

MDM4U – Sample Test 1 – Probability – Sept 20, 2023

Name: Solutions

Knowledge	Application	Communication	Thinking	Total	Percent
20	25	13	20	78	%

Knowledge

1. Fill in the last column with the word, formula or number indicated. /8

- (a) Write down the formula for Conditional Probability.
- (b) Write down the formula for the Additive Principle.
- (c) What is the probability of rolling two 6s on a pair of dice?
- (d) What is the sample space for flipping a coin?
- (e) What is the Excel symbol for exponents?
- (f) What is the probability of perfectly guessing all answers on the SATs?
- (g) What is an application of probability trees on computers?
- (h) Which event is more likely: A: you are a librarian OR B: you are a librarian who loves books?

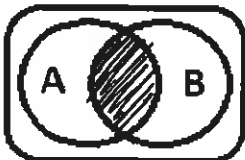
$P(B A) = \frac{P(A \cap B)}{P(A)}$
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$\frac{1}{36}$
$S = \{H, T\}$
^
o
AI decision tree
A: you are a librarian \cap

2. If P(A) is 0.4 and P(B) is 0.5, fill in the probabilities in the following table. /8

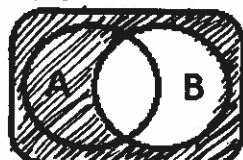
	A and B are independent <i>Not Required, just FYI</i>		A and B are mutually exclusive	
P(A')	0.6	(1-0.4)	0.6	(1-0.4)
P(A ∩ B)	0.2	(0.4 × 0.5)	0	
P(A ∪ B)	0.7	(0.4 + 0.5 - 0.2)	0.9	(0.4 + 0.5)
P(A B)	0.4	($\frac{0.2}{0.5}$)	0	($\frac{0}{0.5}$)

3. On the Venn diagram, shade in the space indicated. (Use the dots!!!) /4

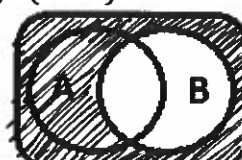
(a) $P(A \cap B)$



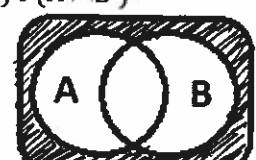
(b) $P(B')$



(c) $P(A \cup B')$



(d) $P(A' \cap B')$





Application

4. This spreadsheet investigates the relationship between homework and test scores.

Based on the data in column B and C, fill in true or false in the cells D2:I5.

/6

	A	B	C	D	E	F	G	H	I
1	Name	Homework	Test 1	A: Homework > 5	B: Test 1 > 80	A and B	A or B	A'	A' and B
2	Bill Ding	7	78	T	F	F	T	F	F
3	Ida Knowe	10	97	T	T	T	T	F	F
4	Stan Dupp	0	56	F	F	F	F	T	F
5	Carrie Oki	4	81	F	T	F	T	T	T

5. Which formulas would appear in the above cells?

/5

D2	= B2 > 5	If the homework was done more than 5 times, display true, otherwise display false.
F2	= AND (D2, E2)	Displays true if A and B are true, false otherwise.
G2	= OR (D2, E2)	Displays true if either A or B is true, false otherwise.
H2	= NOT (D2)	Finds the compliment of A.
I2	= and(<u>H2</u> , <u>E2</u>) A' B	Displays true if A is false and B is true. Otherwise displays false.

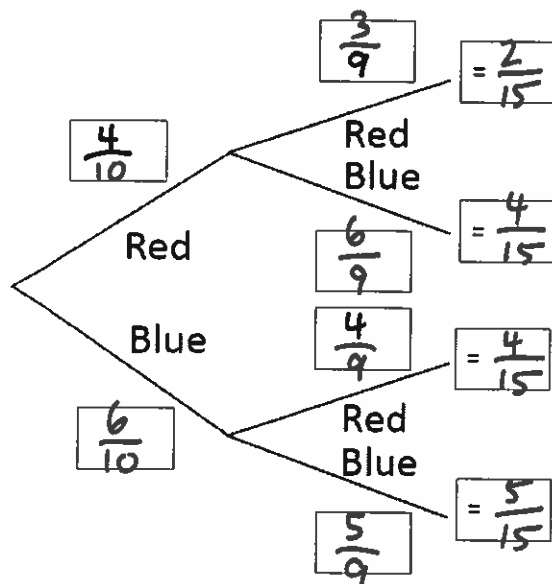
6. Using the above spreadsheet, write a single formula for row 2 to calculate the following:

/4

(a)	B' or A	= OR (NOT (E2), D2)
(b)	Displays true if the homework is over 3 or under 7; false otherwise.	= OR (B2 > 3 , B2 < 7)

7. A hat contains 4 red tickets and 6 blue ones. Two tickets are drawn at random *without replacement*.

/10



(a) Fill in the tree diagram.

