Worked Examples

Sample Space & Counting

Example 1. Five cards are labelled A, B, C, D, and E but are otherwise identical. The cards are shuffled and then one card is chosen. Construct a probability model to describe the selection of the card and calculate the probability of each event.

F: a vowel is selected
G: D or E is selected
H: A is not selected

Solution. The sample space is

$$S = \{A, B, C, D, E\}$$

which is the set of all possible outcomes. Since the cards are identical and shuffled, it is reasonable to assign the probability $\frac{1}{5}$ to each outcome in S. These probabilities add to 1. The event F is

$$F = \{A, E\}$$

and

$$P(F) = \tfrac{1}{5} + \tfrac{1}{5} = \tfrac{2}{5}$$

The event G is

$$G = \{D, E\}$$

and

$$P(G) = \tfrac{1}{5} + \tfrac{1}{5} = \tfrac{2}{5}$$

The event H is

$$H = \{B, C, D, E\}$$

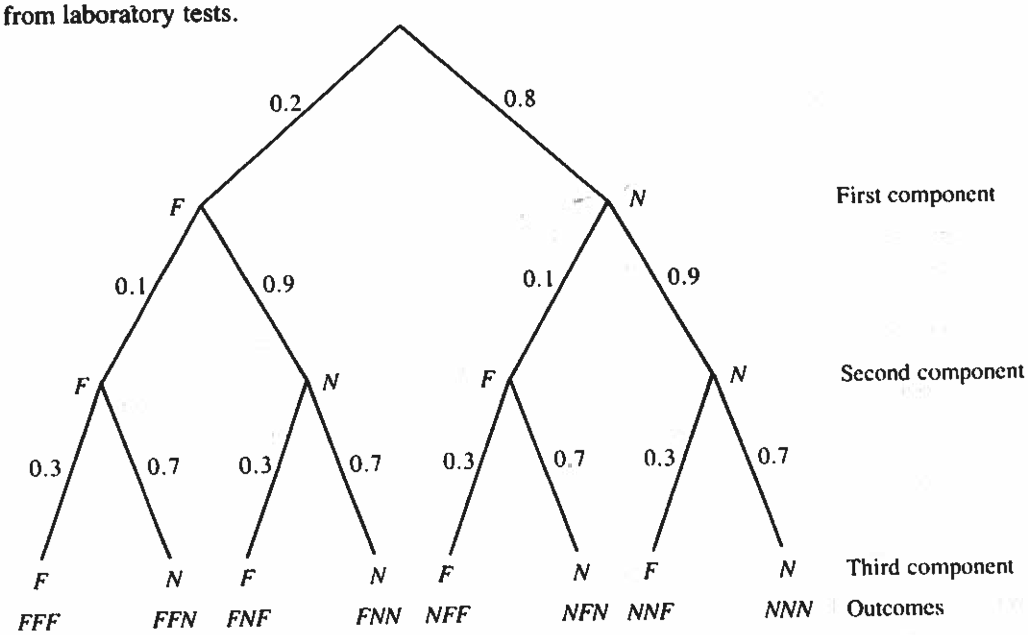
and

$$\begin{aligned} P(H) &= \tfrac{1}{5} + \tfrac{1}{5} + \tfrac{1}{5} + \tfrac{1}{5} \\ &= \tfrac{4}{5} \end{aligned}$$

□

Probability Tree

Example 2. The probability tree shown describes 3 components of an electronic circuit. Failure of a component is denoted by F and non-failure by N. The probabilities were derived from laboratory tests.



Find the probability of each event.

A: all 3 components fail
B: at least 1 component fails
C: component 2 fails

Solution. The event A corresponds to the path FFF and so

$$P(A) = (0.2)(0.1)(0.3) = 0.006$$

There are 7 paths corresponding to the event B, so we use the complement. The event $notB$ has the single path NNN so

$$P(B) = 1 - P(notB) = 1 - (0.8)(0.9)(0.7) = 0.496$$

There are 4 paths, FFF, FFN, NFF, NFN corresponding to C, so

$$P(C) = (0.2)(0.1)(0.3) + (0.2)(0.1)(0.7) + (0.8)(0.1)(0.3) + (0.8)(0.1)(0.7) = 0.1$$

□