Theoretical Probability

A.1.1 probabilities represent the likelihood of a result of an experiment (e.g., spinning spinners; drawing blocks from a bag that contains different coloured blocks; playing a game with number cubes; playing Aboriginal stick-and-stone games) and the likelihood of a real-world event (e.g., that it will rain tomorrow, that an accident will occur, that a product will be defective).

Probability

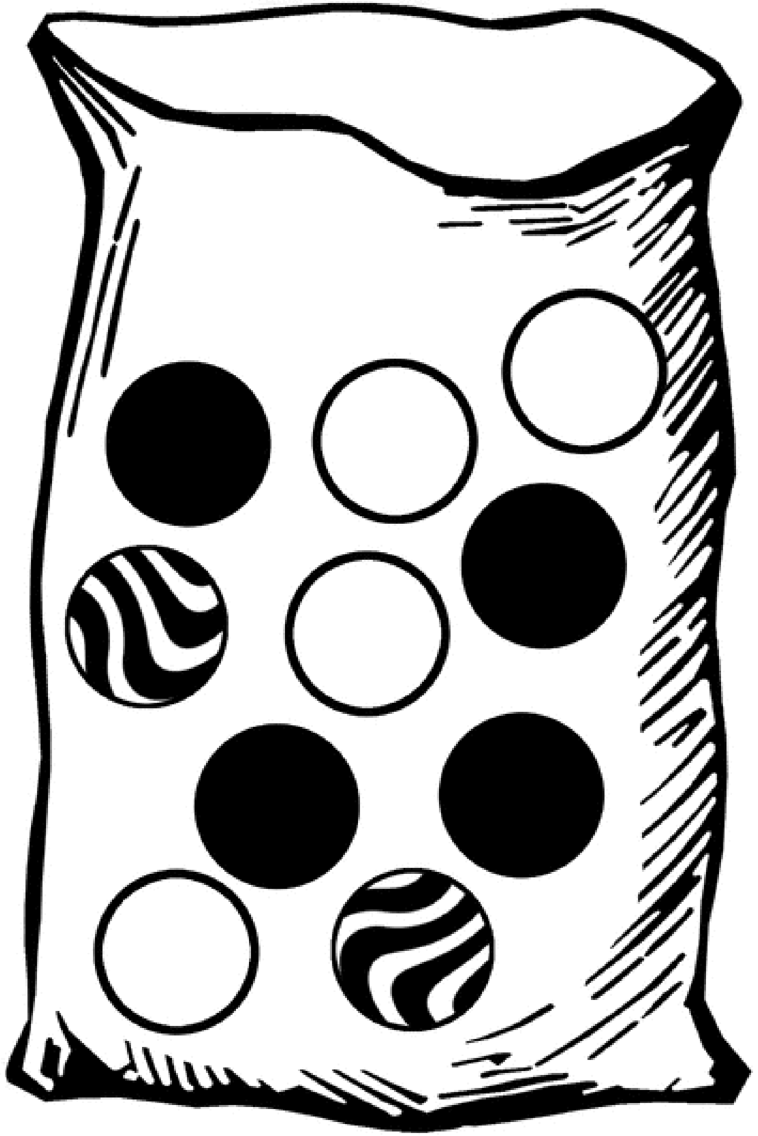
A mathematical statement about how likely something is to occur.

Can be calculated using theoretical probability or using an experiment.

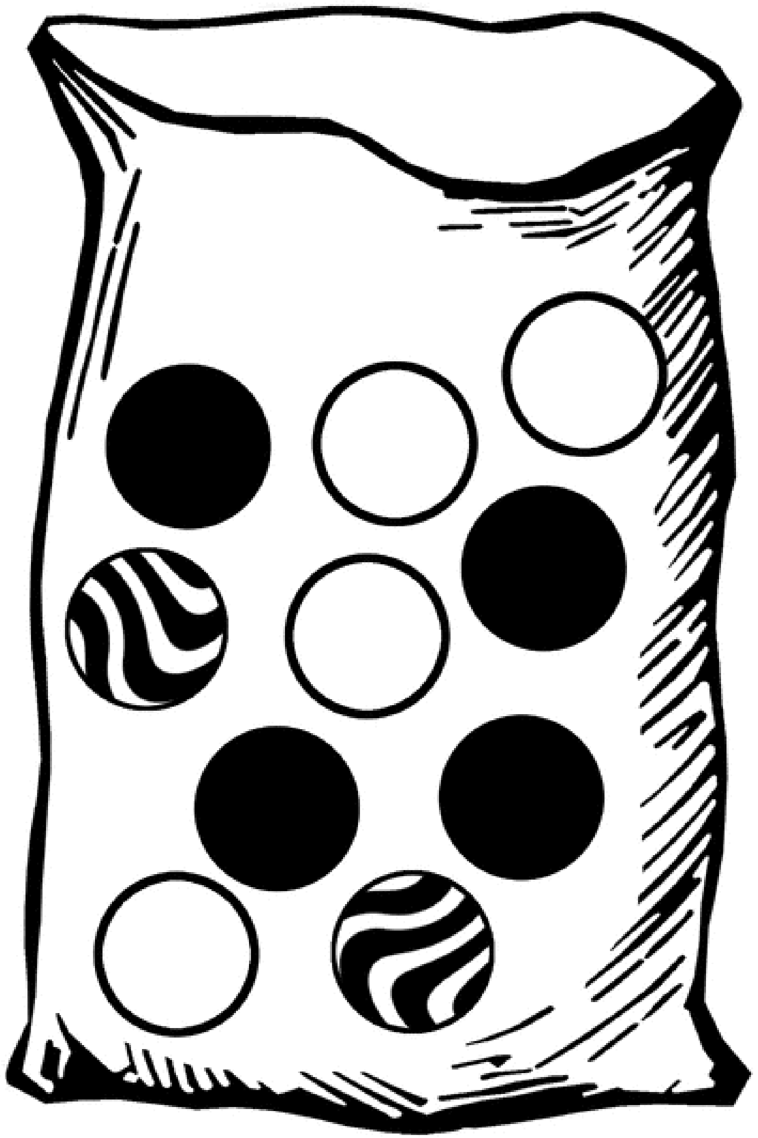
Theoretical Probability

$$P(A) = \frac{n(A)}{n(S)} = \frac{Number\ of\ A\ outcomes}{Total\ number\ of\ outcomes}$$

The total number of outcomes is also known as the sample space. It is written in { }.



Sample Space =
{ Black, White, White,
Striped, White, Black,
Black, Black,
White, Striped }



Sample Space =
{ Black, White, White,
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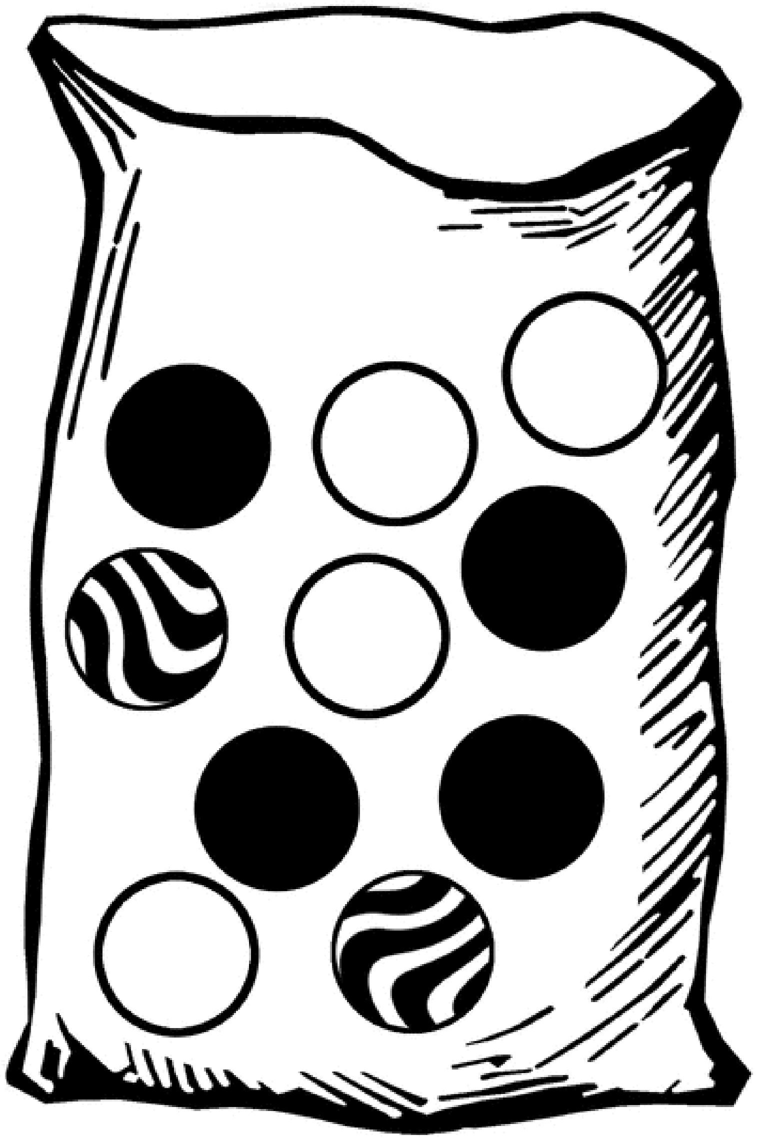
Black, Black,
White, Striped }

What is the probability you draw a striped marble?



Sample Space =
{ Black, White, White,
Striped, White, Black,
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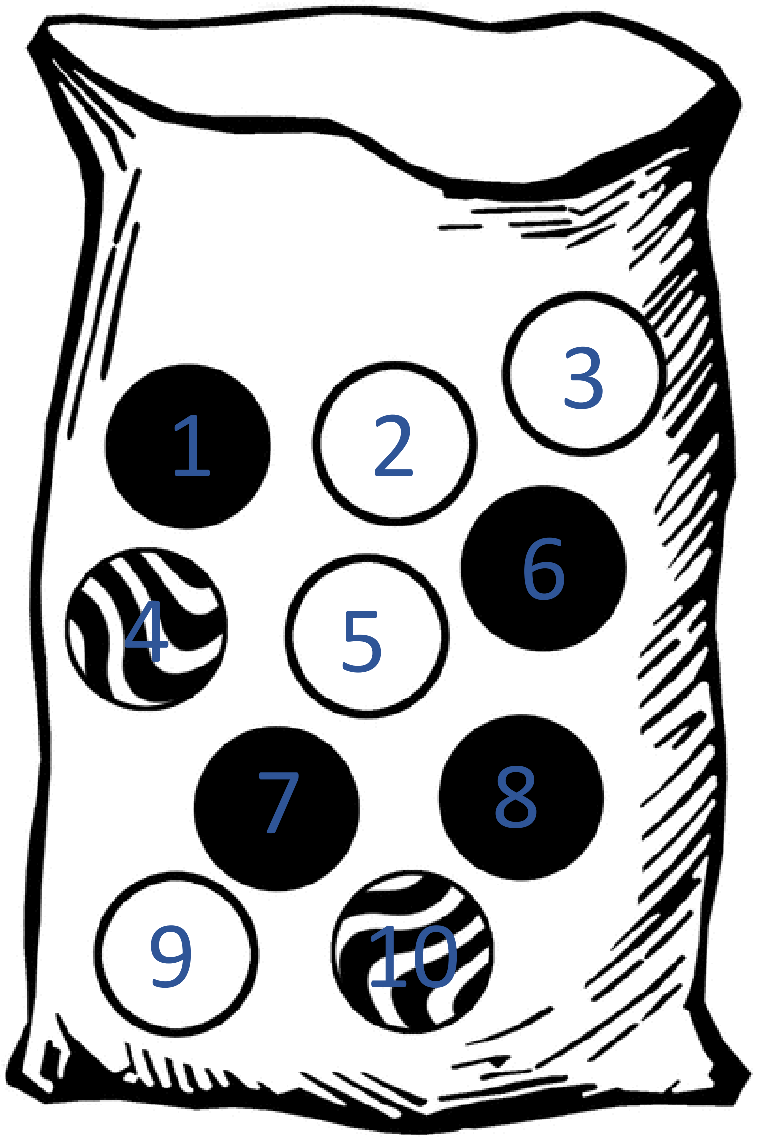
What is the probability you draw a striped marble?

$$P(Stripe) = \frac{n(Striped)}{n(Marbles)}$$



Sample Space =
{ Black, White, White, Striped, White, Black, Black, Black, White, Striped }

