

Binary Search

Searching in Sorted Arrays



Linear Search

- Searching is looking for the location of the item (index).
- **Algorithm:** Start at element 0. Continue until (a) you find it > return index (b) you reach the end > return -1.
- **Speed:** $O(n)$.
- **Trade-off:** Slower search than binary, however it works on **unsorted** data.

Linear Search #1

<https://youtu.be/xDBxVzkUk7E>

Start at the beginning.

Look at each element.

Stop when:

- You find it.
- You get to the end. Return -1. It isn't there.

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]
1	91	3	5	8	7	21	4	33	9	0	10	2

Linear Search #2

<https://youtu.be/Wo-mLpXL9aE>

Start at the beginning.

Look at each element.

Stop when:

- You find it.
- You get to the end. Return -1. It isn't there.

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
Z	X	B	N	M	P	O	J	K	F	R

Binary Search

- Can be coded recursively.
- **Algorithm:** Track the lowest and highest spot where the item might be. Search halfway, adjust.
- **Speed:** $O(\log n)$.
- **Trade-off:** Much faster search than linear search, however only works on sorted data.
- Uses the order of the data to speed up the search.

14

Looking for 14

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
-2	-1	4	6	8	9	10	12	14	16	17	18	20	21	24

↑
L
o
w

↑
M
i
d

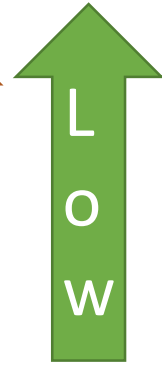
↑
H
i
g
h

In index
7?

Looking for 14

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
-2	-1	4	6	8	9	10	12	14	16	17	18	20	21	24

So, we can adjust our low boundary.



In index
7?

Too
Low.

Looking for 14

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
-2	-1	4	6	8	9	10	12	14	16	17	18	20	21	24

↑
L
o
w

↑
M
i
d

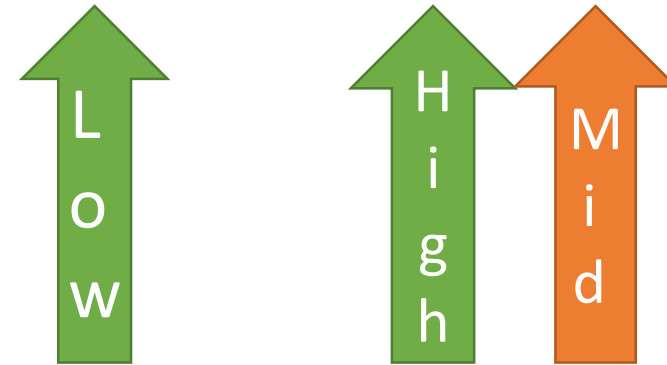
↑
H
i
g
h

In index
11?

Looking for 14

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
-2	-1	4	6	8	9	10	12	14	16	17	18	20	21	24

So, we can adjust our high boundary.

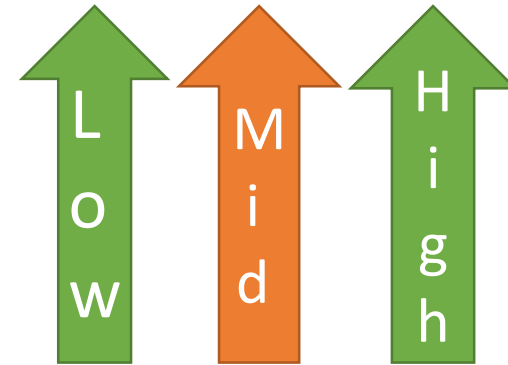


In index 11?

Too High.

Looking for 14

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
-2	-1	4	6	8	9	10	12	14	16	17	18	20	21	24



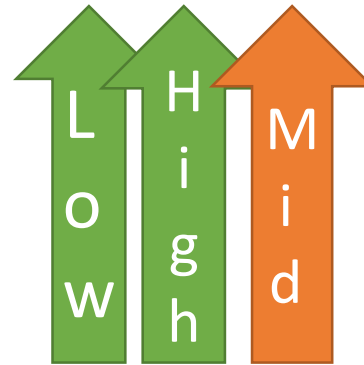
In index
9?

Too High.

Looking for 14

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
-2	-1	4	6	8	9	10	12	14	16	17	18	20	21	24

So, we can adjust our high boundary.

