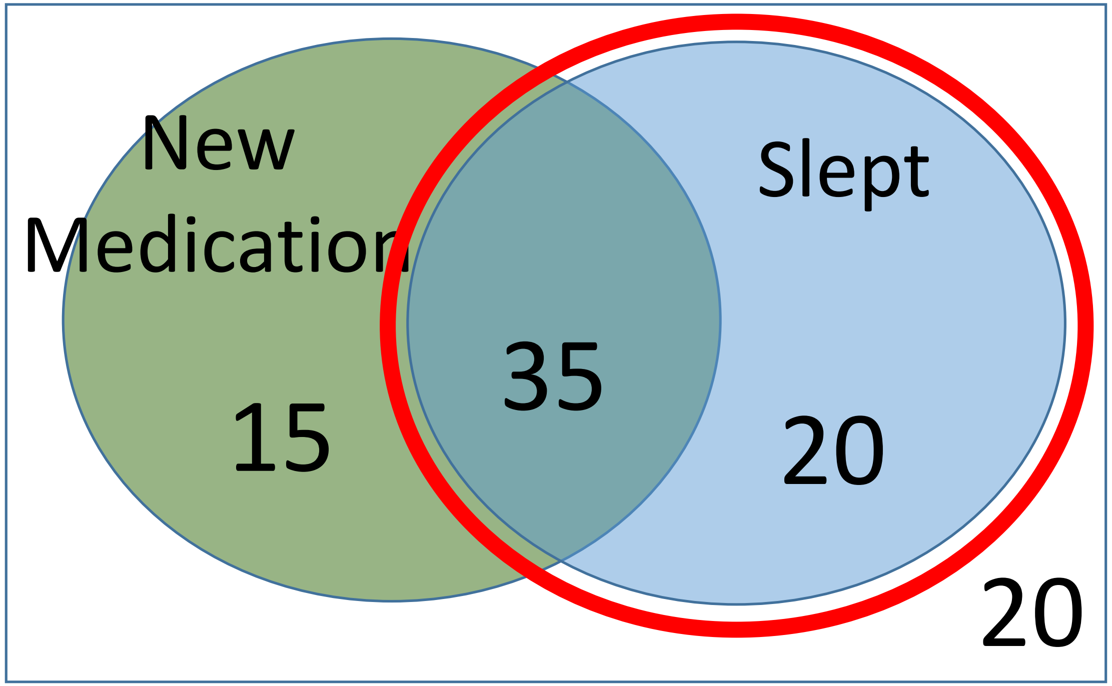
$$= \frac{35}{55} \qquad = 63.6\%$$



Given that they slept well, what was the probability they had medication?

$$= \frac{35}{55} \qquad = 63.6\%$$

Given that they slept well, what was the probability they had a placebo?

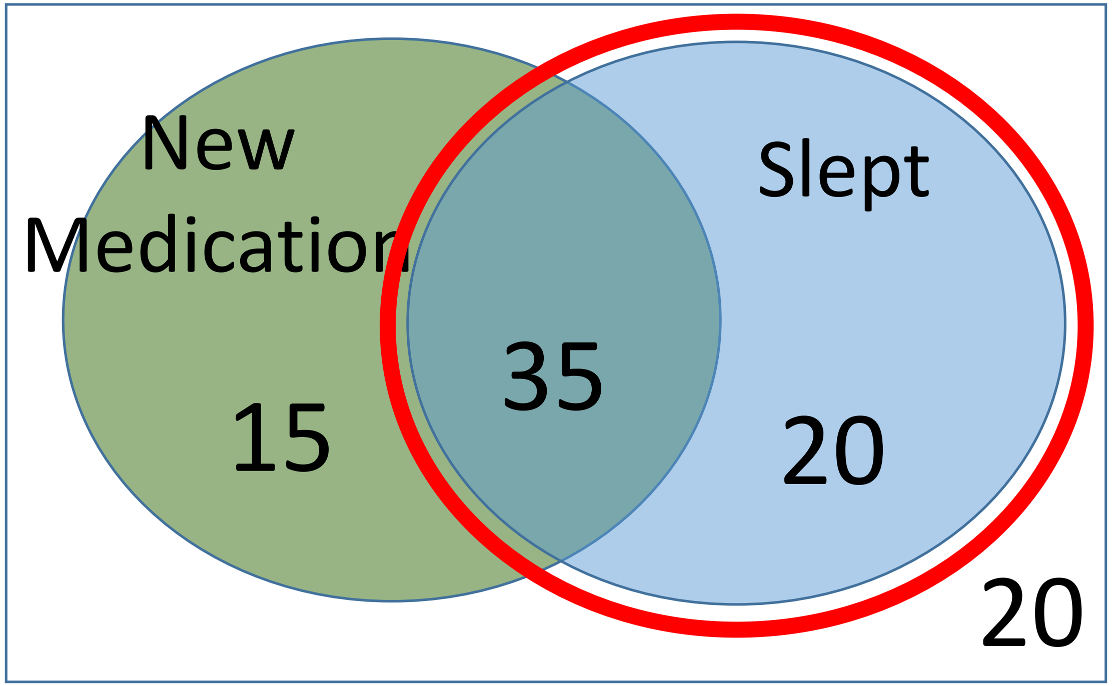


Given that they slept well, what was the probability they had medication?