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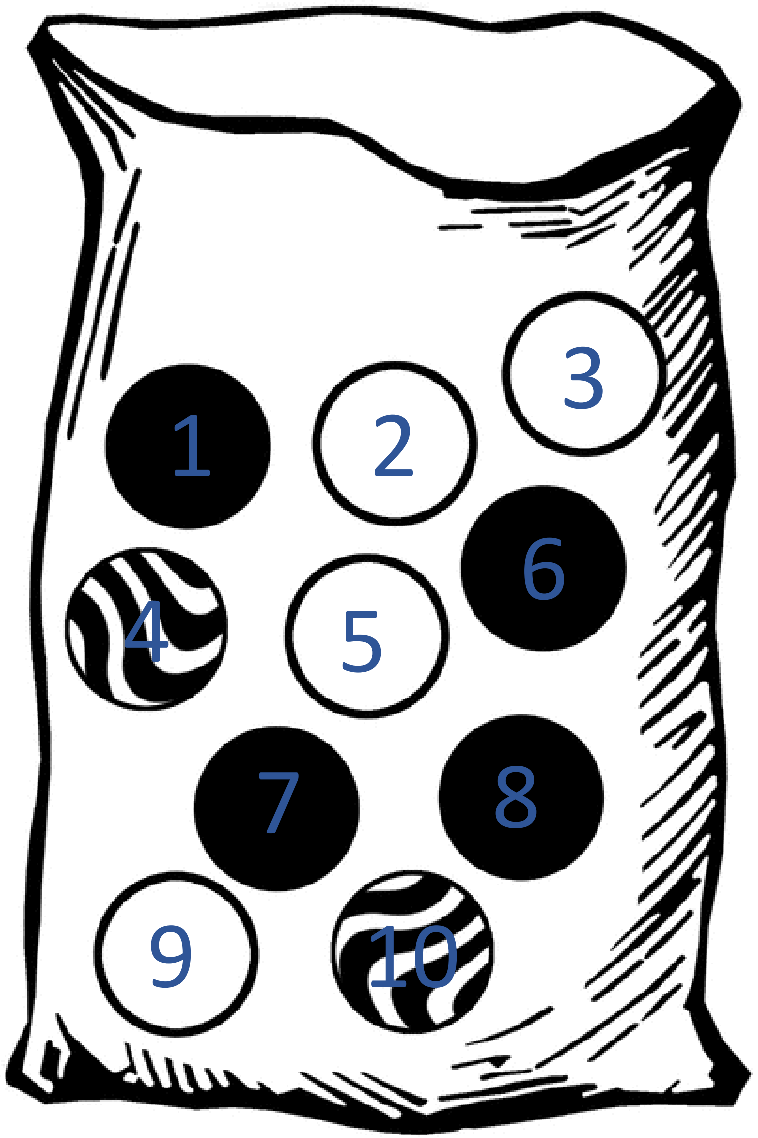
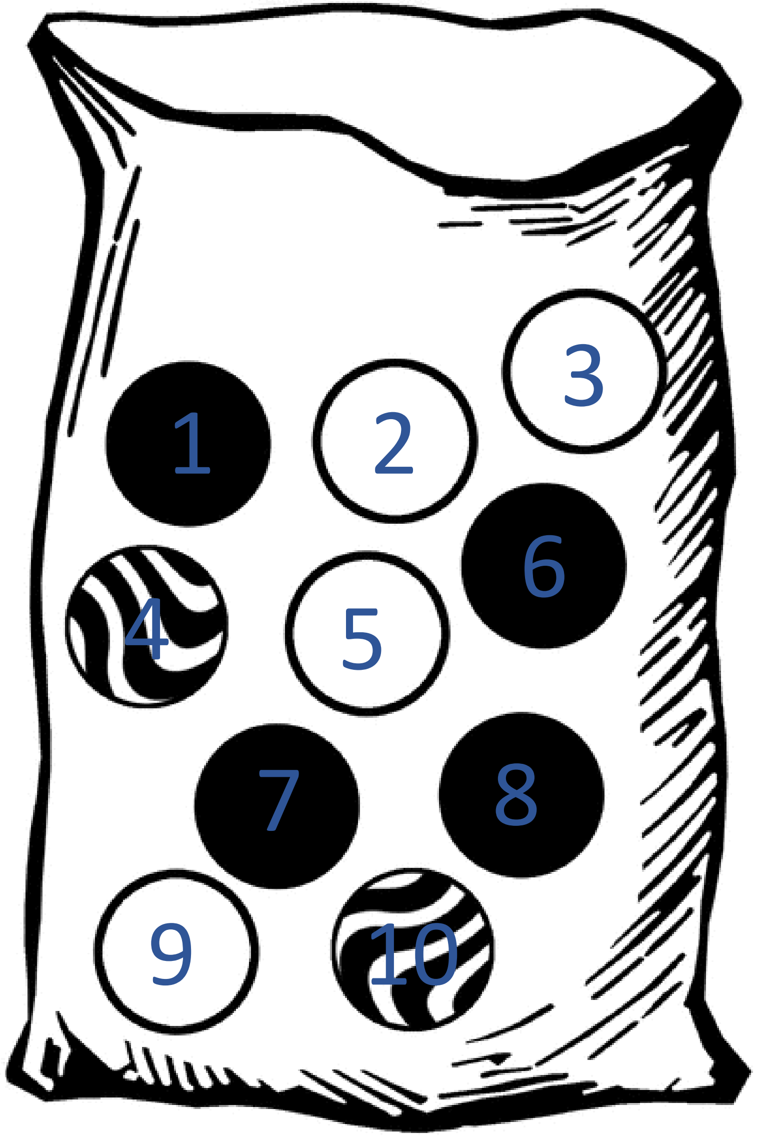


Sample Space =
{ Black, White, White, Striped, White, Black, Black, Black, White, Striped }

What is the probability you draw a striped marble?

$$P(Stripe) = \frac{n(Striped)}{n(Marbles)}$$

$$= \frac{2}{10}$$



Sample Space =
{ Black, White, White, Striped, White, Black, Black, Black, White, Striped }

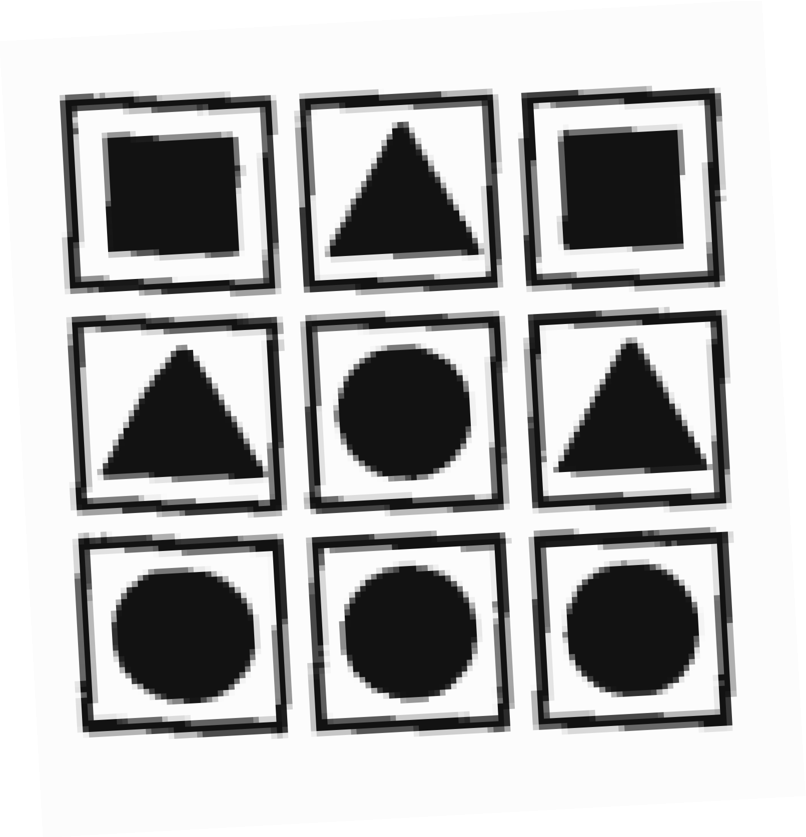
What is the probability you draw a striped marble?

$$P(Stripe) = \frac{n(Striped)}{n(Marbles)}$$

$$= \frac{2}{10}$$

$$= \frac{1}{5}$$

One card is drawn at random from the 9 cards. Calculate the probability that a triangle is drawn.



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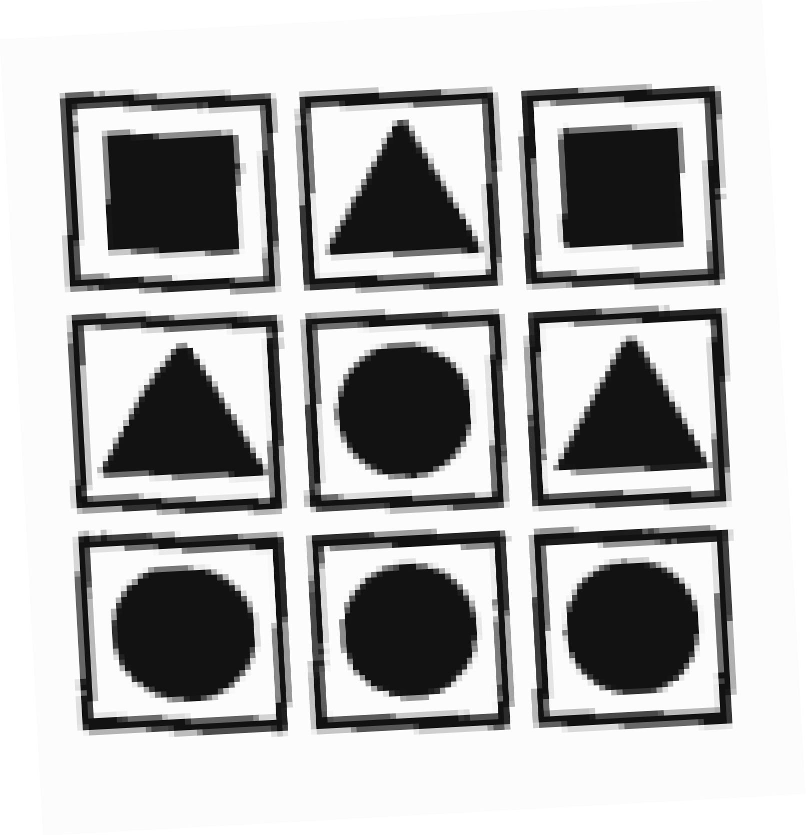
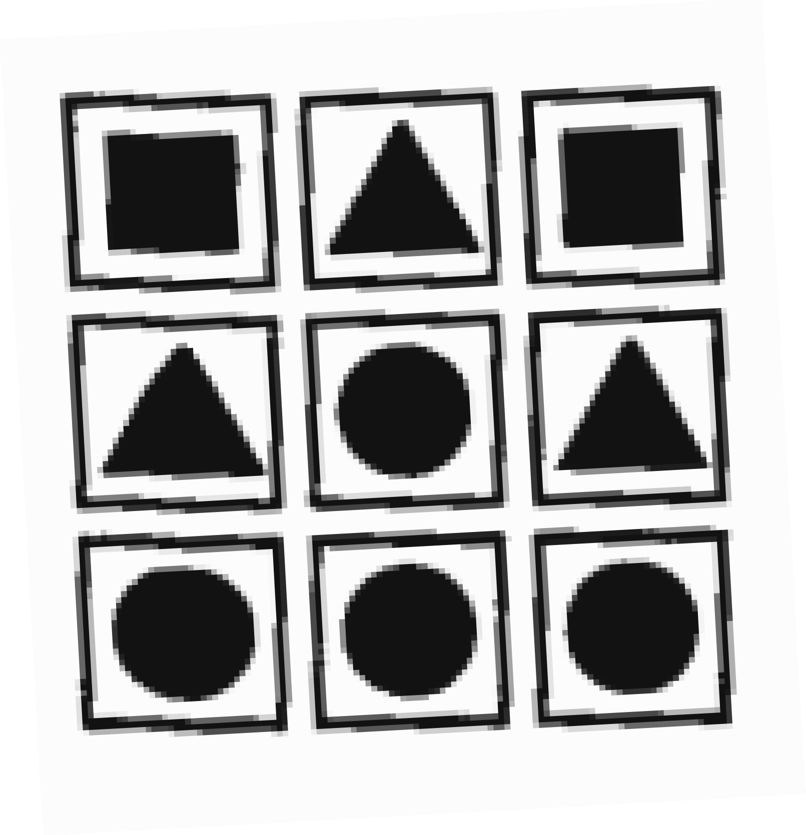