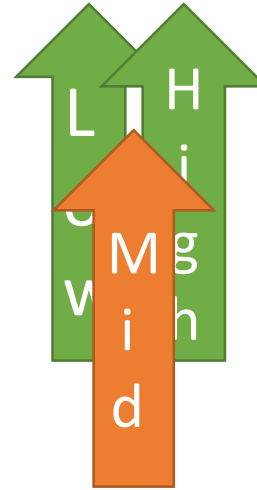


In index 9?

Too High.

Looking for 14

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
-2	-1	4	6	8	9	10	12	14	16	17	18	20	21	24



In index
8?

Yes.



Looking for 13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
-2	-1	4	6	8	9	10	12	14	16	17	18	20	21	24

↑
L
o
w

↑
M
i
d

↑
H
i
g
h

In index
7?

Looking for 13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
-2	-1	4	6	8	9	10	12	14	16	17	18	20	21	24

So, we can adjust our low boundary.

Mid

Low

High

In index
7?

Too
Low.

Looking for 13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
-2	-1	4	6	8	9	10	12	14	16	17	18	20	21	24

↑
L
o
w

↑
M
i
d

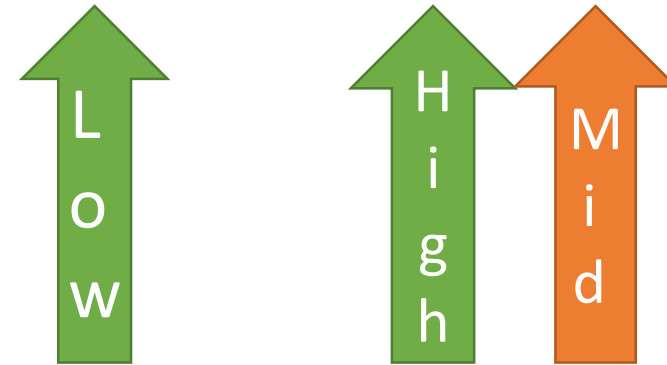
↑
H
i
g
h

In index
11?

Looking for 13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
-2	-1	4	6	8	9	10	12	14	16	17	18	20	21	24

So, we can adjust our high boundary.

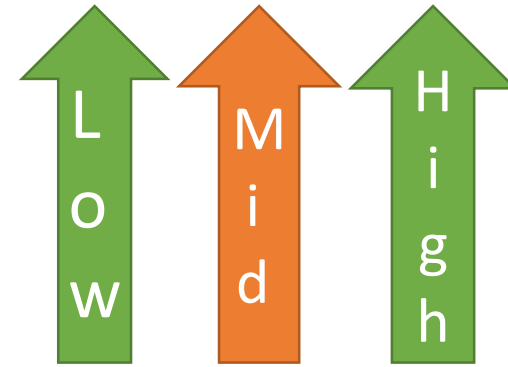


In index 11?

Too High.

Looking for 13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
-2	-1	4	6	8	9	10	12	14	16	17	18	20	21	24



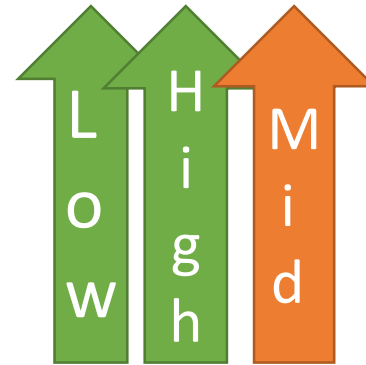
In index
9?

Too High.

Looking for 13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
-2	-1	4	6	8	9	10	12	14	16	17	18	20	21	24

So, we can adjust our high boundary.



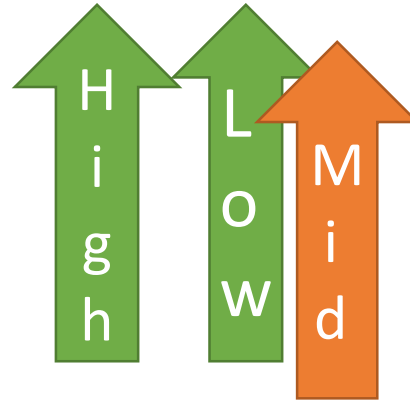
In index 9?

Too High.

Looking for 13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
-2	-1	4	6	8	9	10	12	14	16	17	18	20	21	24

So, we can adjust our high boundary.