$$= \frac{35}{55} \qquad = 63.6\%$$

Given that they slept well, what was the probability they had a placebo?

$$= \frac{20}{55} \qquad = 36.3\%$$



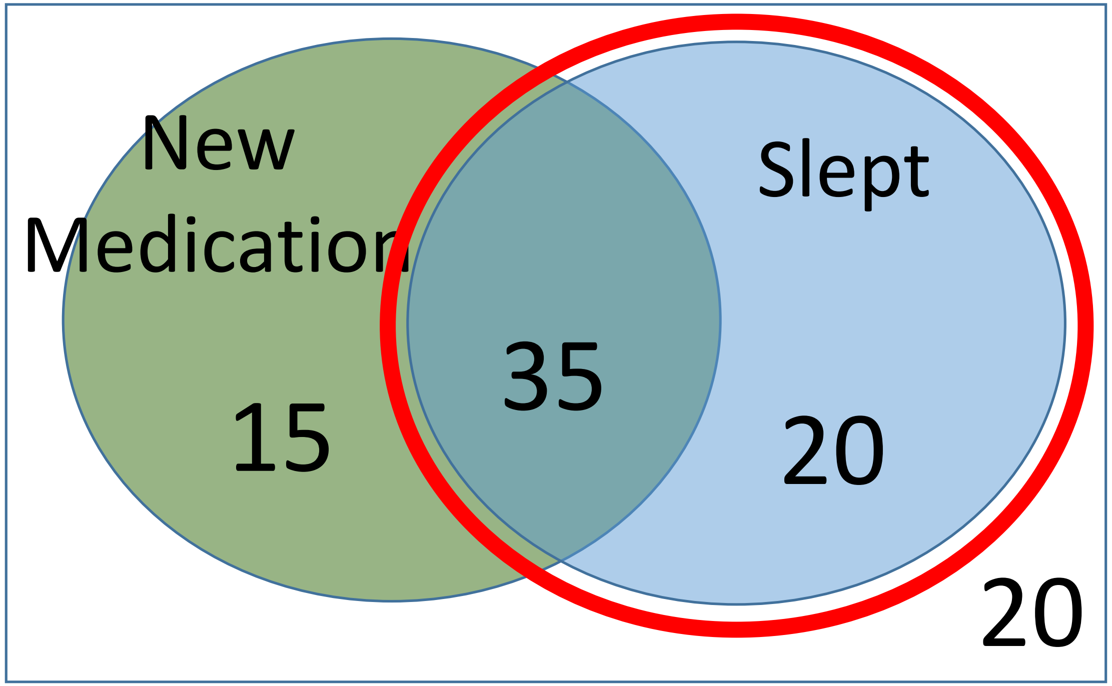
What do we know about the medication? Does it work?

Given that they slept well, what was the probability they had medication?

$$= \frac{35}{55} \qquad = 63.6\%$$

Given that they slept well, what was the probability they had a placebo?

$$= \frac{20}{55} \qquad = 36.3\%$$



What do we know about the medication? Does it work?

Given that they slept well, what was the probability they had medication?

$$= \frac{35}{55} \qquad = 63.6\%$$

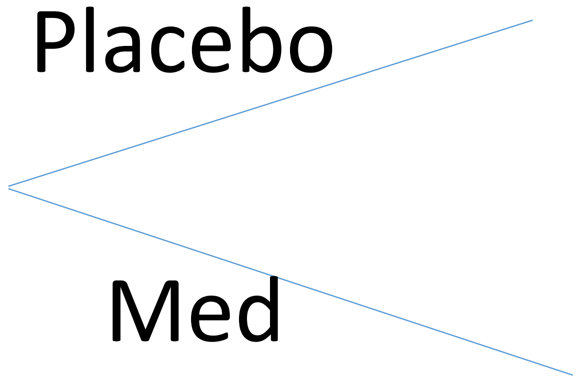
Given that they slept well, what was the probability they had a placebo?

$$= \frac{20}{55} \qquad = 36.3\%$$

Because the people with the medication were more likely to sleep well, we can tell that the medication works better than a placebo.

A randomized study of 100 people was done for a new sleeping pill.