$$P(triangle) = \frac{n(triangle)}{n(total)}$$

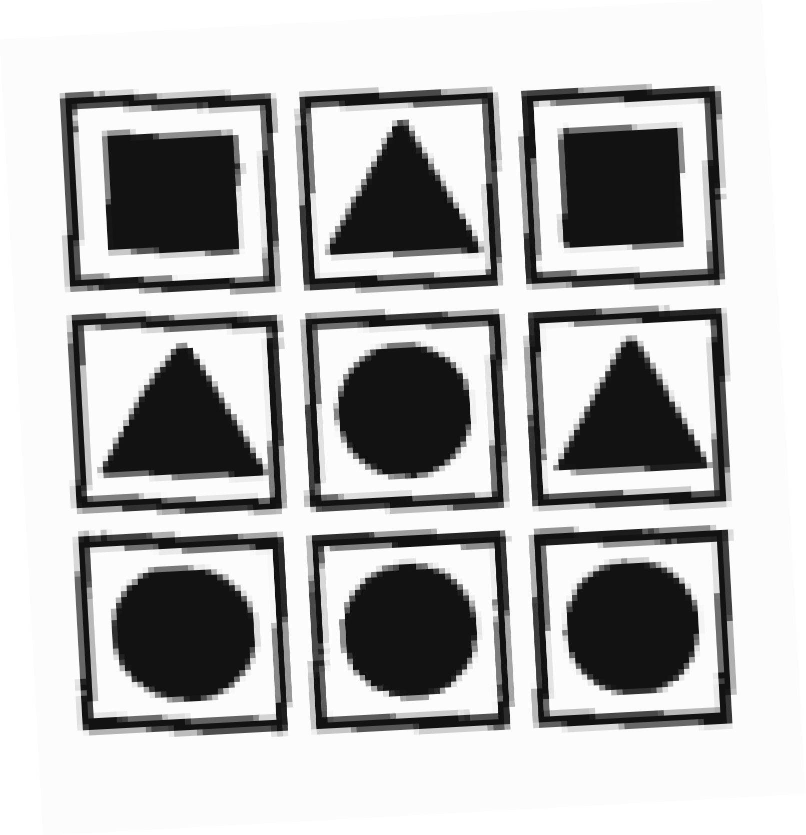
One card is drawn at random from the 9 cards. Calculate the probability that a triangle is drawn.



$$P(triangle) = \frac{n(triangle)}{n(total)}$$

$$= \frac{3}{9}$$

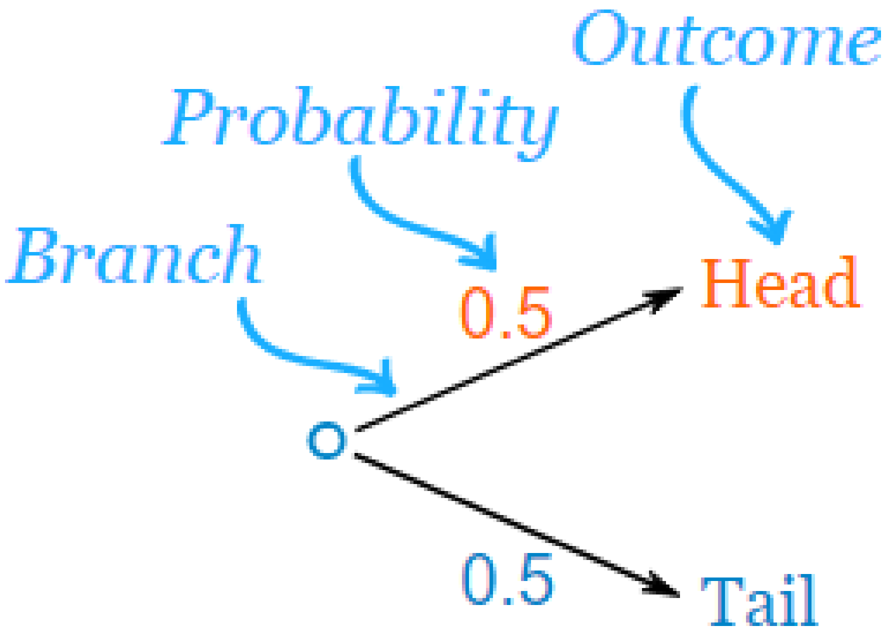
One card is drawn at random from the 12 cards. Calculate the probability that a triangle is drawn.



$$P(triangle) = \frac{n(triangle)}{n(total)}$$

$$= \frac{3}{9}$$

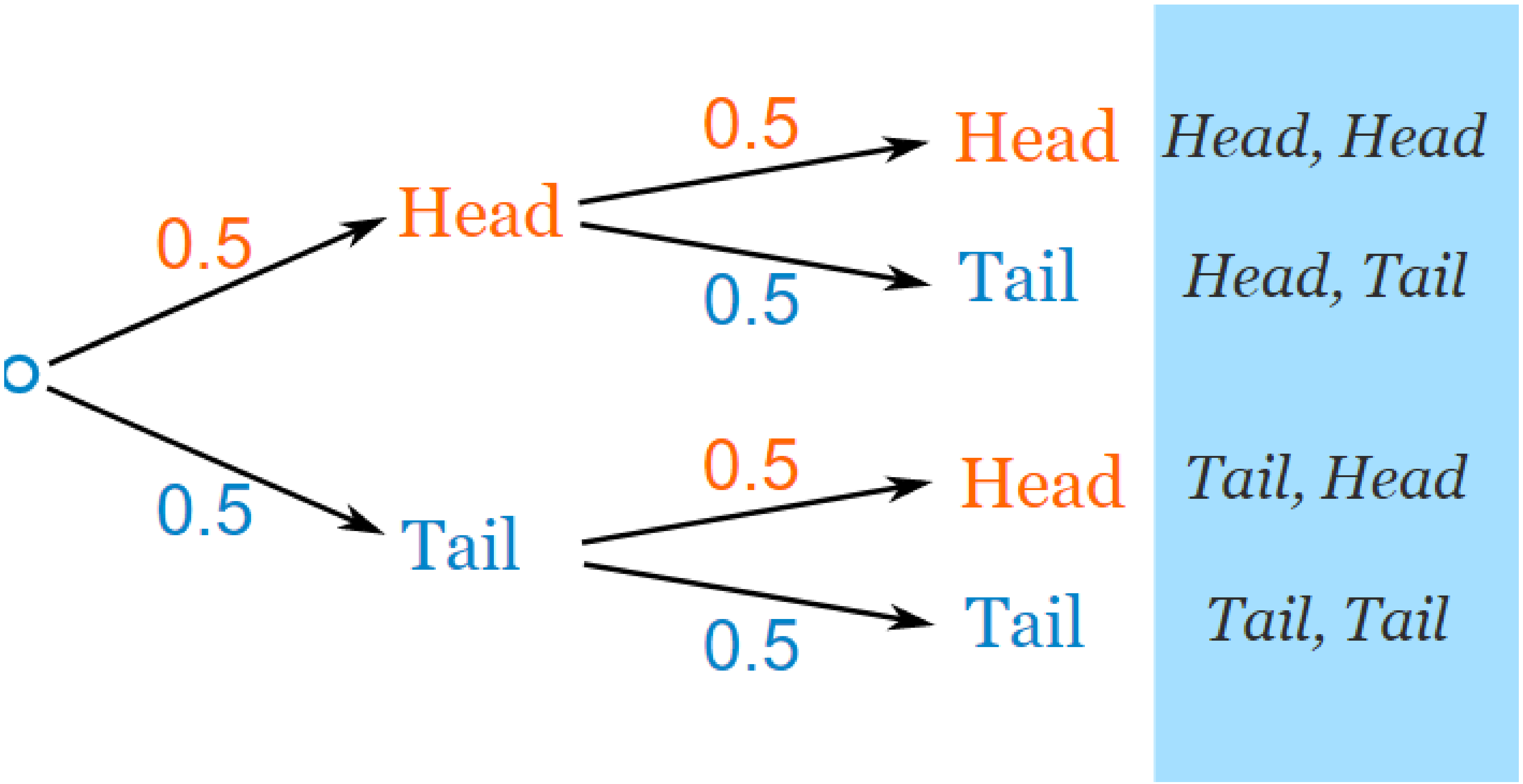
$$= \frac{1}{3}$$



There are two "branches" (Heads and Tails)

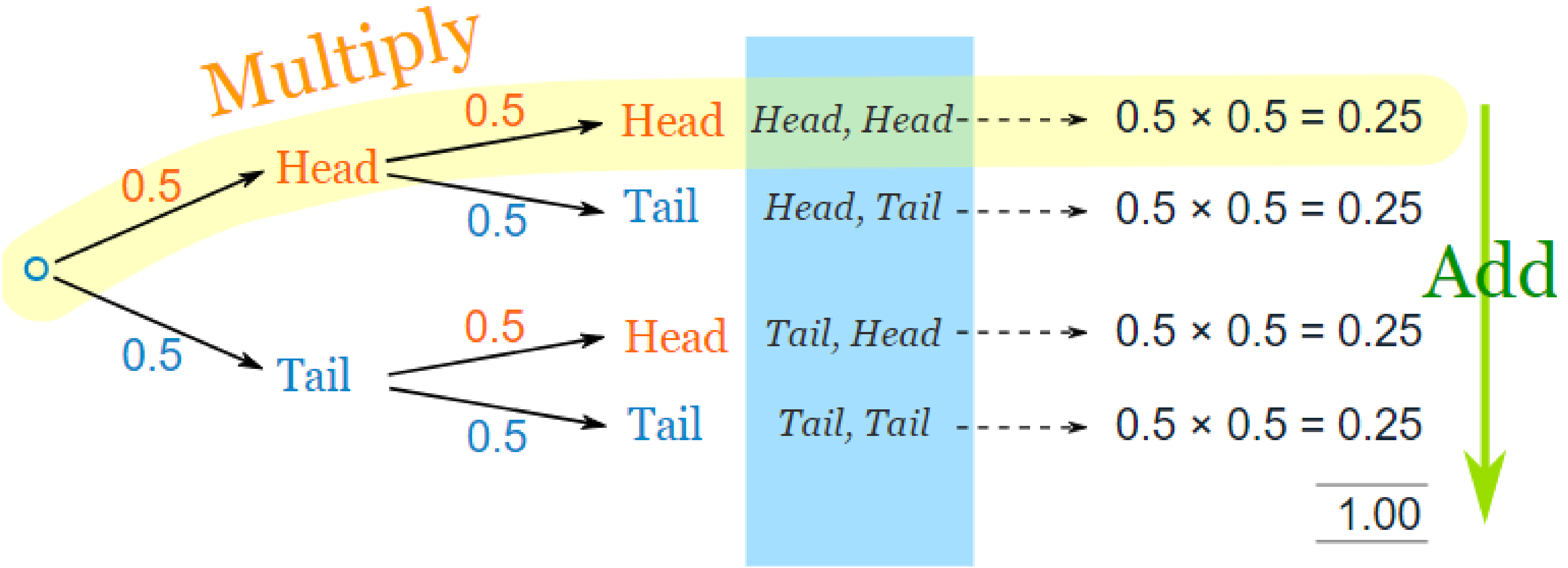
- The probability of each branch is written on the branch
- The outcome is written at the end of the branch

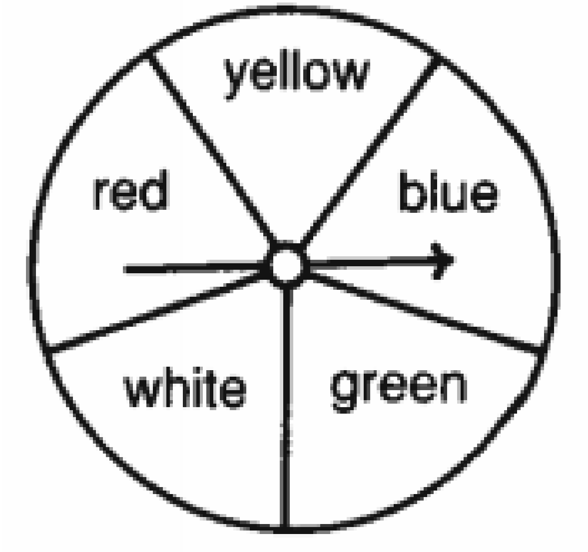
We can extend the tree diagram to two tosses of a coin:



How do we calculate the overall probabilities?