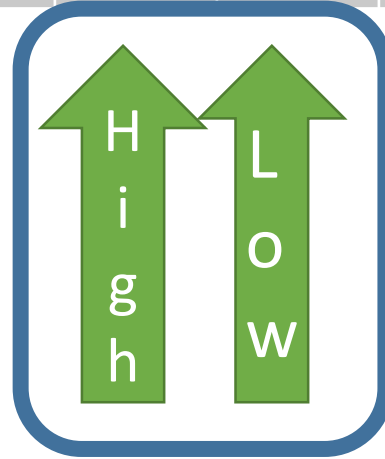
In index
8?

No.

Looking for 13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
-2	-1	4	6	8	9	10	12	14	16	17	18	20	21	24

Out of order. It's not in either half. It's not there.



Is

No.

Binary Search #1

Start Low at 0. Start High at a.length.
Find mid. Look in that position.

- If too low, adjust low to (mid + 1)
- If too high, adjust high to (mid - 1)

Stop when:

- Mid is the right position.
- Low > High. Return -1. It isn't there.

<https://youtu.be/NjGSKXnaFz8>

Low	High	Mid

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
-45	-32	-12	-6	0	1	3	5	7	10	14	21

Binary Search #2

Start Low at 0. Start High at a.length.

Find mid. Look in that position.

- If too low, adjust low to (mid + 1)
- If too high, adjust high to (mid - 1)

Stop when:

- Mid is the right position.
- Low > High. Return -1. It isn't there.

<https://youtu.be/79hICibvntQ>

Low	High	Mid

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]
Apple	Bee	Carrot	Egg	Jam	Mitt	Pet	Zebra

Binary Search #3

Start Low at 0. Start High at a.length.
Find mid. Look in that position.

- If too low, adjust low to (mid + 1)
- If too high, adjust high to (mid - 1)

Stop when:

- Mid is the right position.
- Low > High. Return -1. It isn't there.

<https://youtu.be/Wo-mLpXL9aE>

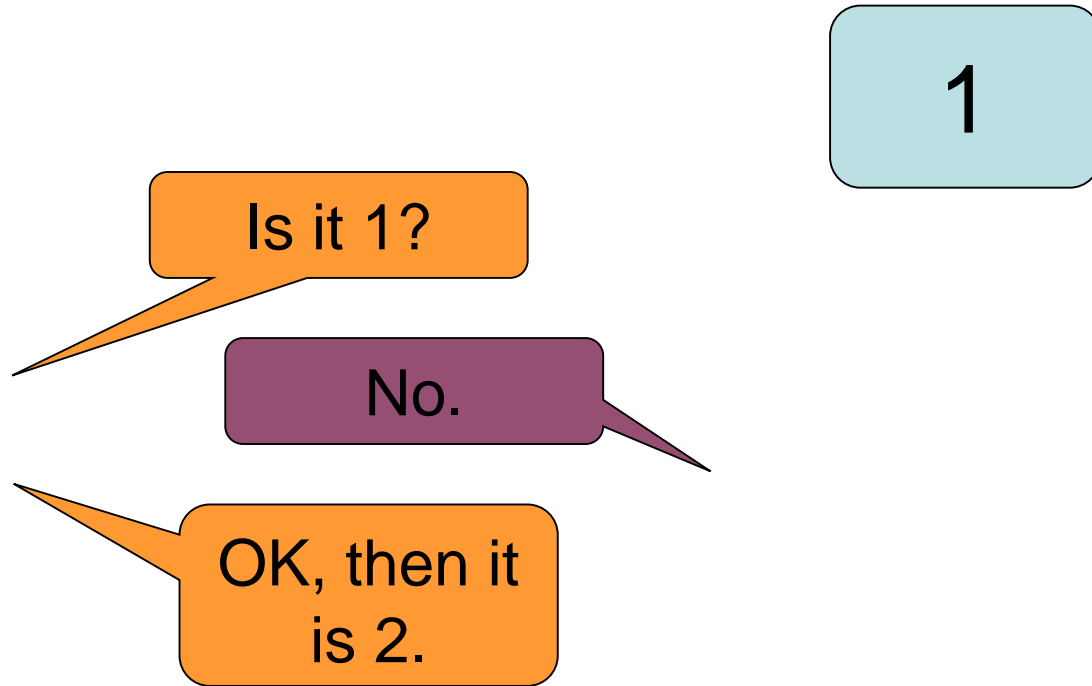
Low	High	Mid

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]
A	B	D	E	G	I	K	M	N	Q	S	U	Z

Why is Binary Search $O(\log n)$?