	Slept Well	**Did not sleep well**
Placebo	35	20
New Medication	30	15

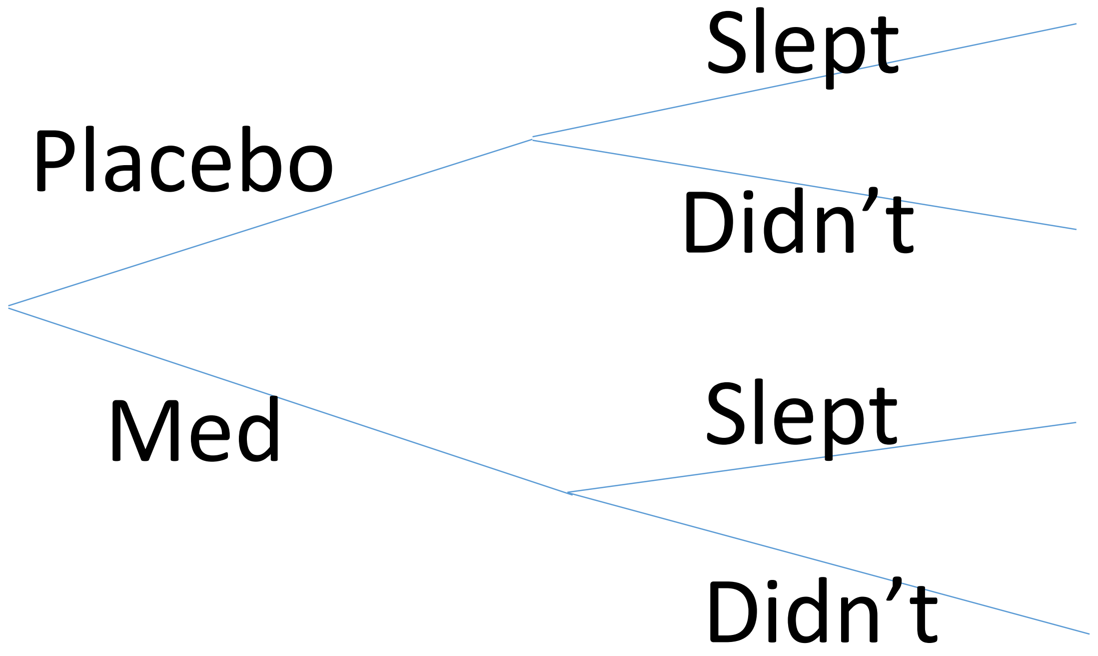


Placebo

Med

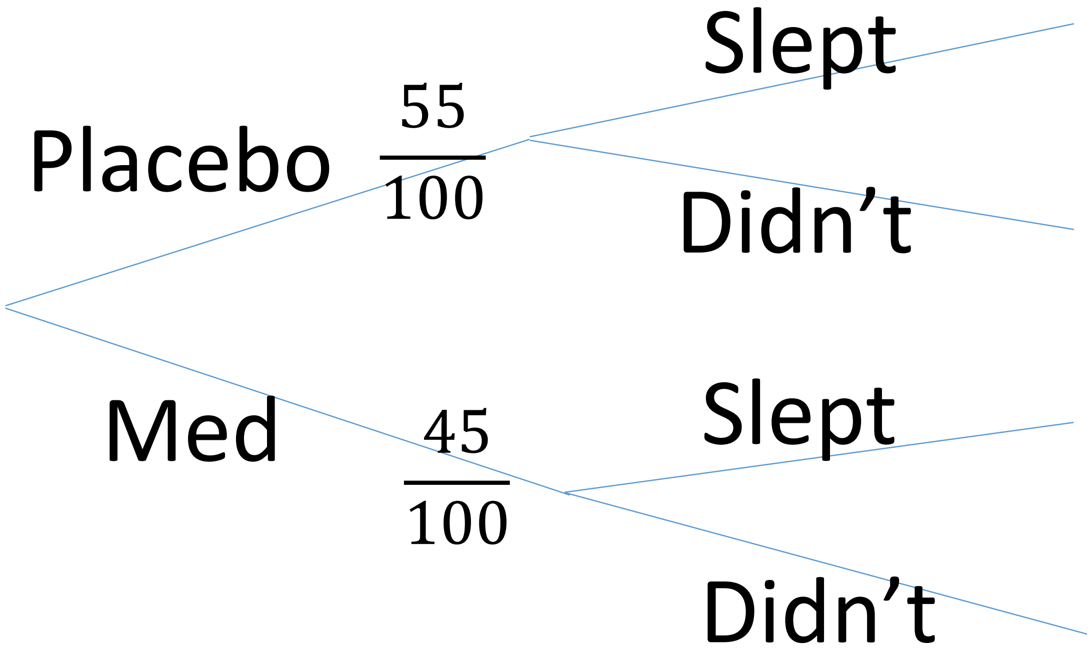
A randomized study of 100 people was done for a new sleeping pill.

	Slept Well	Did not sleep well
Placebo	35	20
New Medication	30	15



A randomized study of 100 people was done for a new sleeping pill.