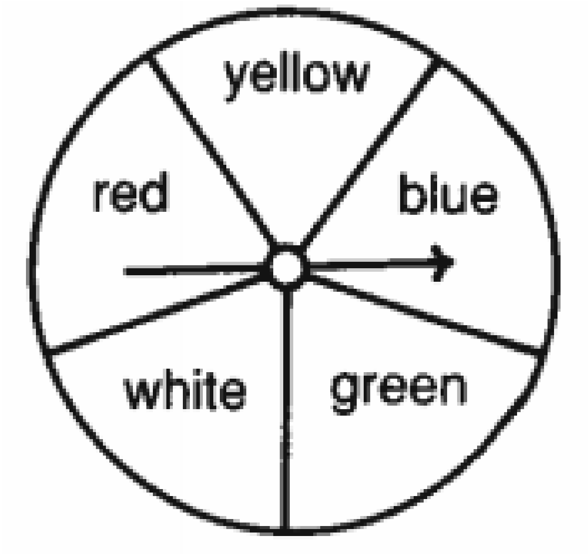
- We **multiply** probabilities **along the branches**
- We **add** probabilities down **columns**





If the arrow is spun twice, what is the probability that it will stop on the blue at least once?

Draw the probability tree.



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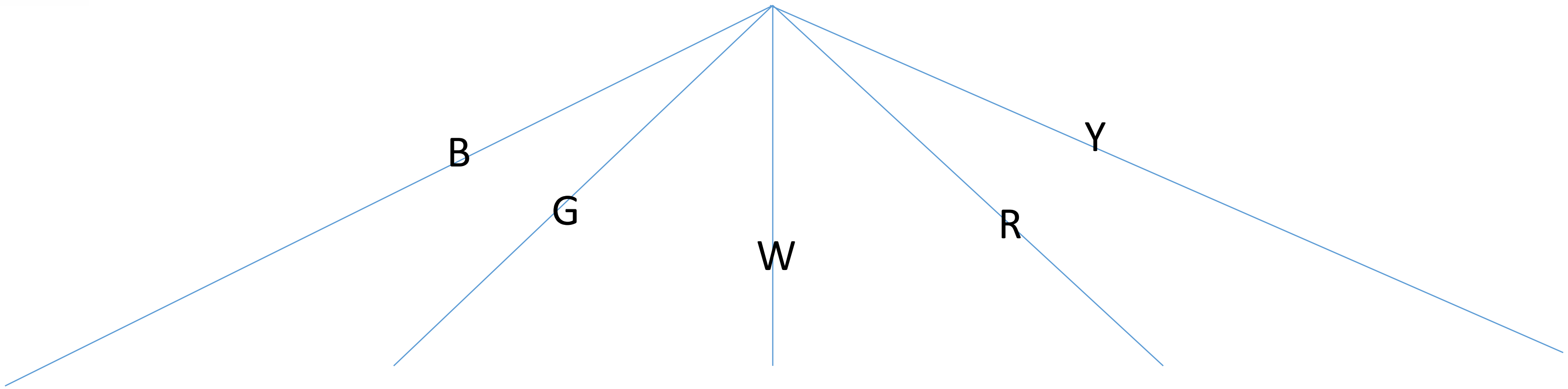

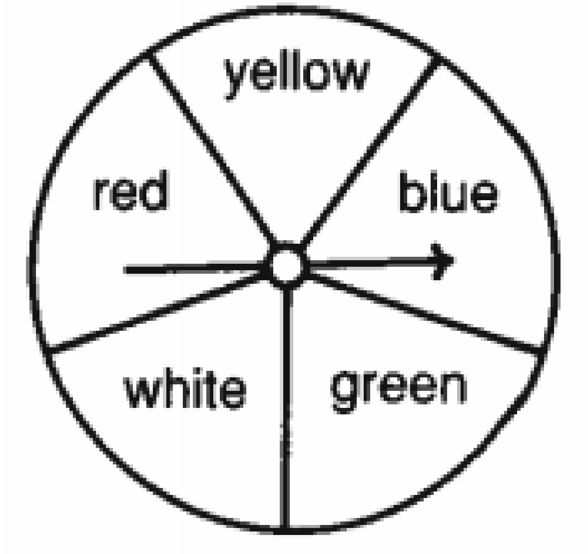

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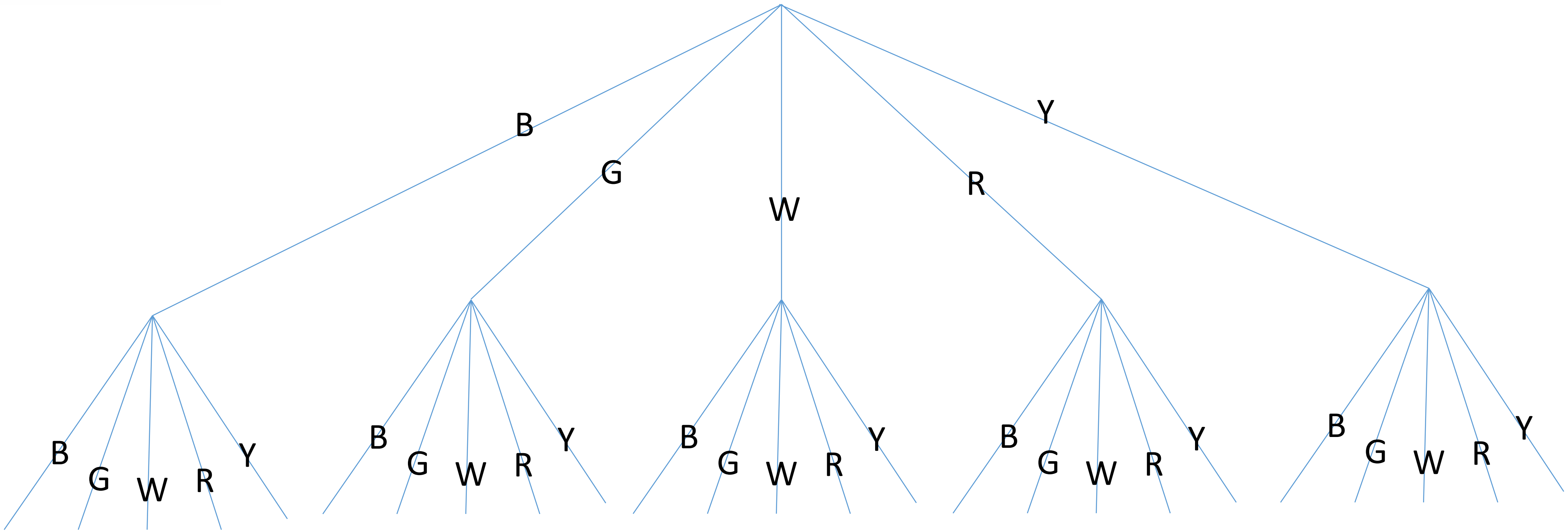
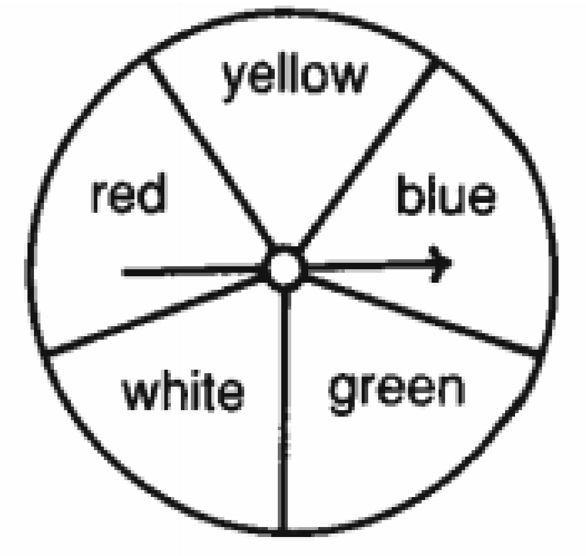
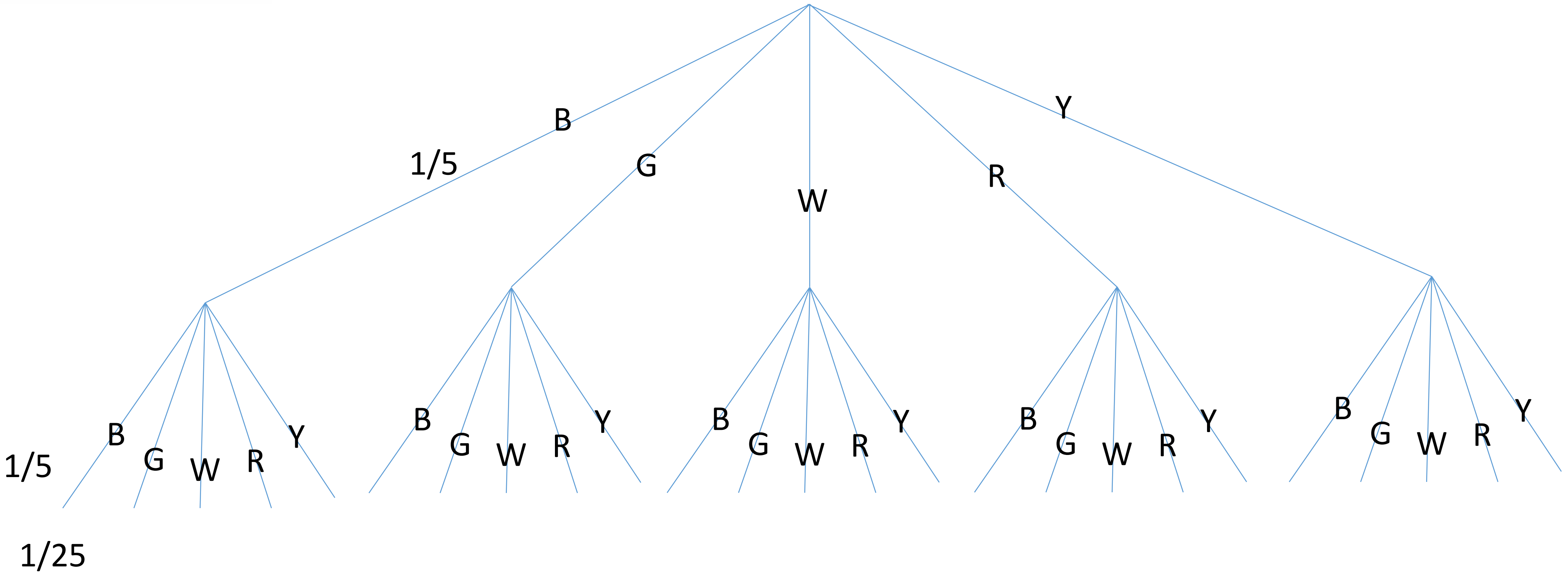
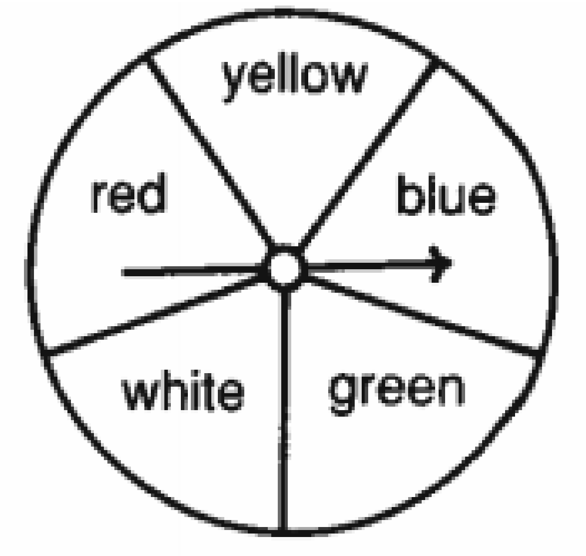




If the arrow is spun twice, what is the probability that it will stop on the blue at least once?

Count up the valid outcomes.





