1.2 Exercises: Probability Trees

Use a probability tree to answer each question.

1. A fair die is rolled. Calculate the probability of each event.

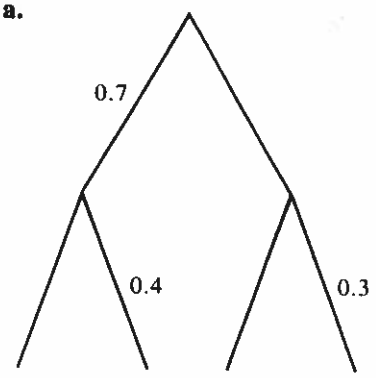
a. A: the top face is even

b. B: the top face is not 6

c. C: the outcome is divisible by 3

2. Copy and complete each probability tree by assigning the correct probabilities to the branches that are unmarked.

a.



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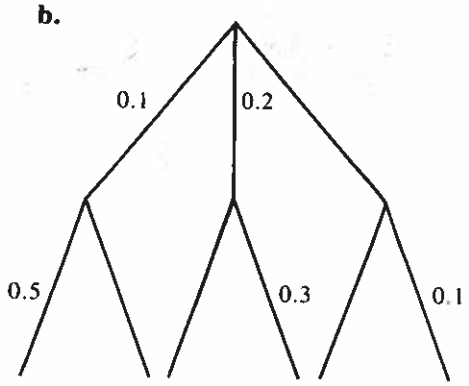


3. A fair die is rolled twice. Calculate the probability of each event.

a. J: the outcomes on the 2 rolls are identical

b. K: the outcome on the first roll is less than the outcome on the second roll

c. L: the outcome on the second roll is 3 times the outcome on the first roll

d. M: the total of the 2 outcomes is 7

e. N: the total of the outcomes is even

4. A fair coin is tossed 3 times. Calculate the probability of each event.

a. D: no heads occurs

b. E: tails occurs at least once

c. F: the numbers of heads and tails differ by 1

d. G: heads occurs at most 2 times

e. H: the second toss results in tails

5. A quality control inspector selects a sample of 2 fuses at random from a box of 100. If the box contains exactly 5 defective fuses calculate the probability that his sample will contain

a. no defective fuses.

b. exactly 1 defective fuse.

c. 2 defective fuses.

6. From a box containing 5 red, 5 blue, and 5 yellow balls, 3 balls are selected at random. Calculate the probability of each event.

   a. Q: there are no red balls in the sample

   b. R: the numbers of blue and yellow balls are equal

   c. V: the sample contains at least 1 yellow ball

   d. T: the numbers of red and blue balls differ by 1

   e. U: the second ball selected is either blue or red

7. Electrical components are packed in a carton with 3 layers of 12 components each. Suppose that the first layer has 1 defective component, the second layer has 2, and the third has none. A layer is chosen at random, and then 2 components are chosen from the selected layer and inspected. Find the probability that at least 1 defective component is found.

8. Abdul plans to purchase 2 lottery tickets, either in Lottery A or in Lottery B. In Lottery A 100 tickets will be sold and 2 prizes will be awarded. Lottery B consists of 2 identical series, B_1 and B_2. Fifty tickets will be sold and 1 prize awarded in each B series. All prize drawings are random and without replacement. Find the probability that Abdul will win at least 1 prize if

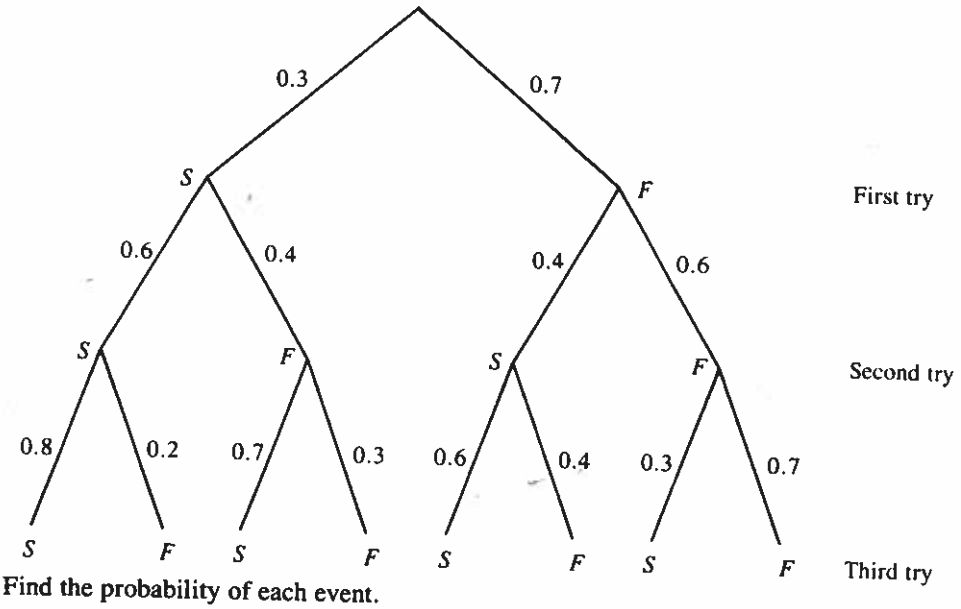
   a. he buys 2 tickets in Lottery A.

   b. he buys 1 ticket in each of B_1 and B_2.

   c. he buys 1 ticket in A and 1 ticket in either B_1 or B_2.

   d. he buys 2 tickets in either of B_1 or B_2.