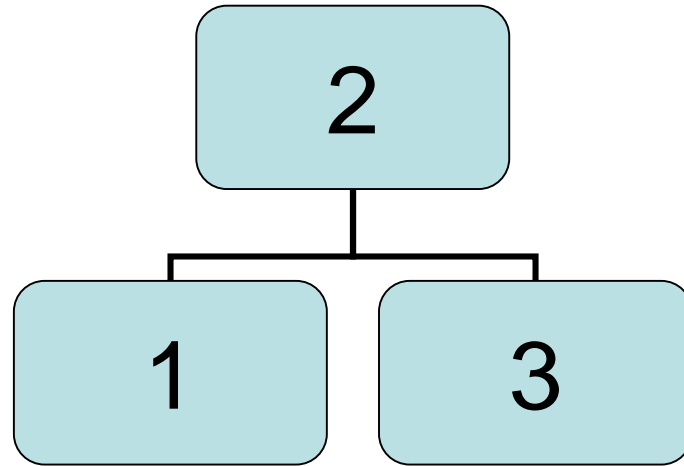
And why do you keep saying that is really fast?





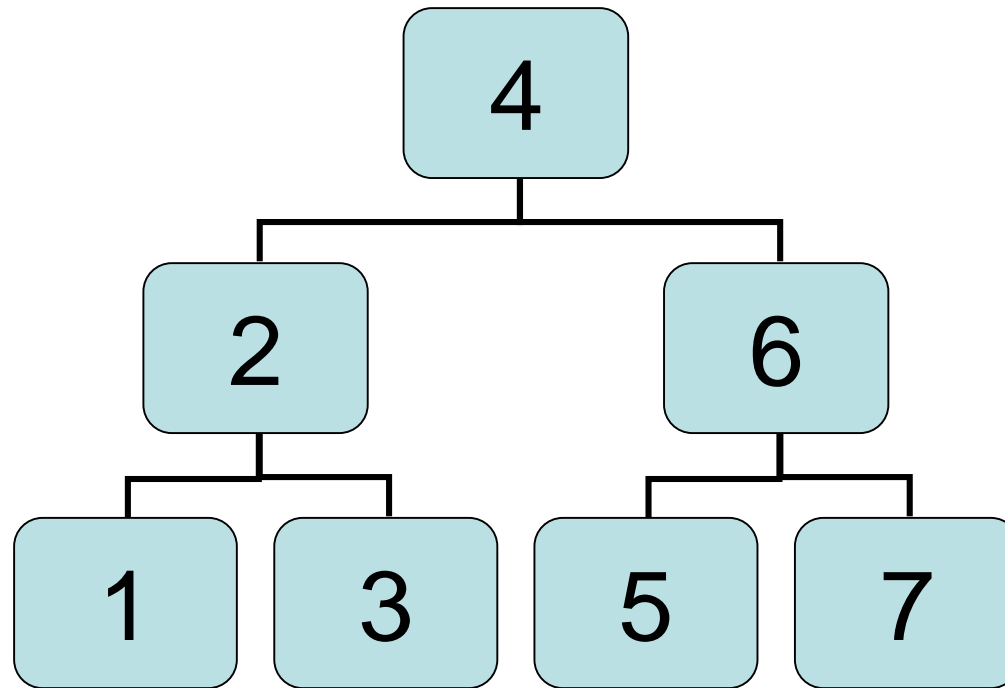
Numbers 1 to 2 = 1 guesses.