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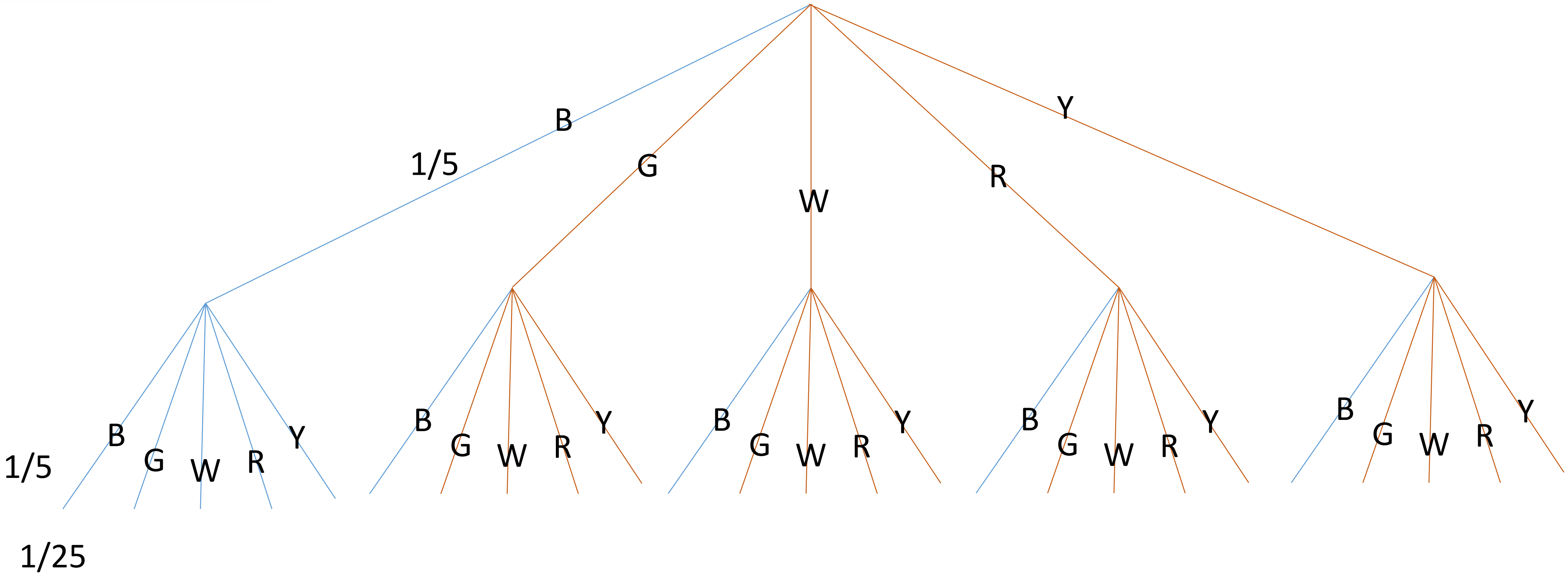
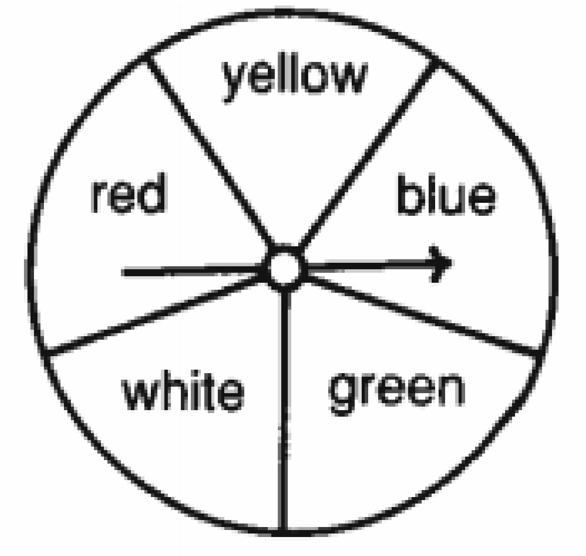


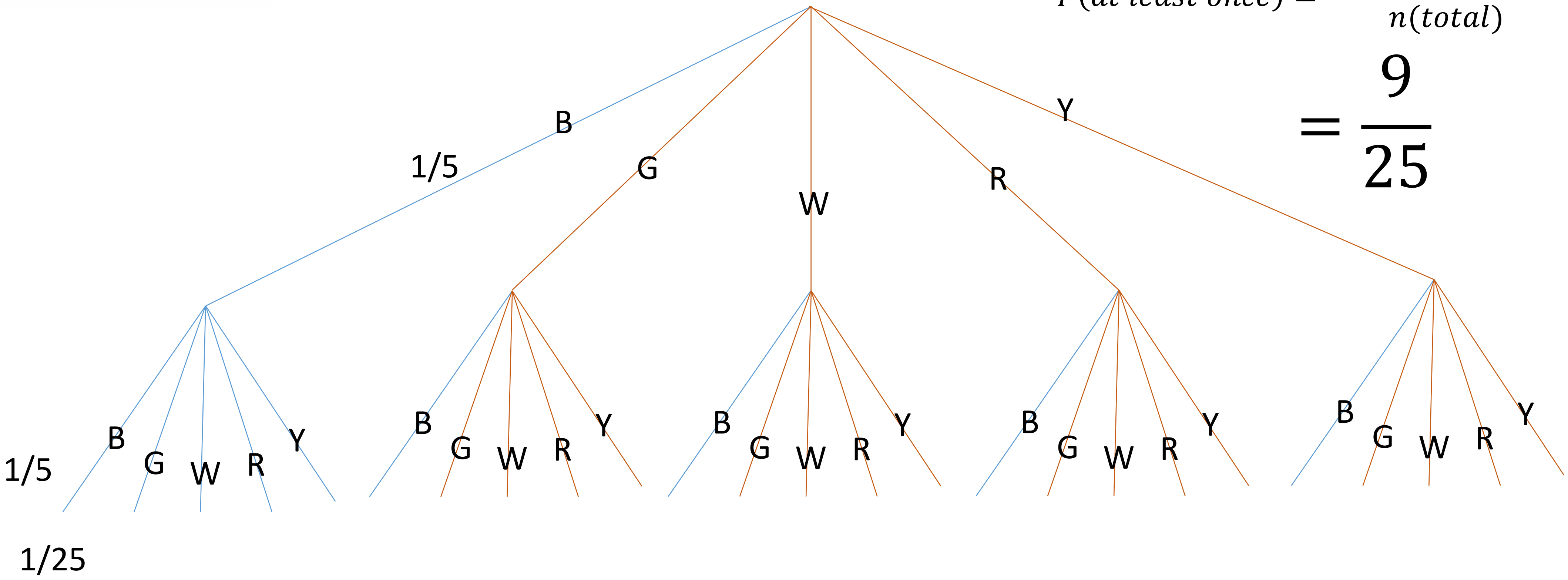


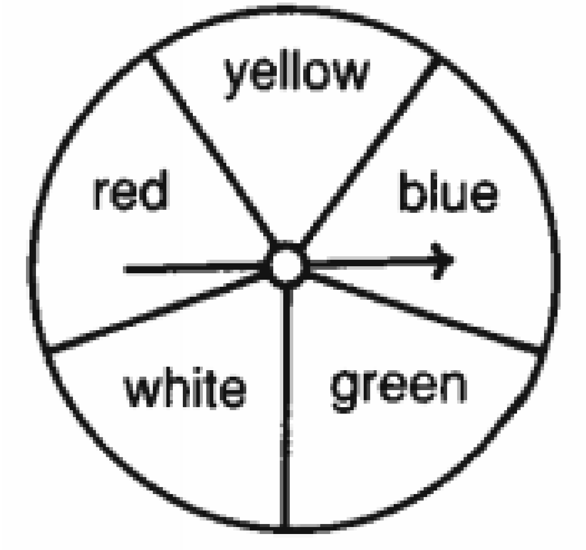
If the arrow is spun twice, what is the probability that it will stop on the blue at least once?

Count up the valid outcomes.

$$P(at\ least\ once) = \frac{n(at\ least\ once)}{n(total)}$$

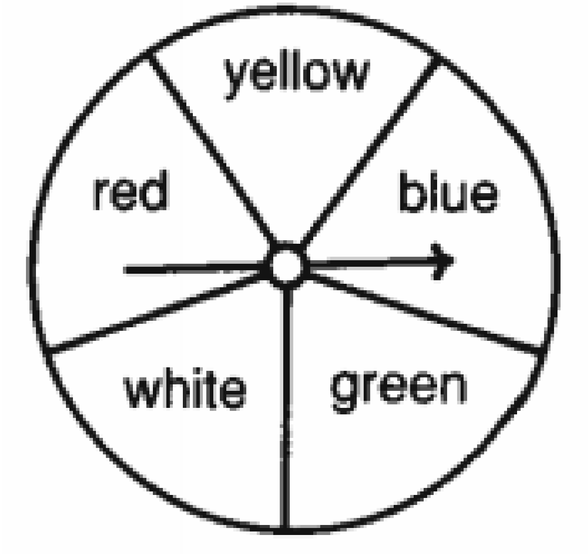
$$= \frac{9}{25}$$





If the arrow is spun twice, what is the probability that it will stop on the blue at least once?

That was slow, we can do better.



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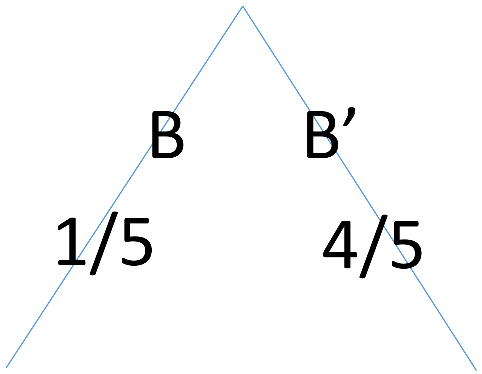