	Slept Well	Did not sleep well
Placebo	35	20
New Medication	30	15



Placebo $\frac{55}{100}$

Slept

Didn’t

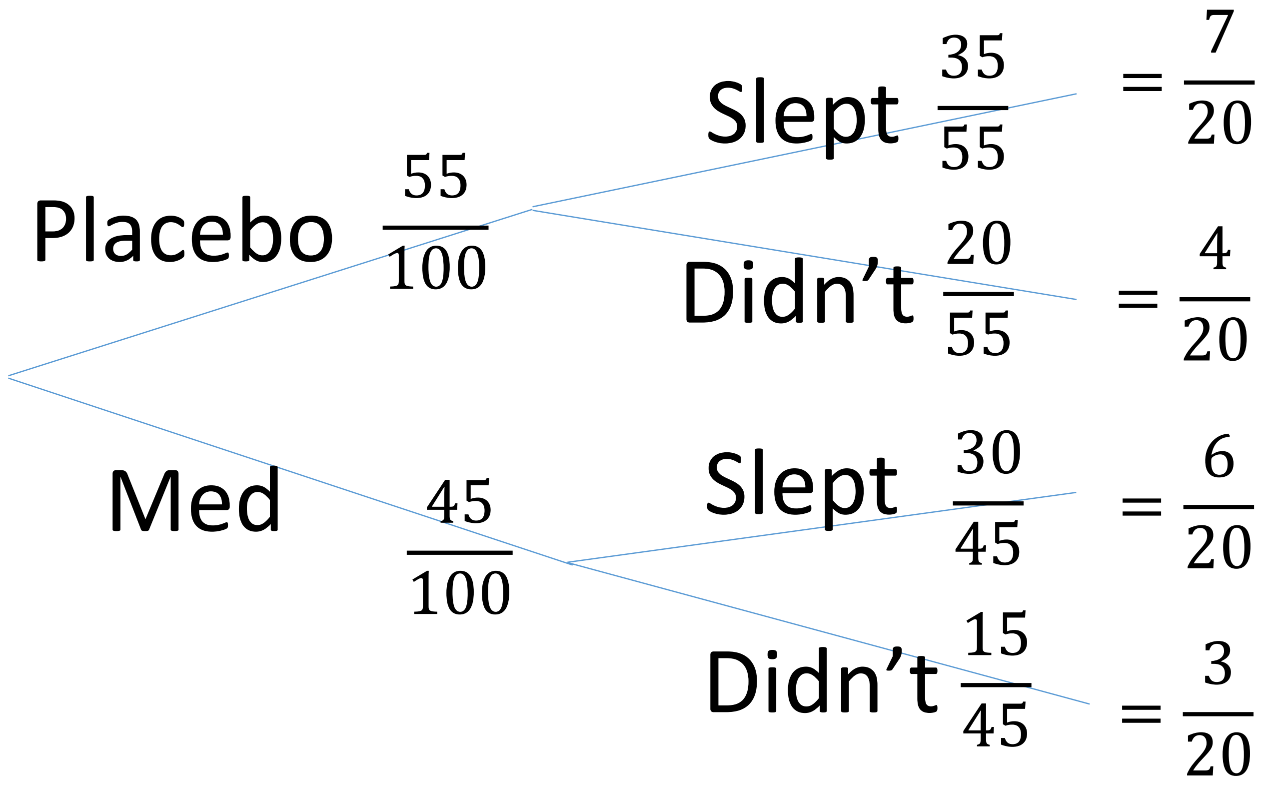
Med $\frac{45}{100}$

Slept

Didn’t

A randomized study of 100 people was done for a new sleeping pill.

	Slept Well	Did not sleep well
Placebo	35	20
New Medication	30	15



Placebo $\frac{55}{100}$

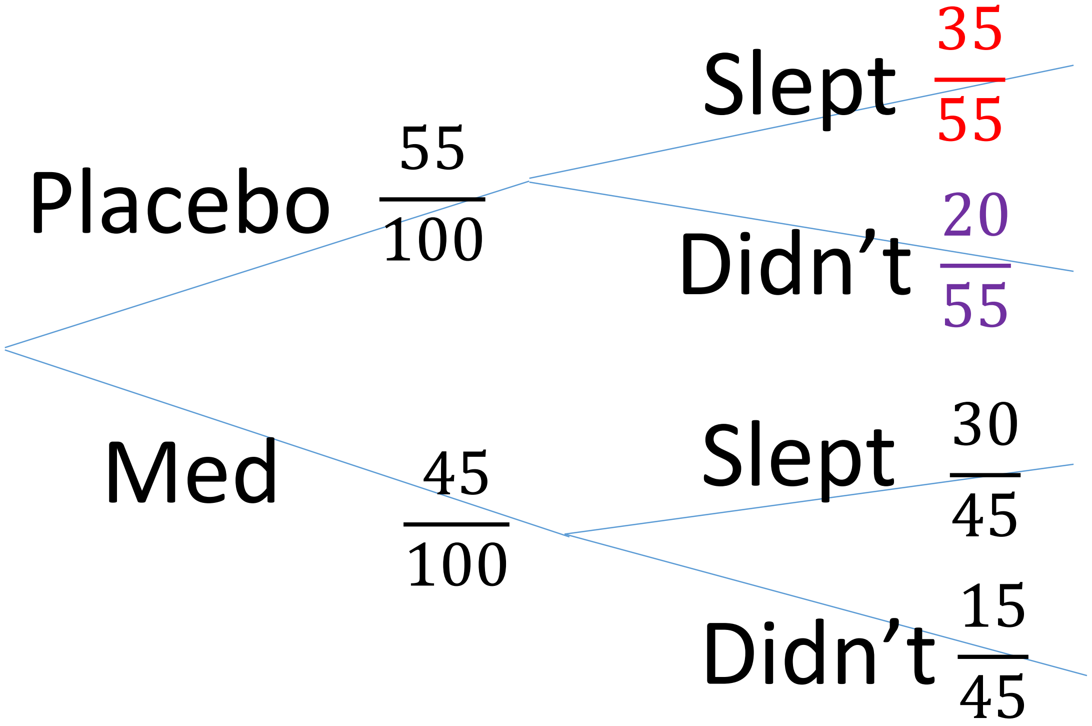
Slept $\frac{35}{55} = \frac{7}{20}$

Didn’t $\frac{20}{55} = \frac{4}{20}$

Med $\frac{45}{100}$

Slept $\frac{30}{45} = \frac{6}{20}$

Didn’t $\frac{15}{45} = \frac{3}{20}$



Given that they slept well, what was the probability they had medication?