$$2^1 = 2$$



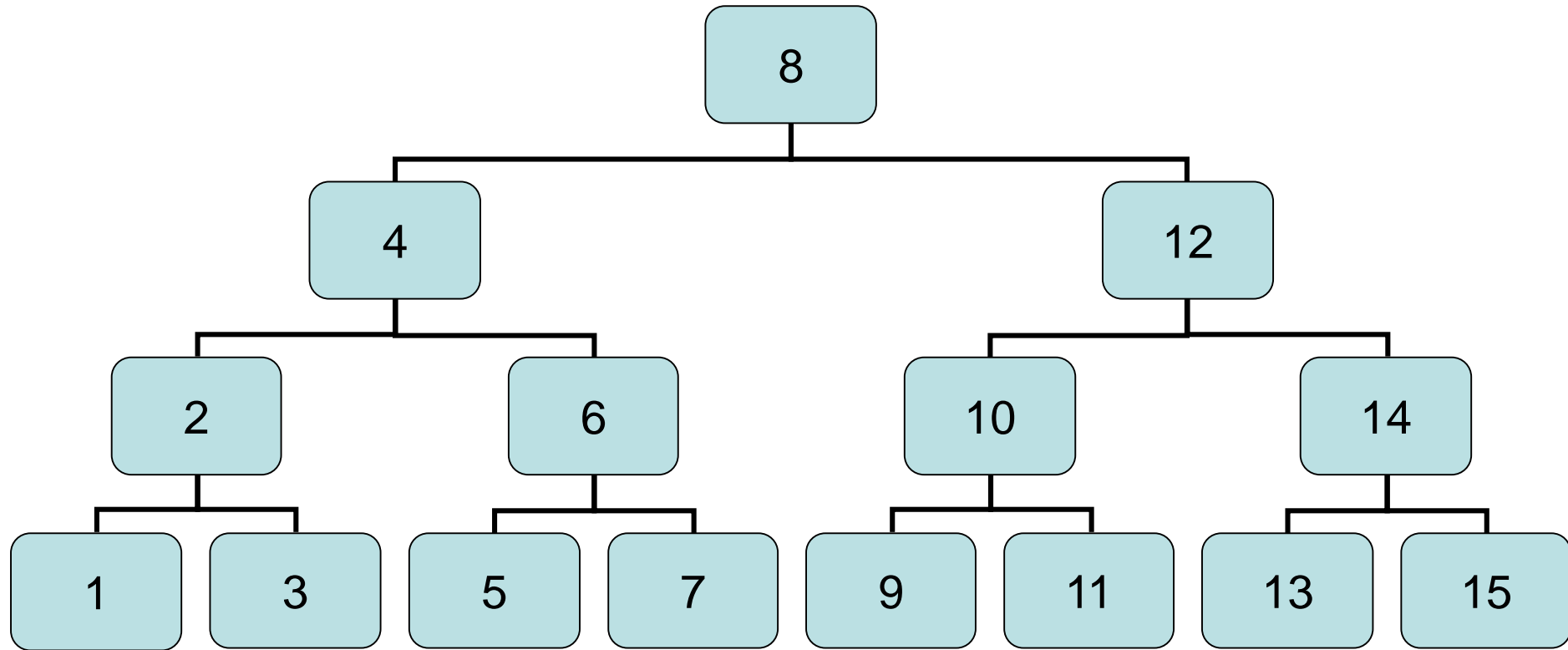
Numbers 1 to 3 = 2 guesses.

$$2^2 = 4$$



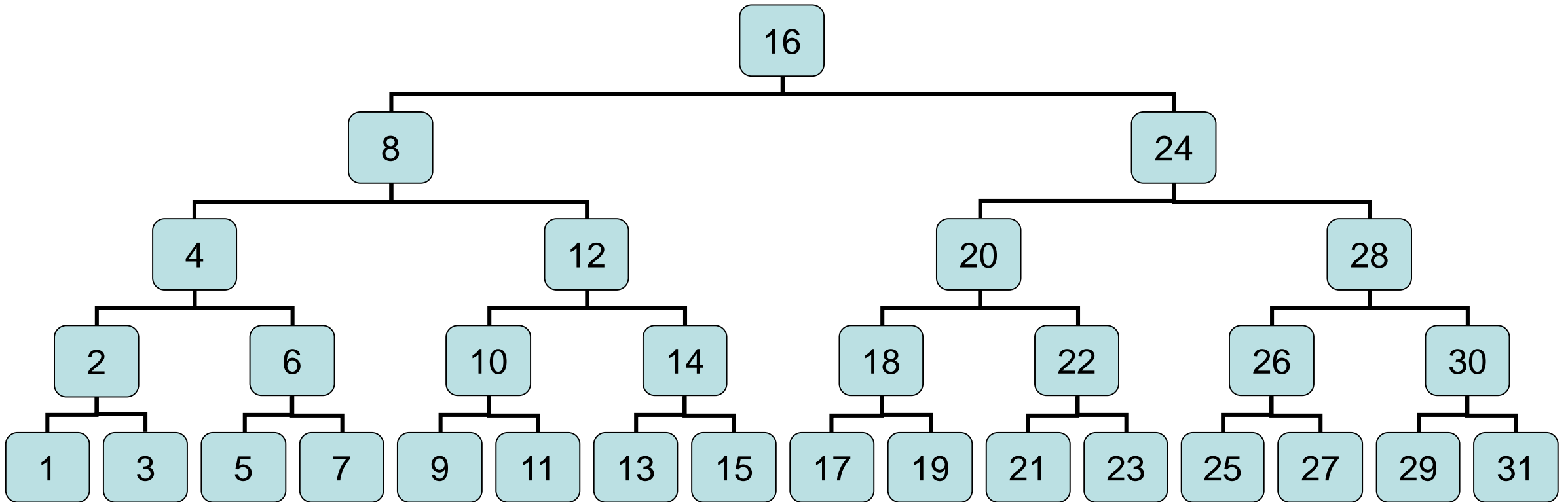
Numbers 1 to 7 = 3 guesses.