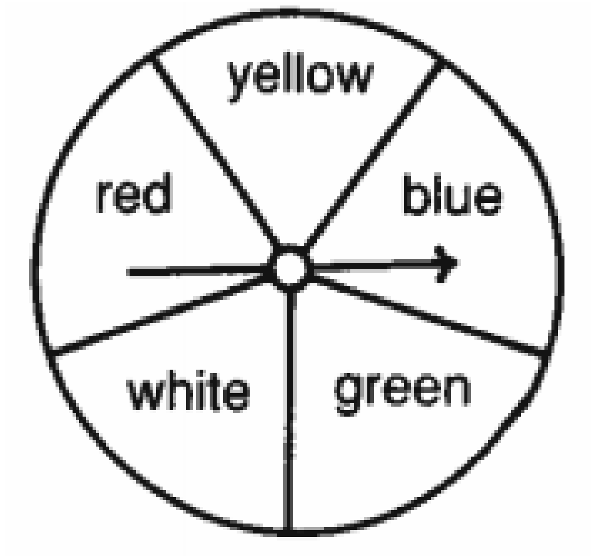

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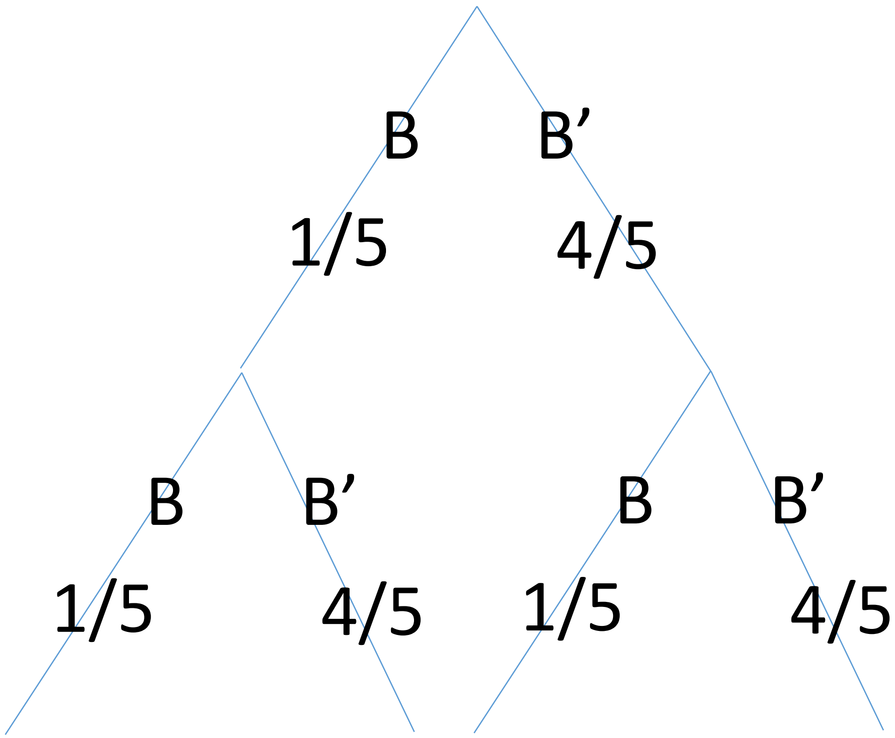
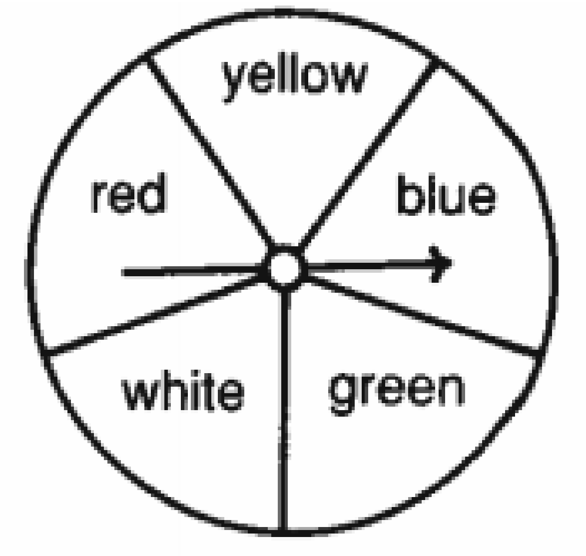
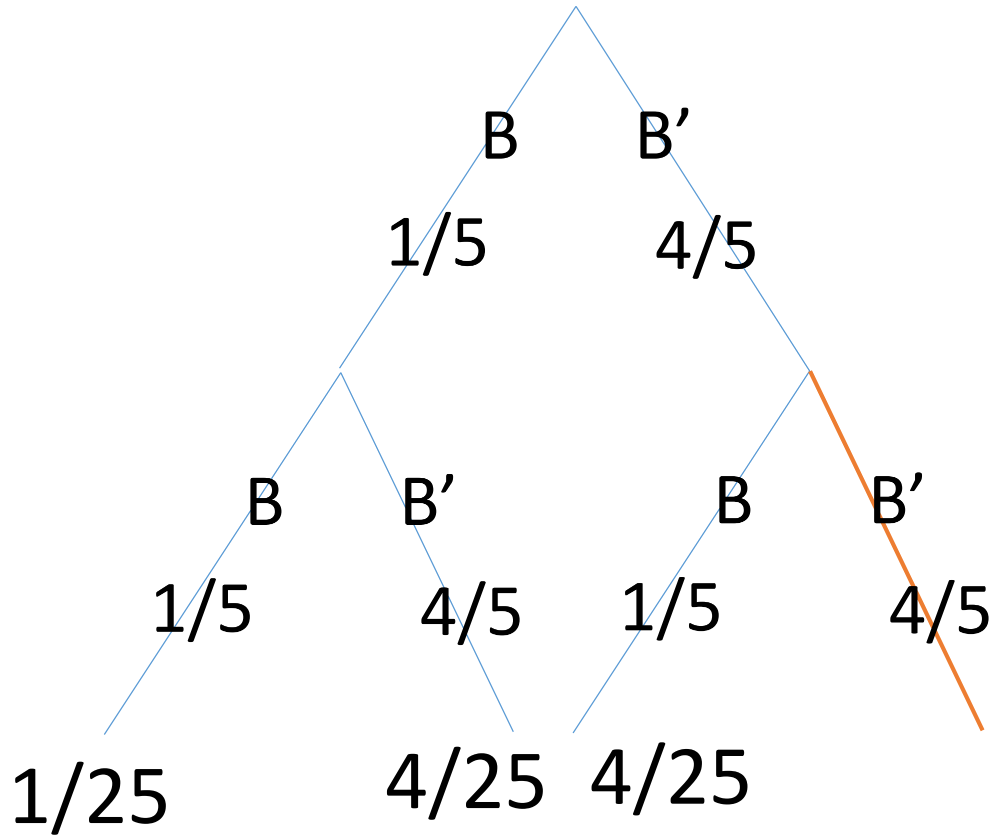
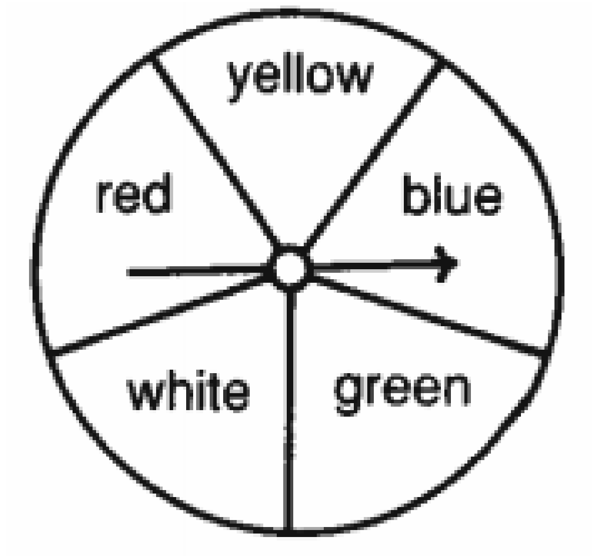




If the arrow is spun twice, what is the probability that it will stop on the blue at least once?

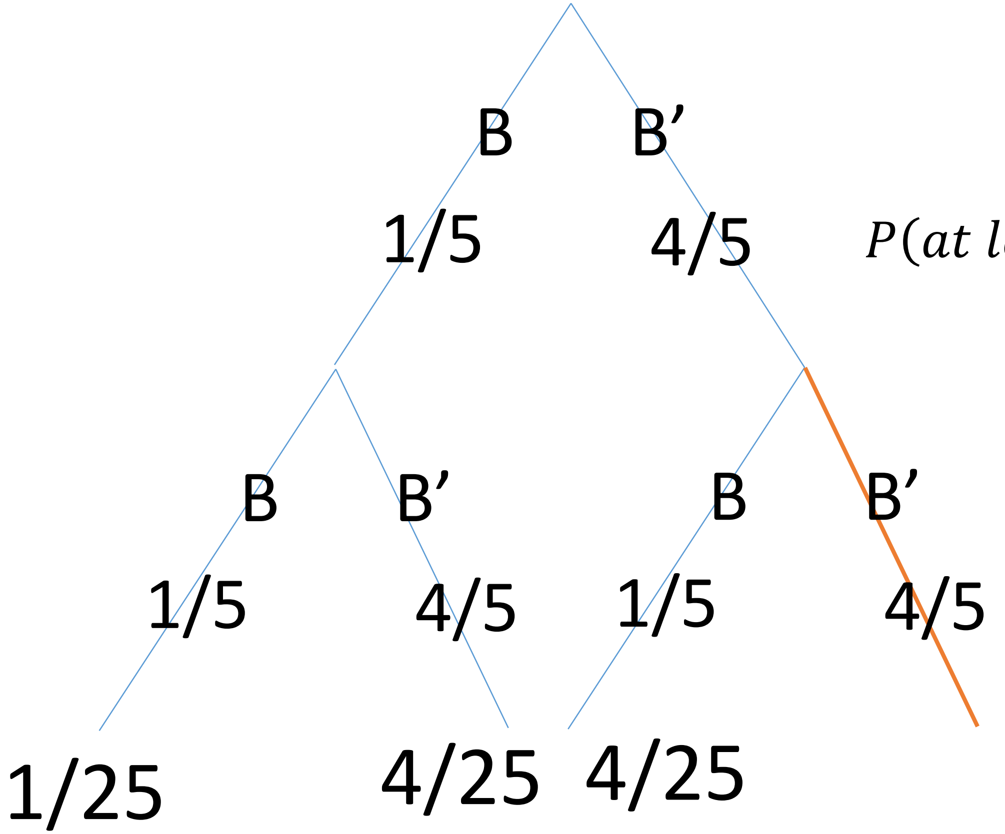
Multiply along the lines you want.





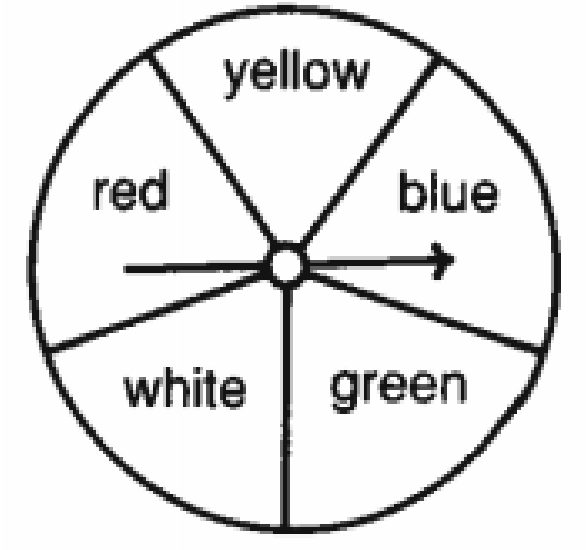
If the arrow is spun twice, what is the probability that it will stop on the blue at least once?

Add up the lines you want.



$$P(at\ least\ once) = \frac{1}{25} + \frac{4}{25} + \frac{4}{25}$$

$$= \frac{9}{25}$$



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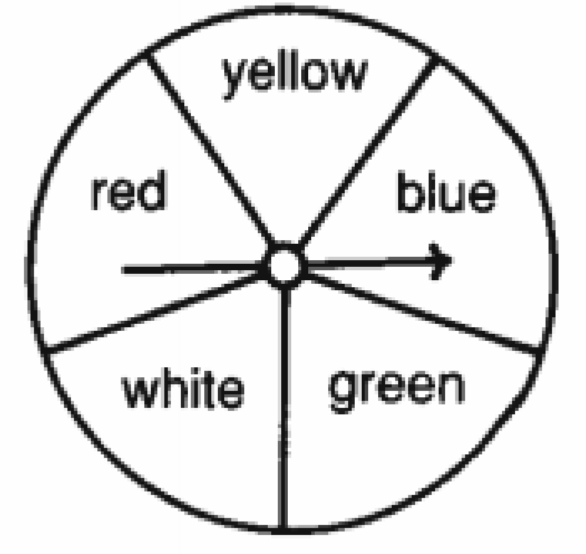