(b) What is the probability that both tickets are red?

$$P(RR) = \frac{2}{15}$$

(c) What is the probability that at least one ticket is blue?

$$\begin{aligned}
 P(\text{at least 1B}) &= 1 - P(RR) \\
 &= 1 - \frac{2}{15} \\
 &= \frac{13}{15}
 \end{aligned}$$

Communication

8. Classify each pair of events as mutually exclusive or non-mutually exclusive.

/3

	Event A	Event B	Classification
(a)	Jelly bean is red	Jelly bean is yellow	Mutually Exclusive
(b)	Jelly bean is sour	Jelly bean is green	Not
(c)	It is boiling hot.	It is snowing.	Mutually Exclusive

9. Classify each pair of events as dependent or independent.

/3

	Event A	Event B	Classification
(a)	Stayed up late.	Failed test.	Dependent
(b)	It is cloudy.	It is raining.	Dependent
(c)	You like cheese.	The dice rolled a 4.	Independent

10. What is the $P(A' \cup A)$? Explain why that is using a real-world example.

/3

A' and A are opposites, one occurs or the other one does. Because they are joined with an OR, that means $P(A' \cup A)$ is 1. For example, if $P(A) = \text{win lottery}$, $P(A') = \text{don't win}$ the Prob you win OR you don't win is 100%. one of those 2 things must happen because they are opposites

11. A student was asked "If you are a golfer, what is the probability you will be hit by lightning?" Explain why their answer, found below, is incorrect.

/4

$$\begin{aligned}
 &P(\text{golfer} \cap \text{lightning}) \\
 &= P(\text{golfer}) \times P(\text{lightning}) \\
 &= \frac{1,500,000}{37,000,000} \times \frac{250}{37,000,000} \\
 &= 0.0406 \times 0.000068 \\
 &= 0.000000274
 \end{aligned}$$

② This is the independent AND formula. However, if you are a golfer, you are outside more often and you are more likely to be hit by lightning. These are dependent events so you can use that formula.

① this is a given that situation, so we need conditional probability. $P(\text{Light} | \text{Golf}) = \frac{P(L \cap G)}{P(G)}$ should be used.

③ should be 0.0405. Rounding error.

④ Actual Answer is 2.74×10^{-07} . They have written the wrong number of zeros. They've written 2.74×10^{-08} .

FYI: you wouldn't need all of these. Item + Explain = 2 marks.

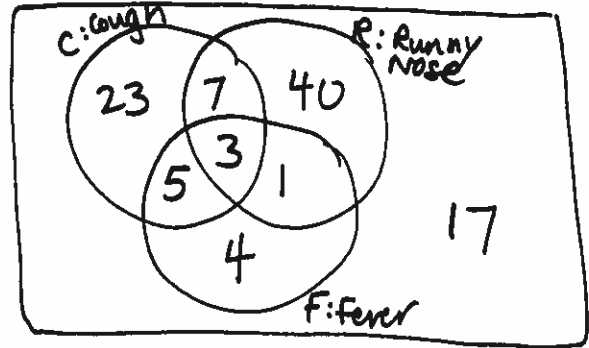
Thinking

12. In a recent medical survey of 100 randomly selected people in Calgary during the month of January, the following observations were made:

/10

- 38 people are coughing
- 51 people have runny noses.
- 13 people have fevers.
- The number of people coughing with runny noses is 10.
- The number of people with runny noses and fevers is 4.
- The number of people coughing with fevers is 8.
- The number of people coughing with fevers and runny noses is 3.

(a) Draw the Venn Diagram for this situation.



(b) What is probability that a Calgarian is healthy?

$$P(\text{Healthy}) = \frac{17}{100}$$

outside region

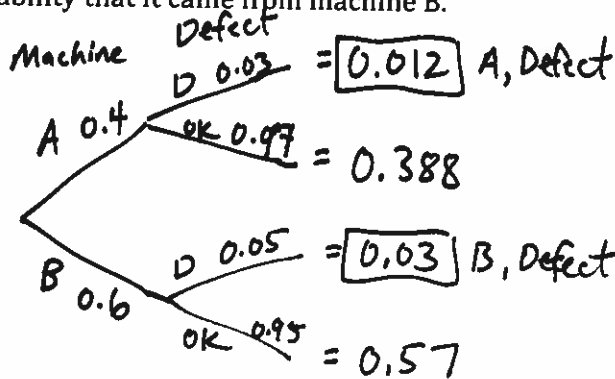
(c) What is the probability that a Calgarian only has a cough?

$$P(\text{only cough}) = \frac{23}{100}$$

(d) What is the probability that a Calgarian has a cough if you see their nose running?

$$P(\text{cough} | \text{Running}) = \frac{P(\text{cough} \cap \text{Running})}{P(\text{Running})} = \frac{10}{51}$$

13. Two machines, A and B produce 40% and 60% of the daily output respectively. Each machine also produces a total of 3% (A) and 5% (B) of items that are defective. An item is selected and is found to be defective. Find the probability that it came from machine B.



$$P(\text{Machine B} | \text{Defect}) = \frac{P(B \cap \text{Defect})}{P(\text{Defect})} = \frac{0.03}{0.012 + 0.03} = \frac{0.03}{0.045} = 0.7143 \text{ or } \frac{5}{7}$$

14. If Events A and B are mutually exclusive, what is $P(A|B)$?

/4

Prove it using algebra AND (Venn diagrams OR a tree diagram). You will have algebra AND a diagram.

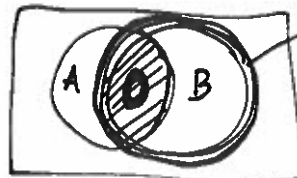
Algebra:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

mutually exclusive means $P(A \cap B) = 0$

$$= \frac{0}{P(B)} = 0$$

Venn:



The question asks: Given you are in the B Bubble, what is the probability you are in A (shaded)? Since there is no crossover there is no chance of being in A and B.