$$2^3 = 8$$



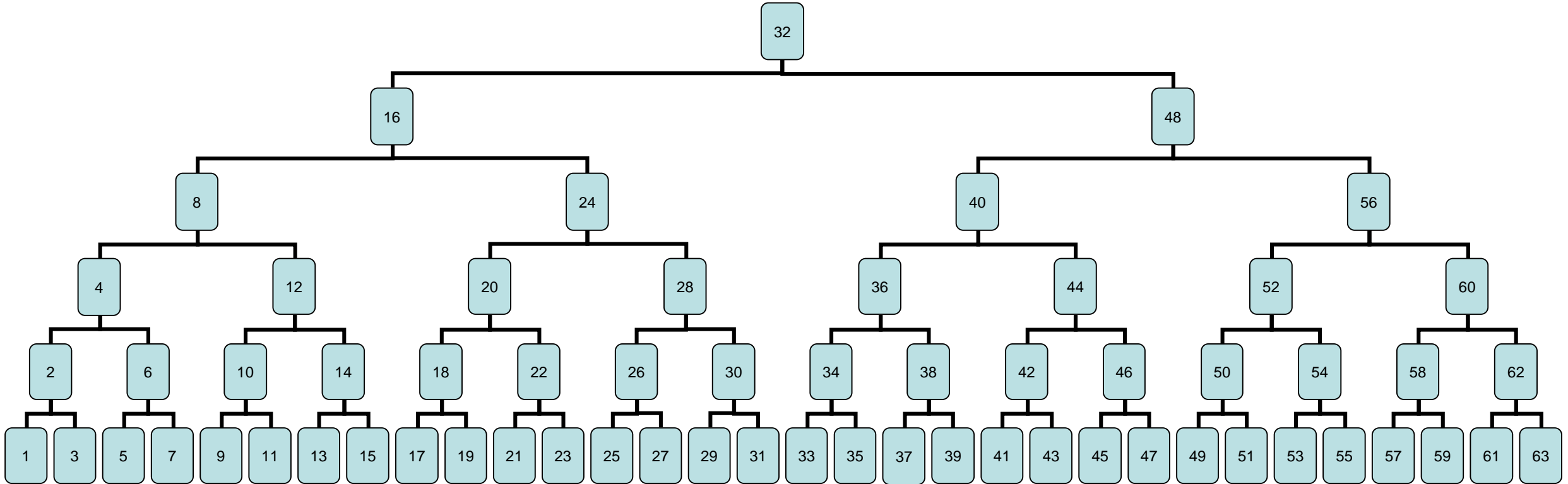
Numbers 1 to 15 = 4 guesses.

$$2^4 = 16$$



Numbers 1 to 31 = 5 guesses.

$$2^5 = 32$$



Numbers 1 to 63 = 6 guesses.

$$2^6 = 64$$

Binary Search is Logarithmic

Highest Number (n)	Binary Expression	Number of Searches	
2	$2^1 = 2$	1	$\log_2(2)=1$
3	$2^2 = 4$	2	$\log_2(4)=2$
7	$2^3 = 8$	3	$\log_2(8)=3$
15	$2^4 = 16$	4	$\log_2(16)=4$
31	$2^5 = 32$	5	$\log_2(32)=5$
63	$2^6 = 64$	6	$\log_2(64)=6$
127	$2^7 = 128$	7	$\log_2(128)=7$

Binary Search is a **lot** faster

	A	B	C
1	# of Records	Avg Linear	Max Binary
2	1	1	1
3	10	5	4
4	100	50	7
5	1000	500	10
6	10000	5000	14
7	100000	50000	17
8	1000000	500000	20
9	10000000	5000000	24
10	100000000	50000000	27
11	1000000000	500000000	30
12	10000000000	5000000000	34

BINÄRY SEARCH

idea-instructions.com/binary-search/
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IDEA