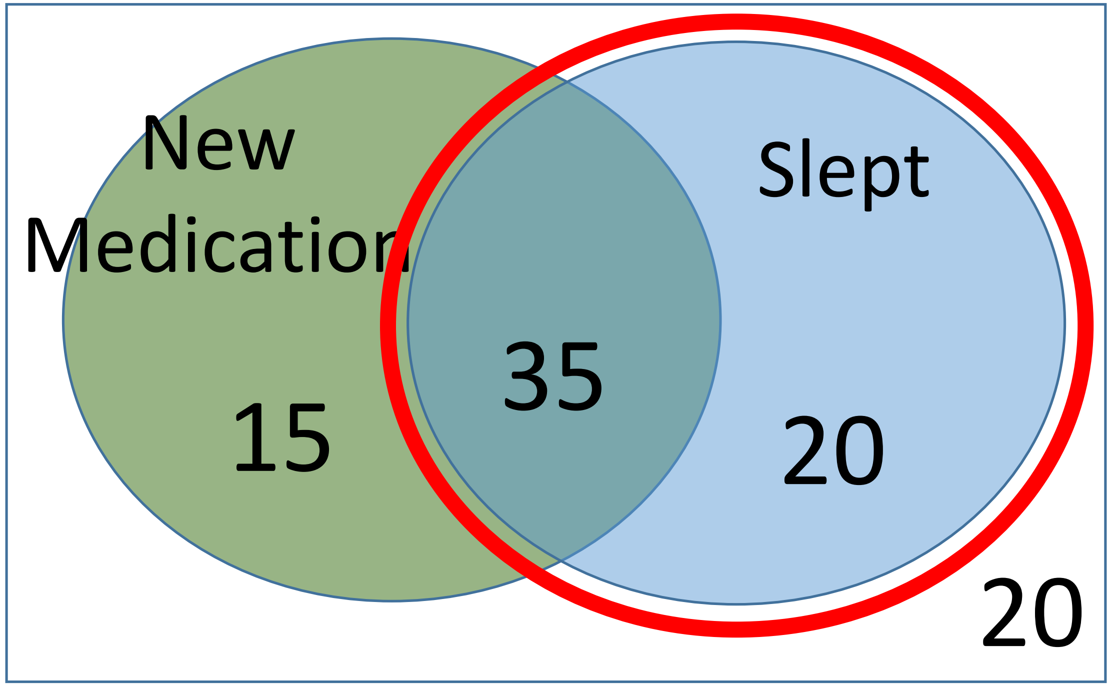
$$= \frac{35}{55} \qquad = 63.6\%$$

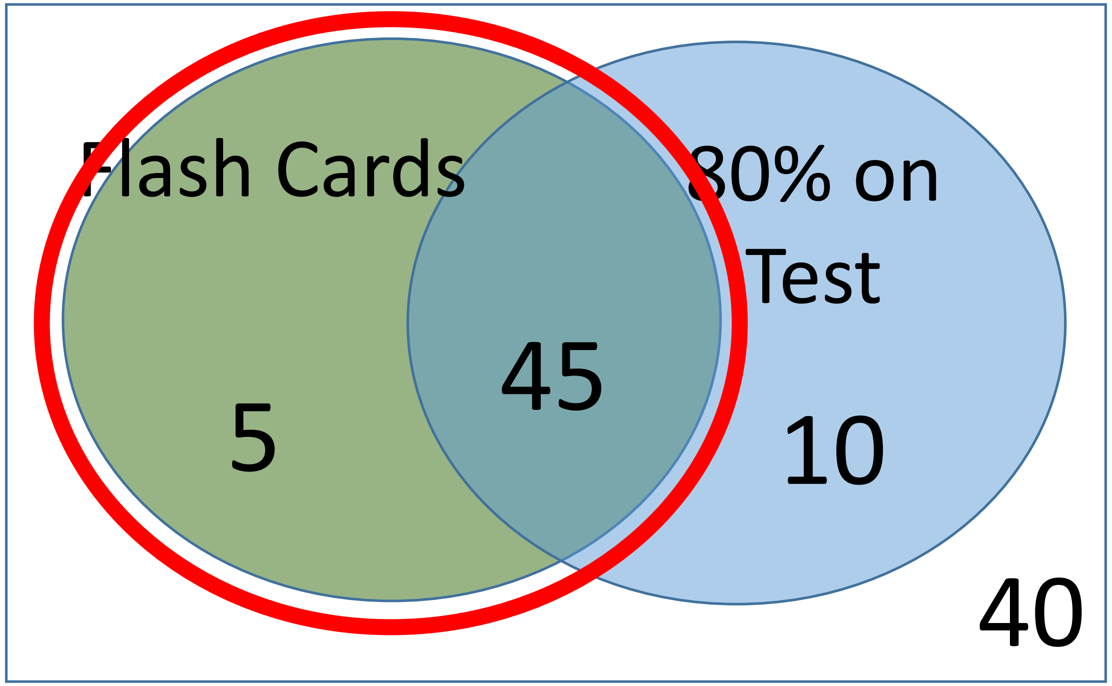
Given that they slept well, what was the probability they had a placebo?