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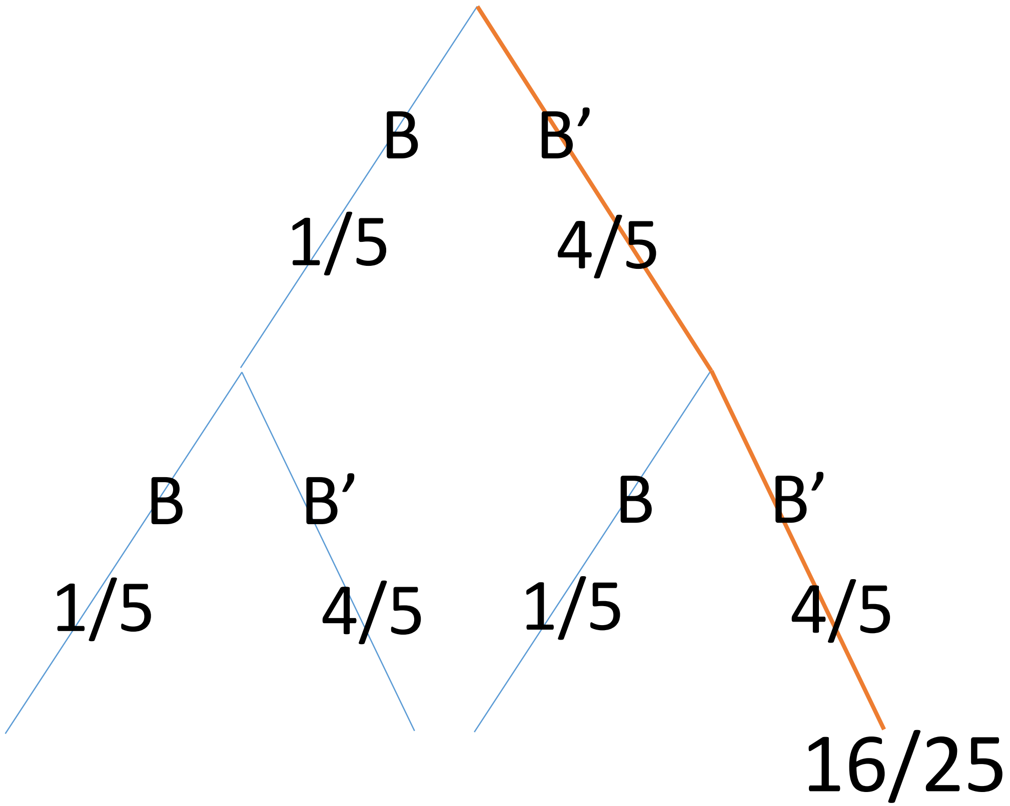
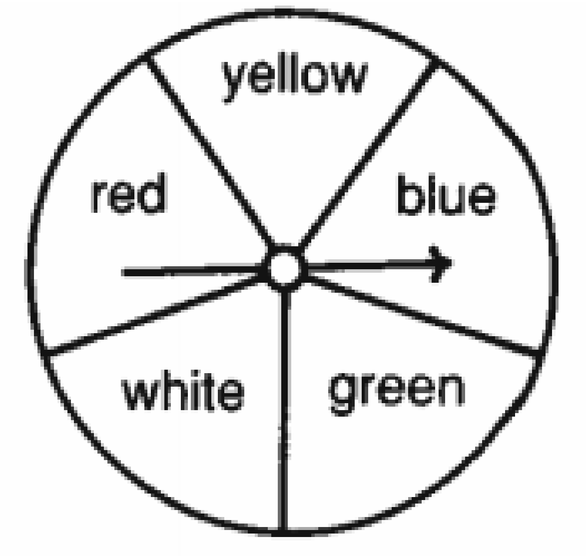
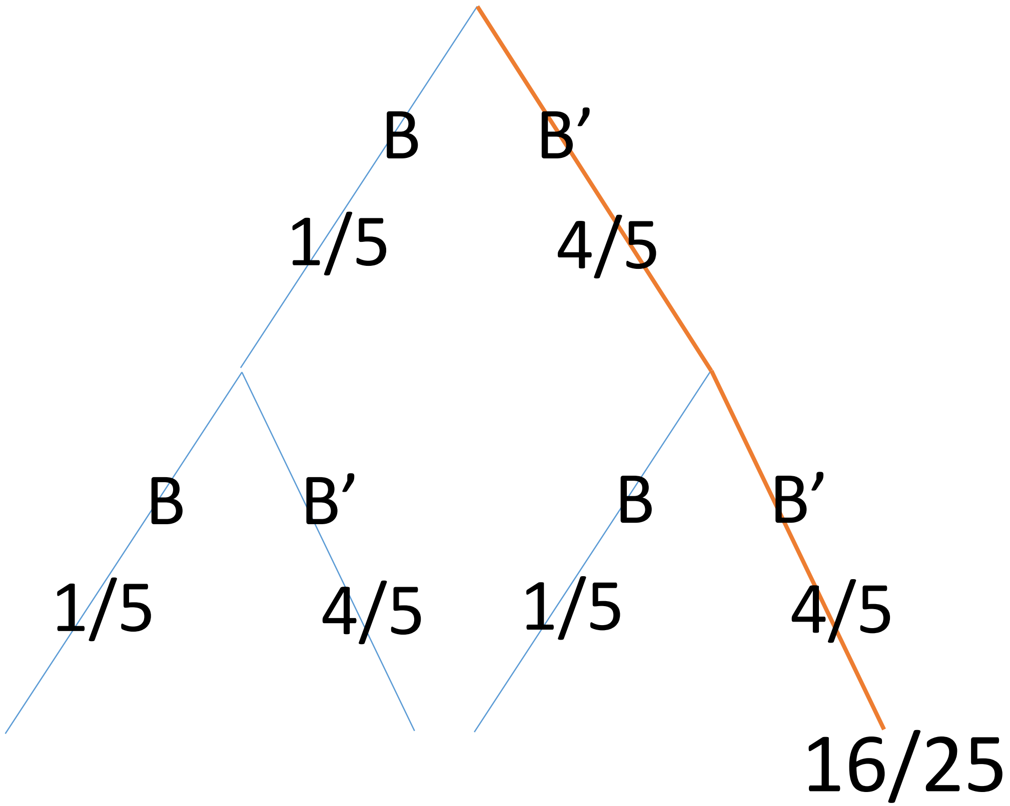


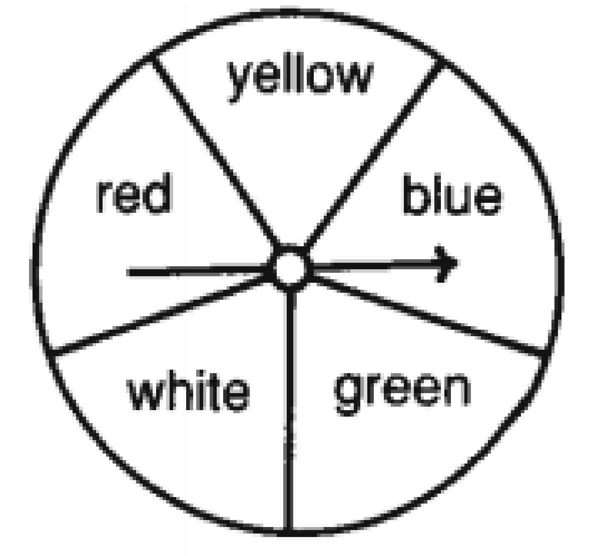


If the arrow is spun twice, what is the probability that it will stop on the blue at least once?

P(at least 1) = 1 – (P none)

= 1 - 16/25





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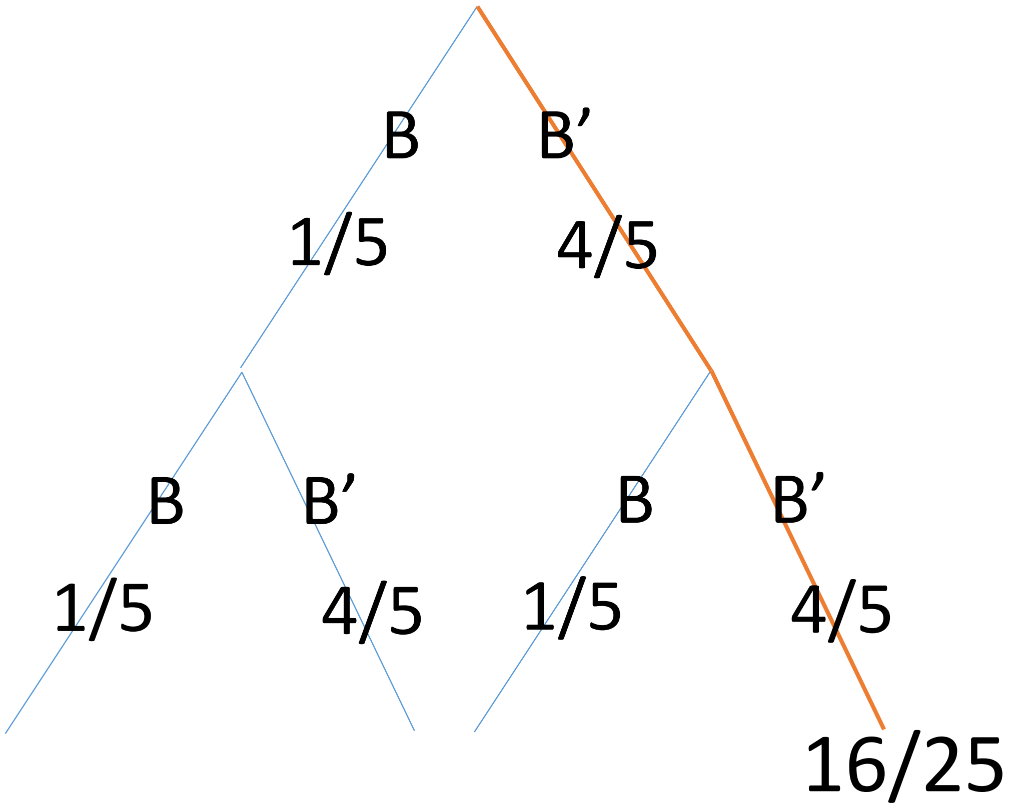
$$= \frac{9}{25}$$



Some notation:

P(event)	The probability of an event.
n(event)	The number of times an event occurs.
P(event’)	The event doesn’t occur. The opposite of the event.
P(event1 and event2)	Both events occur.
P(event1 or event2)	One event OR the other OR both events occur.

Some formulas:

$$P(event) = \frac{n(event)}{n(total)}$$

$$P(event) = 1 - P(event')$$

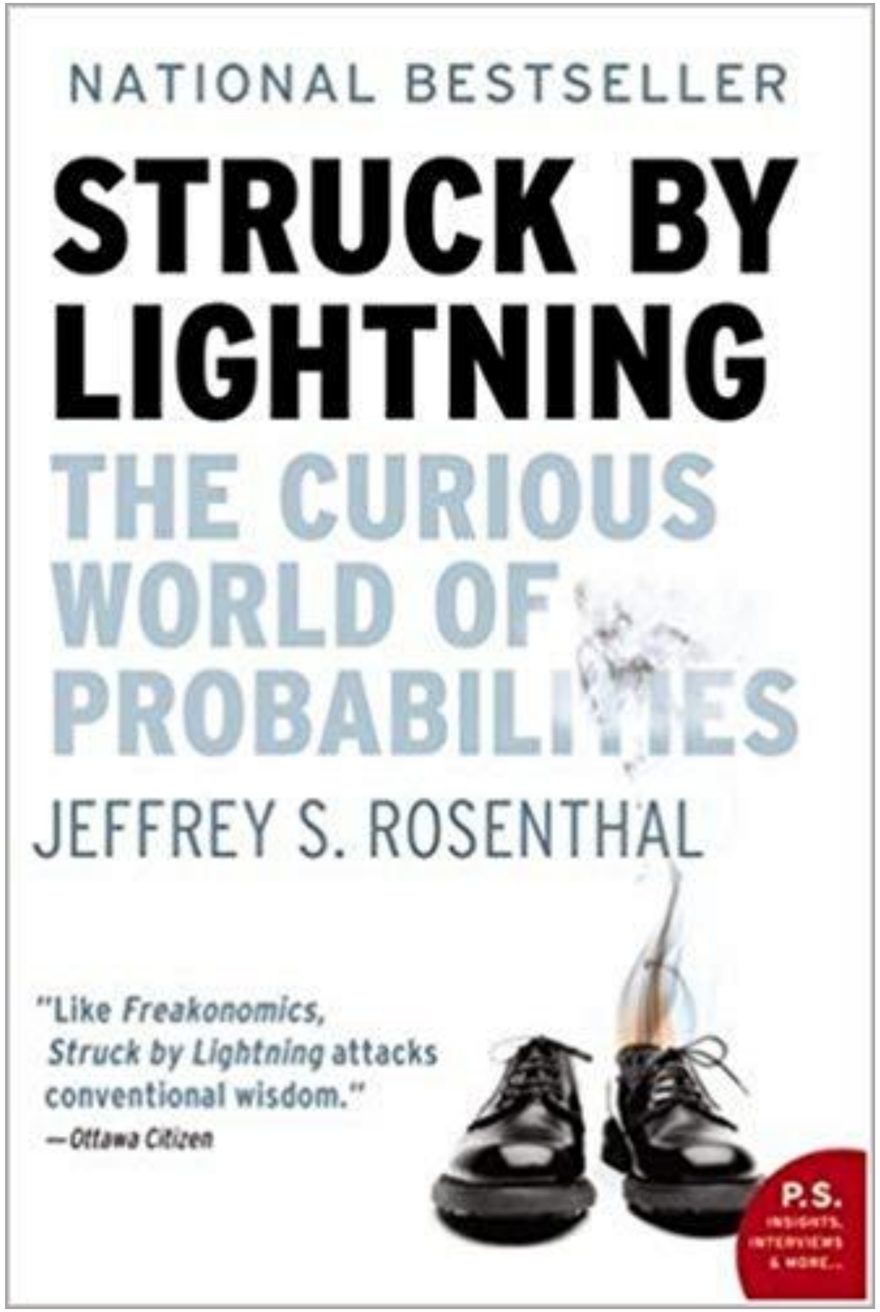
$$P(event1\ and\ event2) = P(event1)\ x\ P(event2)$$

If the events are independent

A warning:

If it seems like it is taking a long time, you might have picked a really inefficient way to do the problem.

Is the 1-P(A’) method easier?



Met my cousin at Disney world.
How amazing.

$$P(Meeting\ Phil) = \frac{n(Phil)}{n(People\ in\ USA)}$$

$$= \frac{1}{230\ million}$$

$$P(Meeting\ Phil) = \frac{n(Phil)}{n(People\ in\ USA)}$$

$$= \frac{1}{230\ million}$$

$$P(Surprize) = \frac{n(People\ we\ saw\ closely)}{n(People\ in\ USA)}$$

$$= \frac{2000}{230\ million}$$

$$= \frac{1}{115{,}000}$$

$$P(Surprize) = \frac{n(We\ know\ well)}{115{,}000}$$

$$= \frac{500}{115{,}000}$$

$$= \frac{1}{230}$$

$$P(Surprize) = \frac{n(We\ know\ well)}{115{,}000}$$

$$= \frac{500}{115{,}000}$$

$$= \frac{1}{230}$$

But, over a lifetime of visiting places, you will go to 230 places and it will probably happen.