9. The probability tree shown describes an experiment in which a subject tries a complicated task 3 times. If the task is failed at any stage, new instructions are given; otherwise the subject attempts the task again without help. On the tree, S stands for successful completion of the task and F for failure.



Find the probability of each event.

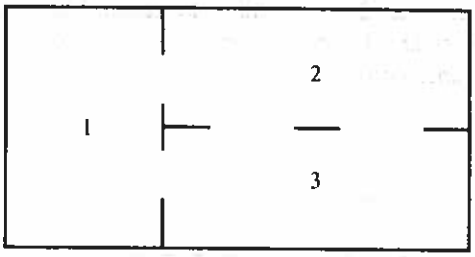
   a. A: the subject fails on the second try

   b. B: the subject succeeds at least twice

   c. C: the subject succeeds on the second but not on the third try

   d. D: the subject fails on the first try but succeeds on the third try

10. At the beginning of an experiment, a rat is placed in room 1 in the maze shown.



If the rat is in any room, he selects the next room by randomly selecting a door. Draw a probability tree that describes the rat's first 3 moves. Calculate the probability of each event.

a. R: the rat is in room 3 after 3 moves

b. A: the rat visits room 2 at least once

c. T: the rat does not visit any room twice

Problems 6.3

1. An experiment has at least 2 distinct outcomes, A and B, where $P(A) = p$, $P(B) = q$, and $p + q \leq 1$. If the experiment is repeated until either A occurs or B occurs, determine the probability that A occurs before B. Construct a probability tree in which the second stage attached to the outcome "neither A nor B occurs" records only if A occurs before B or B occurs before A.

2. In a certain dice game a player rolls 2 dice and wins the game immediately if the total of the showing faces is 7 or 11. He loses immediately if the total is 2, 3, or 12; otherwise he continues to roll the dice. On any subsequent roll he either wins the game by throwing his initial total a second time or loses by rolling 7; otherwise he rolls again. Calculate the probability of the player winning the game.

3. Hamish and Jenny play a game with 4 fair dice labelled as follows:

A	B	C	D
4, 4, 4, 4, 0, 0	3, 3, 3, 3, 3, 3	6, 6, 2, 2, 2, 2	5, 5, 5, 1, 1, 1

Each player chooses a die and rolls it; the winner is the player who rolls the higher number. If Hamish chooses his die first, show that Jenny can always choose a die that gives her 2:1 odds of winning the game.

4. Consider a probability tree in which each of the new trees added at each stage has only 2 branches. This is called a binary tree.

a. If n is the number of stages in the tree, how many distinct paths does the tree contain?

b. Let M_n represent the least number of multiplications required to calculate the probabilities of all the distinct paths in an n-stage binary probability tree. Note that a multiplication means multiplying 2 numbers together. Show that $M_1 = 0$, $M_2 = 4$ and, in general, $M_{n+1} = M_n + 2^{n+1}$ for $n = 1, 2 \ldots$.

c. Show that $M_{n+1} = 4(2^n - 1)$ for $n = 1, 2, \ldots$.

5. A family name tree becomes extinct as soon as a generation occurs with no male descendants. Suppose that each male in a family tree is likely to give rise to 0, 1, or 2 male descendants with probabilities p, q, and r respectively, where $p + q + r = 1$. Suppose that e is the probability that a family name which begins with a single ancestor eventually becomes extinct.

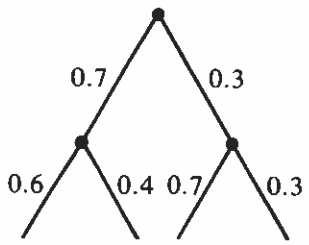
a. Show that e is a solution of the quadratic equation $rx^2 + (q - 1)x + p = 0$.

b. When does the equation in part (a) have a root which is less than 1?

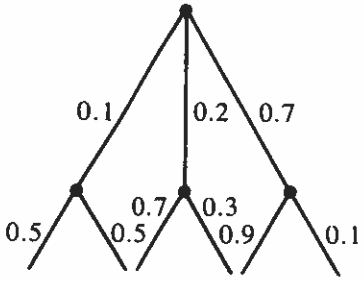
Exercises 6.3

1. a. $\frac{1}{2}$ **b.** $\frac{5}{6}$ **c.** $\frac{1}{3}$

2. a.



b.



3. a. $\frac{1}{6}$ **b.** $\frac{5}{12}$ **c.** $\frac{1}{18}$ **d.** $\frac{1}{6}$ **e.** $\frac{1}{2}$

4. a. $\frac{1}{8}$ **b.** $\frac{7}{8}$ **c.** $\frac{3}{4}$ **d.** $\frac{7}{8}$ **e.** $\frac{1}{2}$

5. a. 0.9025 **b.** 0.095 **c.** 0.0025

6. a. $\frac{24}{91}$ **b.** $\frac{27}{91}$ **c.** $\frac{67}{91}$ **d.** $\frac{40}{91}$ **e.** $\frac{2}{3}$

7. $\frac{16}{99}$ **8. a.** $\frac{99}{2475}$ **b.** $\frac{99}{2500}$

c. $\frac{99}{2500}$ **d.** $\frac{1}{25}$ **9. a.** 0.54 **b.** 0.432

c. 0.148 **d.** 0.294

10. a. $\frac{7}{18}$ **b.** $\frac{11}{12}$ **c.** $\frac{7}{18}$