$$= \frac{20}{55} \qquad = 36.3\%$$

Conditional Probability is like “Pruning a Tree”

Conditional Probability is like “Zooming in on A Venn Diagram”



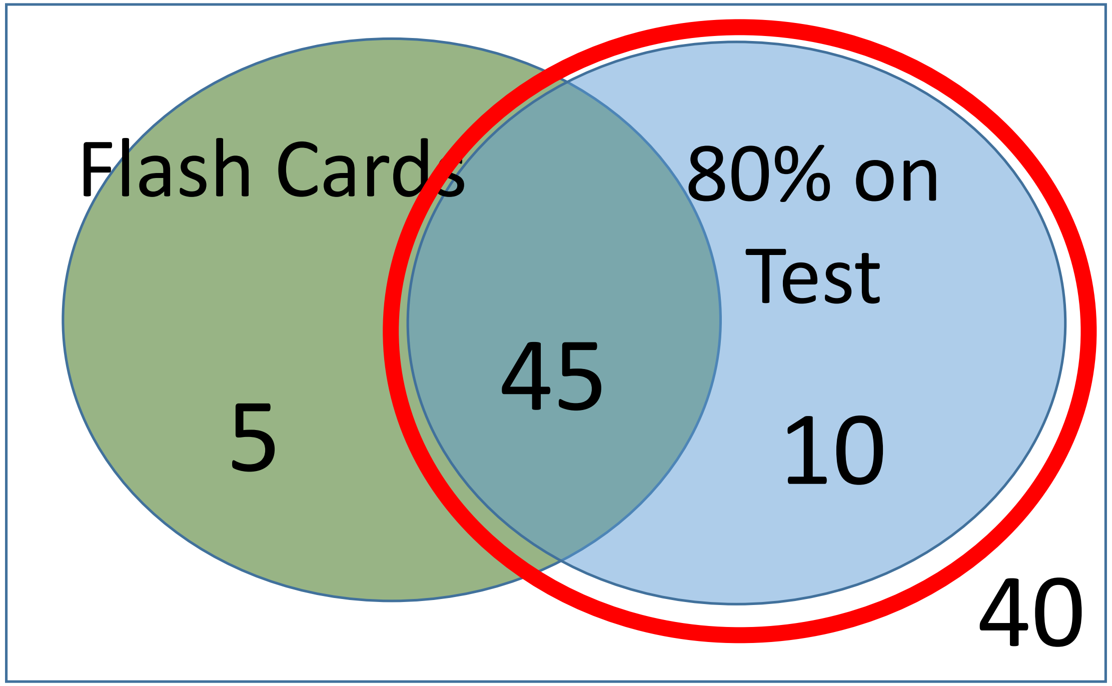


Given that you used flash cards, what is the probability you got 80% on the test?

$$= \frac{45}{50} \qquad = 90\%$$

Given that you used flash cards, what is the probability you didn't get 80% on the test?

$$= \frac{5}{50} \qquad = 10\%$$

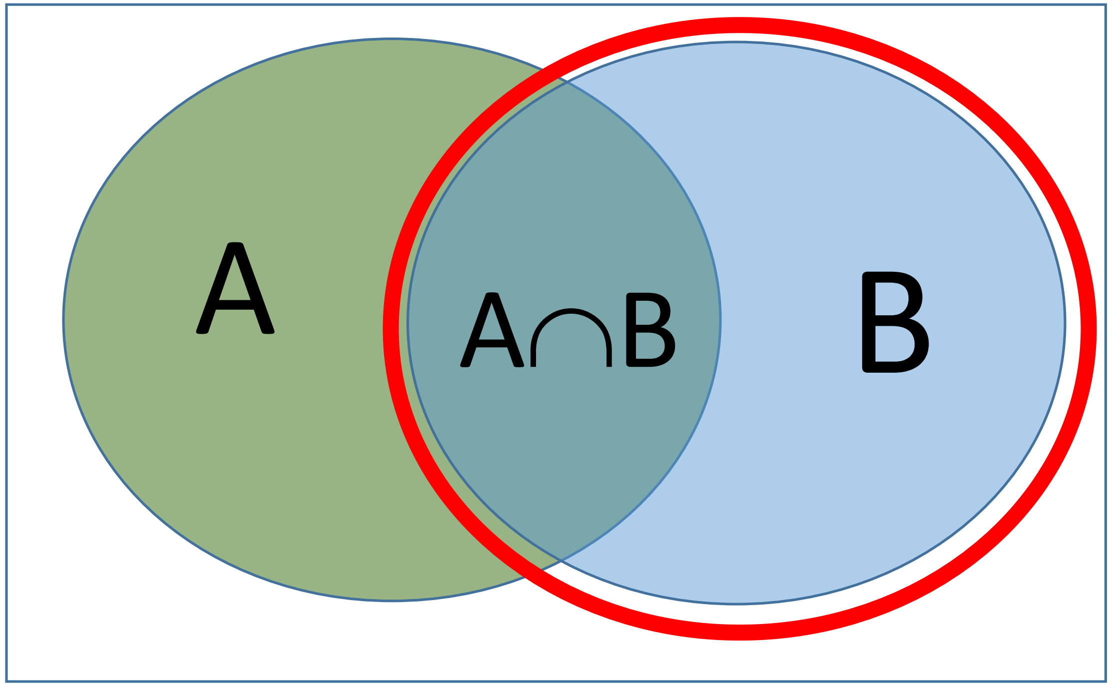


Given that you got 80% on the test, what is the probability you used flash cards?

$$= \frac{45}{55} \qquad = 82\%$$

Given that you got 80% on the test, what is the probability you didn’t use flash cards?

$$= \frac{10}{55} \qquad = 18\%$$



$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

In a class of 40 students, 34 like bananas, 22 like pineapple, and 2 dislike both fruits. A student is randomly selected. Find the probability that the student:

a likes both fruits

b likes at least one fruit

c likes bananas given that he or she likes pineapple

d dislikes pineapple given that he or she likes bananas.