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

Determine the members of the following sets.

a. The set of vowels in the English alphabet

b. The set of month names beginning with J

c. The set of national capital cities in North America

d. The set of multiples of 11 which are less than 100

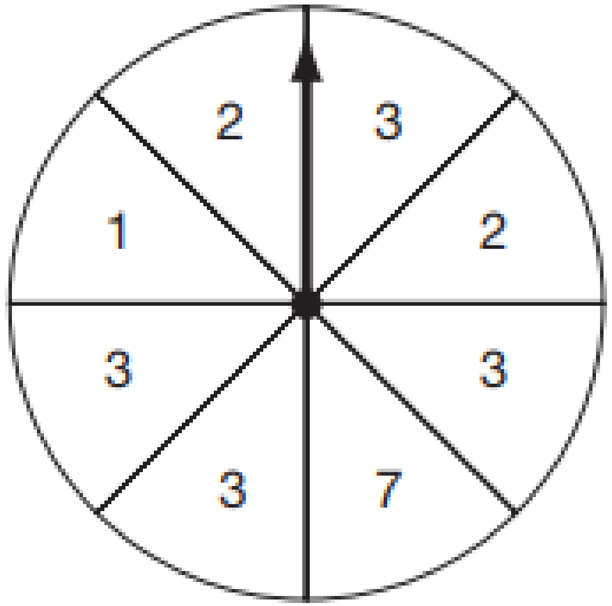
What is the smallest probability we can have?

What that value mean?

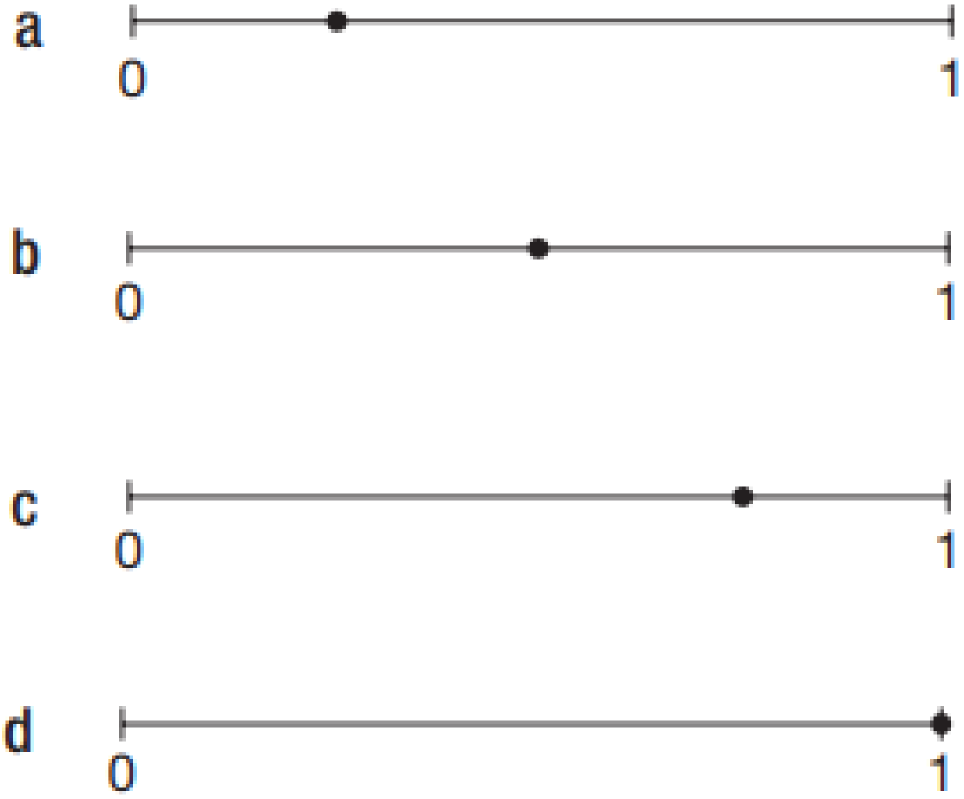
What is the largest probability we can have?

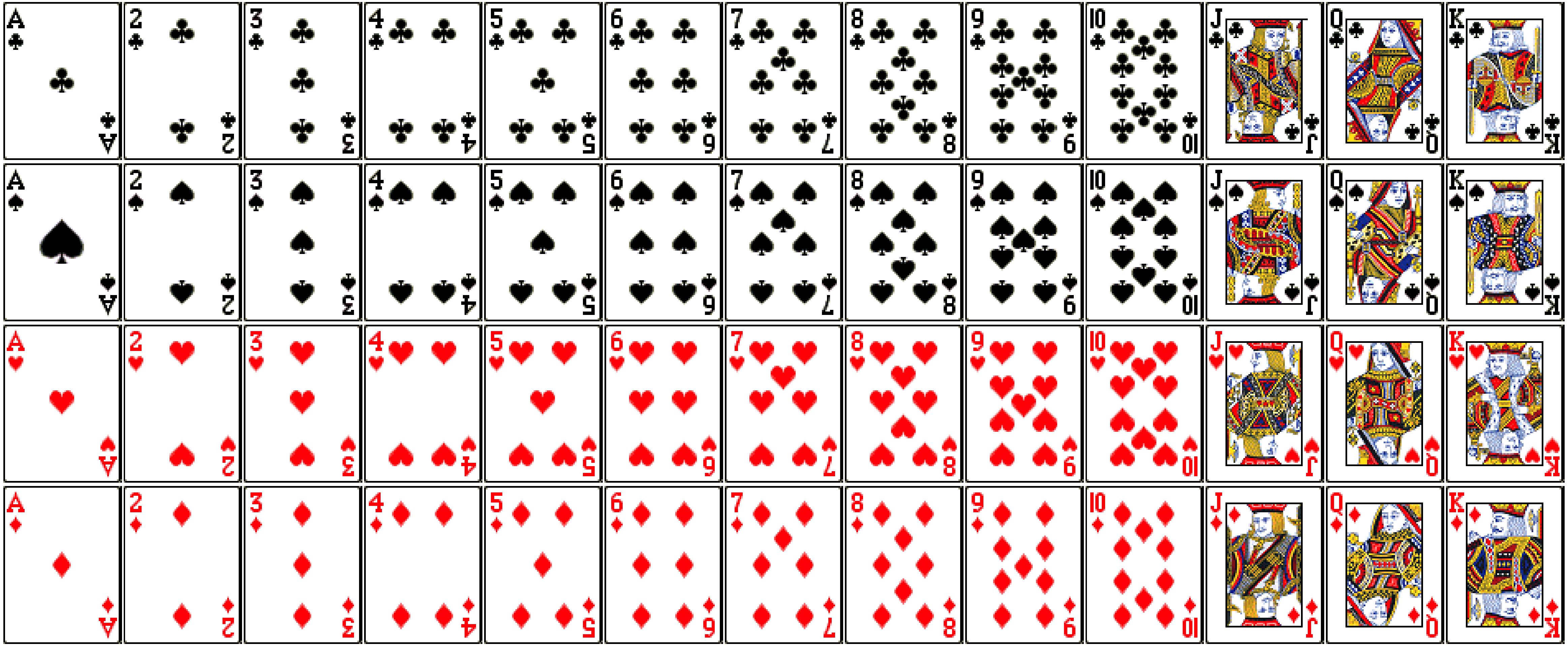
What that value mean?

15 A spinner is shown below.



On which of the following number lines does the point represent the probability of spinning an even number?





What is the probability of drawing a face card?

A fair die is thrown. List the sample space and calculate the probability of observing a multiple of three.

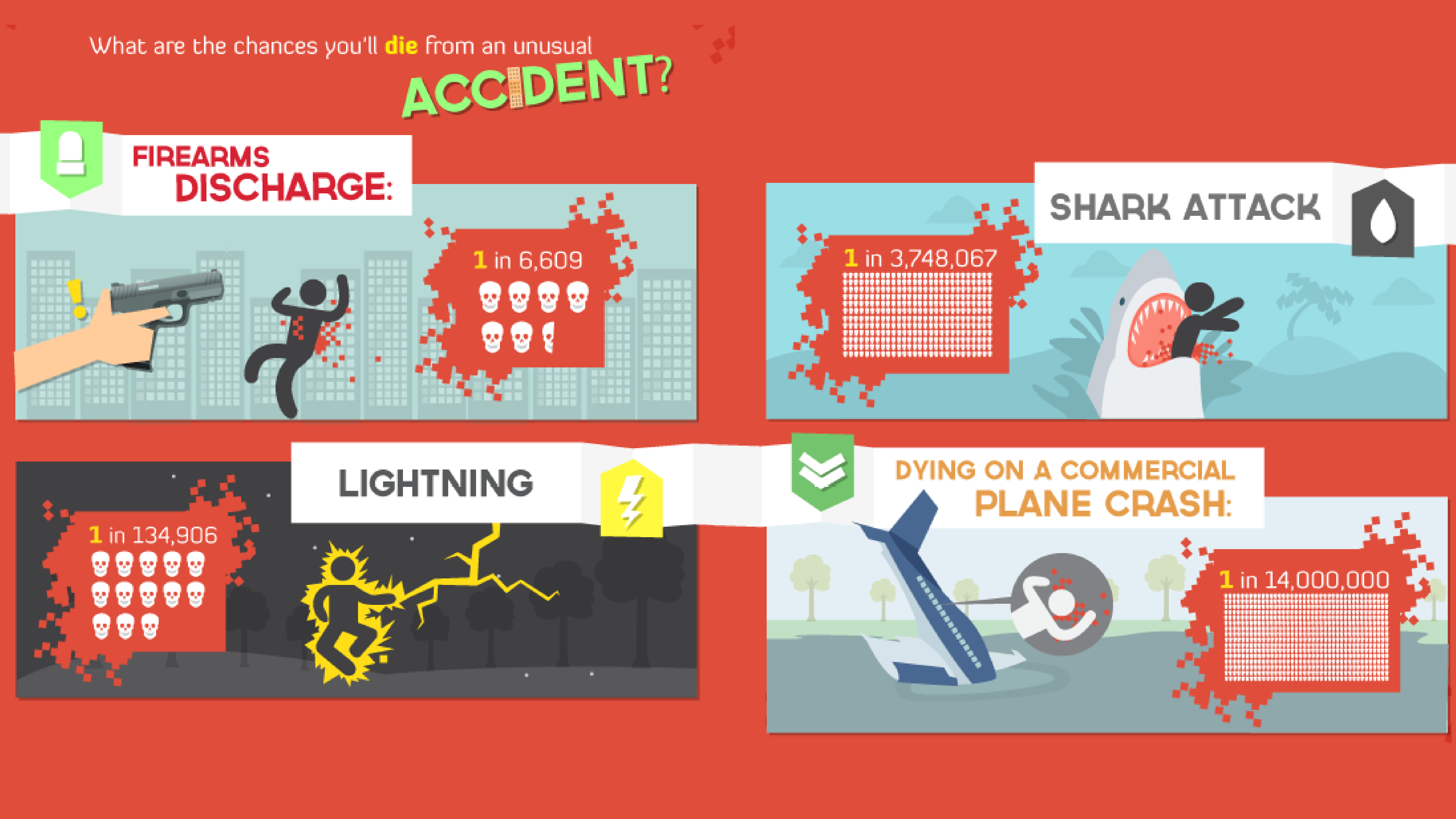
Phishing

WHY DO WE FALL FOR THESE SCAMS?

- Urgency
- Desire to please
- Greed
- Curiosity
- Complacency
- Fear

PROBABILITY THAT A PHISHING MESSAGE SUCCEEDS

1 out of 10!


What are the chances you'll die from an unusual
ACCIDENT?
FIREARMS DISCHARGE:
1 in 6,609
SHARK ATTACK
1 in 3,748,067
LIGHTNING
1 in 134,906
DYING ON A COMMERCIAL PLANE CRASH:
1 in 14,000,000


LIGHTNING
1 in 134,906


ASTEROID IMPACT
1 in 700,000
These odds are based on the number of asteroids that can hit the Earth and how much damage they can do (depending on size).


SHARK ATTACK
1 in 3,748,067