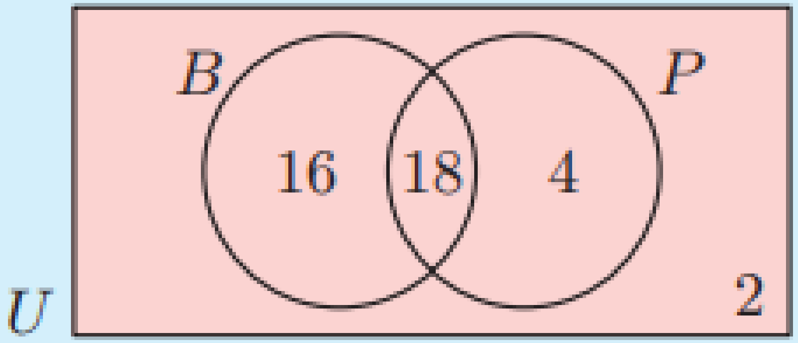
In a class of 40 students, 34 like bananas, 22 like pineapple, and 2 dislike both fruits. A student is randomly selected. Find the probability that the student:

a likes both fruits

b likes at least one fruit

c likes bananas given that he or she likes pineapple

d dislikes pineapple given that he or she likes bananas.



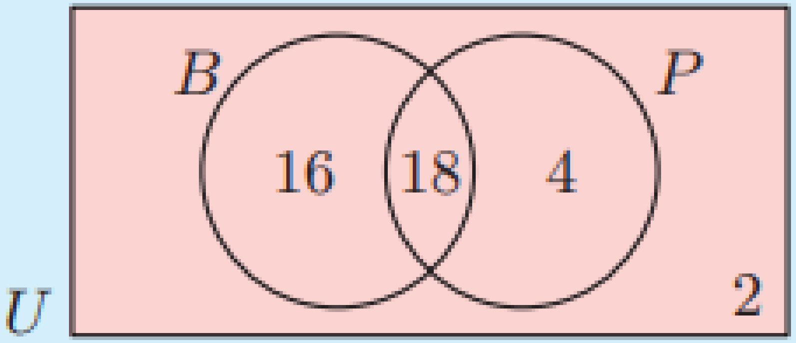
In a class of 40 students, 34 like bananas, 22 like pineapple, and 2 dislike both fruits. A student is randomly selected. Find the probability that the student:

a likes both fruits

b likes at least one fruit

c likes bananas given that he or she likes pineapple

d dislikes pineapple given that he or she likes bananas.



a P(likes both)

$$= \frac{18}{40}$$

$$= \frac{9}{20}$$

b P(likes at least one)

$$= \frac{38}{40}$$

$$= \frac{19}{20}$$

c $\mathrm{P}(B \mid P)$

$$= \frac{18}{22}$$

$$= \frac{9}{11}$$

d $\mathrm{P}(P' \mid B)$

$$= \frac{16}{34}$$

$$= \frac{8}{17}$$

The top shelf in a cupboard contains 3 cans of pumpkin soup and 2 cans of chicken soup. The bottom shelf contains 4 cans of pumpkin soup and 1 can of chicken soup.

Lukas is twice as likely to take a can from the bottom shelf as he is from the top shelf. Suppose Lukas takes one can of soup without looking at the label. Determine the probability that it:

a is chicken

b was taken from top shelf given that it is chicken.

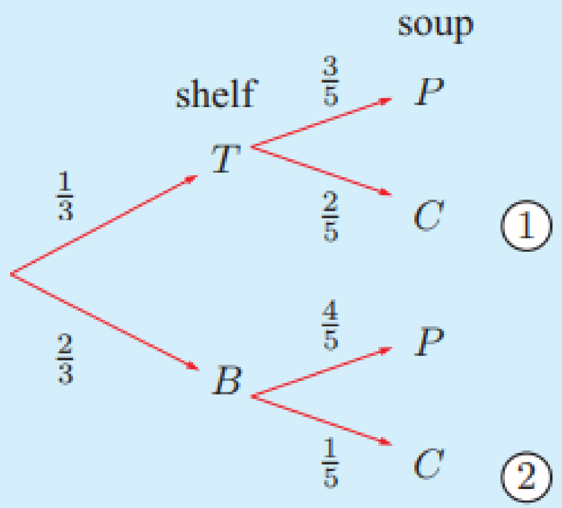
The top shelf in a cupboard contains 3 cans of pumpkin soup and 2 cans of chicken soup. The bottom shelf contains 4 cans of pumpkin soup and 1 can of chicken soup.

Lukas is twice as likely to take a can from the bottom shelf as he is from the top shelf. Suppose Lukas takes one can of soup without looking at the label. Determine the probability that it:

a is chicken

b was taken from top shelf given that it is chicken.

T represents the top shelf.
B represents the bottom shelf.
P represents the pumpkin soup.
C represents the chicken soup.



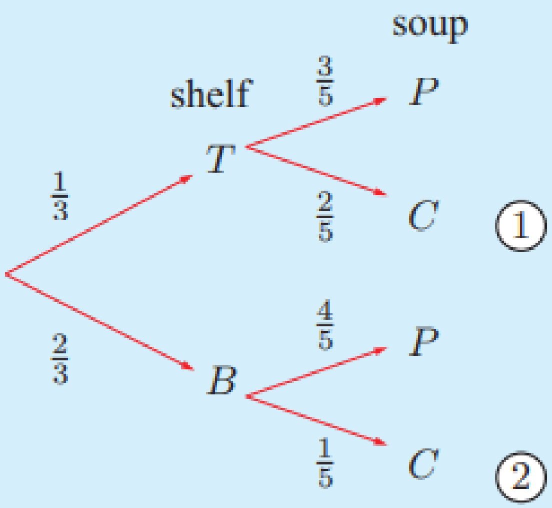
The top shelf in a cupboard contains 3 cans of pumpkin soup and 2 cans of chicken soup. The bottom shelf contains 4 cans of pumpkin soup and 1 can of chicken soup.

Lukas is twice as likely to take a can from the bottom shelf as he is from the top shelf. Suppose Lukas takes one can of soup without looking at the label. Determine the probability that it:

a is chicken

b was taken from top shelf given that it is chicken.

T represents the top shelf.
B represents the bottom shelf.
P represents the pumpkin soup.
C represents the chicken soup.



a P(soup is chicken)

$$= \tfrac{1}{3}\times\tfrac{2}{5}+\tfrac{2}{3}\times\tfrac{1}{5} \quad \{\text{paths ① and ②}\}$$

$$= \tfrac{4}{15}$$

b P(top shelf | chicken)

$$= \frac{\text{P(top shelf and chicken)}}{\text{P(chicken)}}$$

$$= \frac{\frac{1}{3}\times\frac{2}{5}}{\frac{4}{15}} \begin{array}{l} \longleftarrow \text{path ①} \\ \longleftarrow \text{from } \mathbf{a} \end{array}$$

$$= \tfrac{1}{2}$$

Identify as independent or dependent.

(a) Student has	a chocolate bar	poor hearing
(b) A worker	is well trained	meets the production quota
(c) A dog	Likes running	Has a name that starts with “M”
(d) A person is	Late	Had trouble sleeping
(e) Playing a game	Draw an Ace of Spades	Roll a pair of 6’s with dice
(f) A person is	left handed	has blonde hair
(g) A person plays	squash	tennis

If independent:

$$P(A \cap B) = P(A) \times P(B)$$

A coin and die are tossed. Determine the probability of getting a head and a 3 without using a tree diagram.

$$P(H \cap Roll\ 3) = P(H) \times P(Roll\ 3)$$

$$= \frac{1}{2} \times \frac{1}{6}$$

$$= \frac{1}{12}$$

When two coins are tossed, A is the event of getting 2 heads. When a die is rolled, B is the event of getting a 5 or 6. Show that A and B are independent events.

When two coins are tossed, A is the event of getting 2 heads. When a die is rolled, B is the event of getting a 5 or 6. Show that A and B are independent events.

$\mathrm{P}(A) = \frac{1}{4}$ and $\mathrm{P}(B) = \frac{2}{6}$.

Therefore, $\mathrm{P}(A)\,\mathrm{P}(B) = \frac{1}{4} \times \frac{2}{6} = \frac{1}{12}$

$$\begin{aligned} &\mathrm{P}(A \cap B) \\ &= \mathrm{P}(2 \text{ heads } \textbf{and} \text{ a 5 or a 6}) \\ &= \frac{2}{24} \\ &= \frac{1}{12} \end{aligned}$$

Since $\mathrm{P}(A \cap B) = \mathrm{P}(A)\,\mathrm{P}(B)$, the events A and B are independent.

We know a lot of formulas with AND:

Additive Principle?

Mutually Exclusive?

Conditional probability?

Independent?

6 Two events are defined such that $\mathrm{P}(A) = 0.11$ and $\mathrm{P}(B) = 0.7$. $n(B) = 14$.

a Calculate: **i** $\mathrm{P}(A')$ **ii** $n(U)$

b If A and B are independent events, find: **i** $\mathrm{P}(A \cap B)$ **ii** $\mathrm{P}(A \mid B)$

c If instead, A and B are mutually exclusive events, find $\mathrm{P}(A \cup B)$.

6 Two events are defined such that $P(A) = 0.11$ and $P(B) = 0.7$. $n(B) = 14$.

a Calculate: **i** $P(A')$ **ii** $n(U)$

b If A and B are independent events, find: **i** $P(A \cap B)$ **ii** $P(A \mid B)$

c If instead, A and B are mutually exclusive events, find $P(A \cup B)$.

6 **a** **i** 0.89 **ii** 20

b **i** 0.077 **ii** 0.11

c 0.81